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Bayesian extraction of the parton distribution amplitude from the Bethe–Salpeter wave function

Fei Gao [b,c], Lei Chang [a,*], Yu-xin Liu [b,c,d]

[a] School of Physics, Nankai University, Tianjin 300071, China
[b] Department of Physics and State Key Laboratory of Nuclear Physics and Technology, Peking University, Beijing 100871, China
[c] Collaborative Innovation Center of Quantum Matter, Beijing 100871, China
[d] Center for High Energy Physics, Peking University, Beijing 100871, China

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ABSTRACT

We propose a new numerical method to compute the parton distribution amplitude (PDA) from the Euclidean Bethe–Salpeter wave function. The essential step is to extract the weight function in the Nakanishi representation of the Bethe–Salpeter wave function in Euclidean space, which is an ill-posed inversion problem, via the maximum entropy method (MEM). The Nakanishi weight function as well as the corresponding light-front parton distribution amplitude (PDA) can be well determined. We confirm prior work on PDA computations, which was based on different methods.

1. Introduction

The parton distribution amplitudes (PDAs) of mesons, encoding the meson structure on the light front, play an essential role in the hard exclusive processes. The cross sections of the processes can be written as the convolution of the hard-scattering kernel which can be computed perturbatively, and the PDAs of the hadrons involved [1,2]. The leading twist parton distribution amplitudes of mesons are defined by integrating out the transverse momentum $k_\perp$ from the light front wave function, which are obtained through projecting meson's Bethe–Salpeter wave function onto the light-front.

Although some efforts have been made to calculate Bethe–Salpeter equation (BSE) directly in Minkowski space (see, e.g., Ref. [3]) with a simple scattering kernel, many BSE calculations are still carried out in Euclidean space which are much easier to handle. The challenge in the Euclidean scheme is how to project the discrete Euclidean wave function data on the light-front to get light-front quantities. The Nakanishi representation of the wave function provides a natural way to solve this problem. This challenging question amounts to whether it is possible to compute the weight function of the Nakanishi representation if one has an appropriate solution of the BSE in Euclidean space. Nakanishi representation was proposed in Ref. [4] to parameterize the relativistic two-particle bound state in Minkowski space.

Despite lacking a non-perturbative proof of uniqueness of the weight function in the Euclidean case, we suppose that the wave-function can still be parametrized by the following similar form,

$$\Phi(k,P)=\int_{-1}^{1}dz\int_{0}^{\infty}d\gamma\,\frac{g(\gamma,1-2x)}{(k^2+zk\cdot P+\frac{1}{4}P^2+M^2+\gamma)^3}\,, \tag{1}$$

where $k^2>0$ is the space-like momentum and $P^2=-M_{\rm bs}^2$ with $M_{\rm bs}$ the bound state mass and M is an infrared regulated scale. The weight function $g(\gamma,1-2x)$ is a two-dimensional function in real space. The corresponding leading twist two-particle light-front parton distribution can be defined as

$$\varphi(x)=\int d^4k\delta(n\cdot k-xn\cdot P)\Phi(k-\frac{P}{2},P)\,, \tag{2}$$

where n is the light-like vector $n^2=0$. We neglect the possible spin structure for simplification at the moment. With the help of

* Corresponding author.
E-mail address: leichang@nankai.edu.cn (L. Chang).

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m-order moments defined as $\int_0^1 dx x^m \varphi(x)$, one can get the one-dimensional integral representation of the light-front PDA in Euclidean space as

$$\varphi(x) = \mathcal{N} \int_0^\infty d\gamma \frac{g(\gamma, 1-2x)}{\gamma + M^2 - x(1-x)M_{\rm bs}^2}, \tag{3}$$

where $\mathcal{N}$ is the corresponding normalization constant to ensure $\int_0^1 dx \varphi(x) = 1$. Once complete knowledge of the weight function of the Nakanishi representation is at hand, one can plot the light-front PDA. The main aim of this paper is then to provide a practical algorithm to construct the weight function from the numerical data of $\Phi(k, P)$. We would like to emphasize that there is currently no proof available regarding the uniqueness of the weight function.

The techniques for projecting a realistic pion's Bethe–Salpeter (BS) amplitude in Euclidean space onto the light-front have been pioneered recently [5], therein a special Nakanishi representation of each scalar component of the BS amplitude was parameterized to reproduce the corresponding Euclidean functions. In turn the Mellin moments of the PDA can be computed directly, which were then used to reconstruct the PDA. Besides, this Euclidean Nakanishi representation has also been extended to evaluate the pion elastic form factor [6]. This technique has proven successfully for the description of the quantities that have been integrated out of $k_\perp$ (PDA for example), practically at least.

On the other hand, for the case of heavy meson system, owing to a damping influence from the large quark mass, Ref. [7] proposes a 'brute-force' approach to calculate the Mellin moments viz. direct integration using interpolations of the numerical solutions for the quark propagator and Bethe–Salpeter amplitude. A damping factor $1/(1 + k^2 r^2)^m$ has been introduced for each Mellin moment to reduce the oscillation problem in the integration. Such procedure works well for the low order of moments, but uncertainty increases progressively for the higher order ones. The limit number of Mellin moments always brings large uncertainty in reconstructing PDA. For example, the PDA near $x = 1$ behaviour depends on the higher order moments' behaviour. However, Ref. [7] ignored this issue and supposed that the heavy meson PDA exponentially damped at the end-point.

In this letter we provide another numerical method to compute the PDA that is independent of the approach given in Refs. [5] and [7]. The procedure is straightforward: we first extract the weight function of Nakanishi representation (Eq. (1)) via the maximum entropy method (MEM) and then produce the PDA with Eq. (3).

2. Maximum entropy method

As pointed by the authors of Ref. [8], extracting $g(\gamma, 1-2x)$ from the $\Phi(k, P)$ is a typical ill-posed inversion problem. They regulate the linear system with a diagonal term ϵI and show the instability of the solution with respect to different values of ϵ. They appeal that more efficient methods are required to obtain a stable and unique solution in the Euclidean space. In this Letter we will claim that the maximum entropy method (MEM) is an appropriate algorithm to solve this problem. In the following we will firstly illustrate this method by an analytical model of weight function and then produce PDAs of π-meson and η_c-meson.

The MEM [9–11] is an approach that can be used to solve an ill-posed inversion problem, in which the number of data points is much smaller than the number of degrees of freedom available to the function whose reconstruction is sought. Its basis is Bayes' theorem in probability theory [12], which states the probability of an event "A", given that a condition "B" is satisfied:

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)}, \tag{4}$$

where, within the sample space, $P(B|A)$ is the probability that events of type "A" satisfy the condition "B" (usually denoted as likelihood function); $P(A)$ is the total probability that event "A" can occur (commonly referred to prior probability); $P(B)$ is the total probability that condition "B" is satisfied (playing the role of normalisation constant).

In making use of the MEM to reconstruct the weight function, one works with the conditional probability that the weight function $g(\gamma, 1-2x)$ corresponds to a correlation function $\Phi(k_E)$:

$$P[g|\Phi M] = \frac{P[\Phi|gM]P[g|M]}{P[\Phi|M]}, \tag{5}$$

where M represents the set of all definitions and prior knowledge of the weight function.

According to the central limit theorem, the likelihood functional is usually taken as:

$$P[\Phi|gM] = \frac{1}{Z_L} e^{-L[g]}, \tag{6}$$

$$L[g] = \sum_i^{N_{data}} \frac{(\Phi_{data}(k_{i,E}) - \Phi_g(k_{i,E}))^2}{2\sigma_i^2}, \tag{7}$$

where Z_L is a normalisation factor; $\{\Phi_{data}(k_{i,E}), i = 1, \ldots, N_{data}\}$ will be provided numerically, here via DSE and BSE, and $\{\Phi_g(k_{i,E}), i = 1, \ldots, N_{data}\}$ are obtained from Eq. (1) using any given model for $g(\gamma, 1-2x)$. One typically chooses $\sigma_i = s\,\Phi_{data}(k_{i,E})$, with $s \lesssim 0.1$.

The central feature of the MEM is the prior probability, which can be expressed here in terms of the entropy as

$$P[g|M(\alpha, m)] = \frac{1}{Z_S} e^{\alpha S[g,m]}, \tag{8}$$

where Z_S is a normalisation factor, α is a positive-definite scaling factor, and the exponent involves the Shannon–Jaynes entropy [13–15]

$$S[g, m] = \int_{-1}^{1} dz \int_0^\Lambda d\gamma \Big[g(\gamma, 1-2x) - m(\gamma, 1-2x) - g(\gamma, 1-2x) \log \frac{g(\gamma, 1-2x)}{m(\gamma, 1-2x)} \Big]. \tag{9}$$

The quantity $m(\gamma, 1-2x)$ is the "default model" of the weight function, which is usually chosen to be a uniform distribution so as to avoid assumptions about the structure of the weight function; viz.,

$$m(\gamma, 1-2x) = m_0 \theta(\Lambda - \gamma). \tag{10}$$

A MEM result for $g(\gamma, 1-2x)$ is considered reliable if it does not depend on the choices for the m_0 and the cutoff Λ. By adding the entropy functional, the solution of the weight function g then becomes the unique solution which makes the probability $P[g|\Phi M]$ maximal.

Given a value of α, the most probable weight function, $g_\alpha(\gamma, 1-2x)$, is obtained by maximising $P[g|\Phi M(\alpha, m)]$. This may be achieved via the singular-value decomposition algorithm in Ref. [9]; and dependence on the scale factor α can also be eliminated by following Ref. [9] and defining the MEM result for the spectral density as

$$\bar{g}(\gamma, 1-2x) = \int_0^\infty d\alpha \int \mathcal{D}g(\gamma, 1-2x) \times g(\gamma, 1-2x)\, P[g|\Phi M(\alpha, m)] P[\alpha|\Phi M(m)]\,, \tag{11}$$

where the second is a functional integral and $P[\alpha|\Phi M(\alpha, m)]$ is the conditional probability distribution for α. In readily workable cases, $P[g|\Phi M(\alpha, m)]$ is sharply peaked in the neighbourhood of a single function $g_\alpha(\gamma, 1-2x)$, in which case Eq. (11) yields

$$\bar{g}(\gamma, 1-2x) \approx \int_0^\infty d\alpha\, g_\alpha(\gamma, 1-2x)\, P[\alpha|\Phi M(m)]\,. \tag{12}$$

At this point, Bayes' theorem can again be employed to obtain

$$P[\alpha|\Phi M(m)] \propto \int \mathcal{D}g(\gamma, 1-2x)\, P[\alpha|M(m)] P[g|\Phi M(\alpha, m)]\,, \tag{13}$$

where we have used the fact that a sensible result is only achieved if it is independent of the default model, Eq. (10). N.B. The conditional probability $P[\alpha|M(m)]$ is independent of $g(\gamma, 1-2x)$; and if one considers α and M to be independent, then $P[\alpha|M(m)]$ is simply a constant.

As noted above, in practically workable instances, $P[g|\Phi M(\alpha, m)]$ is sharply peaked in the neighbourhood of a single function $g_\alpha(\gamma, 1-2x)$, so the functional integral in Eq. (13) is accurately estimated using Laplace's method, with the result [9]

$$P[\alpha|\Phi M(m)] \approx \frac{1}{Z_\Lambda} \exp\left[\frac{1}{2}\sum_k \ln\left[\frac{\alpha}{\alpha+\lambda_k}\right]\right] P[g_\alpha|\Phi M(\alpha, m)]\,, \tag{14}$$

where Z_Λ is a normalisation constant and $\{\lambda_k\}$ is the set of eigenvalues of the real, symmetric matrix

$$\Lambda_{ij}(g_\alpha) = g_i^{\frac{1}{2}} \frac{\partial^2 L[g]}{\partial g_i \partial g_j} g_j^{\frac{1}{2}} \Big|_{g=g_\alpha} \tag{15}$$

where the set $\{g_i\}$ represents a discretised version of the function $g(\gamma, 1-2x)$; i.e., the set of values of $g(\gamma, 1-2x)$ obtained by evaluating the function on a large but finite number of points.

The successful application of the MEM to extracting the quark spectral density at finite temperature and to explore the properties of strongly interacting matter (see, e.g. Refs. [16–19]) shows that it is a promising approach also to obtain the Nakanishi weight function. To check further the efficiency of the MEM in exploring the weight function in the Nakanishi representation of the BS wave function, we take the Nakanishi weight function model quoted from Ref. [8]

$$g(\gamma, 1-2x) = e^{-(\gamma+1)/(4x(1-x))}\,, \tag{16}$$

in which the dependence on the parameters gamma and/or x is explicitly known. We use this model to create the mock data of wave function $\Phi(k_i, \cos(\theta_j))$ with $\cos\theta = \frac{k\cdot P}{M_{\rm bs}\sqrt{k^2}}$ through the Nakanishi representation, that is, Eq. (1). Practically, we employed $N_{data} = 40$ for the wave function as input, and for simplicity, here we choose the 40 points in a fixed θ_j value and a series of k_i with $i = 1, \ldots, N_{data}$. Then we extract the weight function through MEM with the input of these mock data and the parameter of standard deviation $s \sim 1\%$. The comparison between the numerical result of the Nakanishi weight function we obtained and the demonstration of the above analytical expression is shown in Fig. 1. It is evident that the difference between the numerical result and the analytical one is modest and the point-to-point comparison displays quite good stability against the change of parameters. However, the deviation exists at the end points. It is because the weight of the end points is small compared to the intermediate region, so most of information is located in the intermediate region, and the extraction of the end points' information is more difficult. It weakens the capacity of investigating the property at end points, but the general features of weight function are preserved well in this procedure. We can conclude, by this experience, MEM is an appropriate approach to extract the weight function that can solve the problem pointed out in Ref. [8].

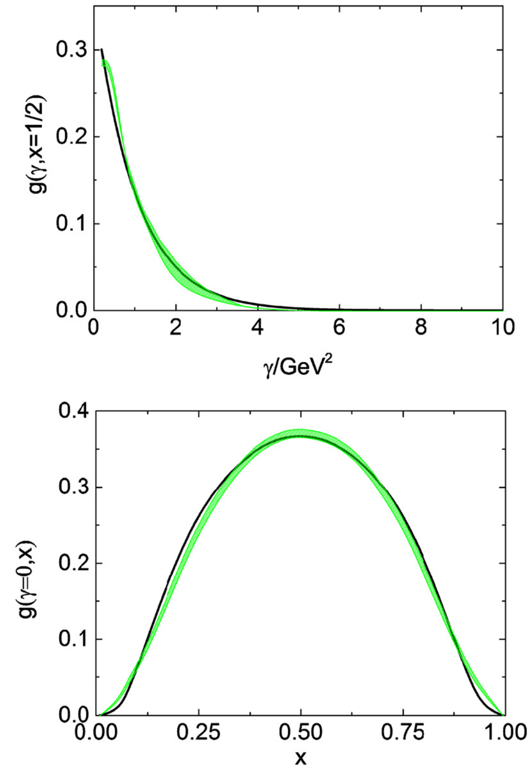

Fig. 1. Calculated Nakanishi weight function $g(\gamma, 1-2x)$ via the MEM and the BSE and DSE in cases of $x = 1/2$ (*upper panel*) and $\gamma = 0$ (*lower panel*) with error band and comparison with the demonstration of the analytical expression in Eq. (16).

3. PDAs of π- and η_c-mesons in realistic model

The success of extracting the weight function via MEM encourages us to compute PDA from a realistic BS wave function. In this section we take the numerical data of pseudoscalar mesons (π and η_c) wave functions that are calculated within BSE and the DSE [20–24] to achieve our aim. The BSE and DSE will be solved in the rainbow-ladder truncation which is the leading order approximation in a systematic, symmetry preserving procedure. The truncation is widely used in the framework of DSEs and will certainly meet our requirements here because it is known to be accurate for ground-state vector- and isospin-nonzero-pseudoscalar-mesons [22–24]. The more sophisticated truncations should certainly be employed in the future on this procedure, especially when people want to generalize this procedure into the scalar, axial vector mesons and also excited states.

The meson wave function will be studied by solving the homogenous BSE in ladder truncation

$$\Gamma(k;P) = -\frac{4}{3}Z_2^2\int_{dq}^{\Lambda}\Big[\mathcal{G}((k-q)^2)D^f_{\alpha\beta}(k-q) \times \gamma_\alpha S(q_+)\Gamma(q;P)S(q_-)\gamma_\beta\Big], \tag{17}$$

where k and P are the $q\bar{q}$ state's relative and total momenta, respectively, $q_\pm = q \pm P/2$. The notation $\int_{dq}^{\Lambda} = \int^{\Lambda} d^4q/(2\pi)^4$ stands for a Poincaré invariant regularization of the integral, with Λ the regularization mass-scale. The regularization can be removed at the end of all calculations by taking the limit $\Lambda \to \infty$. $D^f_{\alpha\beta}$ represents the free gluon propagator, $\mathcal{G}$ denotes the effective interaction and S the quark propagator. This equation has solutions at discrete values of $P^2 = -m_H^2$, where m_H is the meson mass. The equation also determines completely the BS amplitude $\Gamma(k;P)$ together with the appropriate normalization condition for the bound states. The normalization of the meson's BS amplitude is usually taken with the condition [25]

$$2P_\mu = \frac{N_c}{N_J}\frac{\partial}{\partial P_\mu}\int_{dq}^{\Lambda}\mathrm{tr}[\Gamma(q;-K) \times S(q+)\Gamma(q;K)S(q-)]\big|_{P^2=K^2=-M_{bs}^2}, \tag{18}$$

where $N_c = 3$ is the color number and $N_J = 2J+1$ is the number of the polarization directions of a meson with angular momentum J.

The rainbow truncated DSE for the quark propagator in Euclidean space reads

$$S(p)^{-1} = Z_2 i\gamma\cdot p + Z_4 m_q(\mu) + \frac{4}{3}Z_2^2\int_{dq}^{\Lambda}\mathcal{G}((p-q)^2) \times D^f_{\alpha\beta}(p-q)\gamma_\alpha S(q)\gamma_\beta, \tag{19}$$

where Z_2 and Z_4 are the wave functions and mass renormalization constant, respectively, $m_q(\mu)$ is the current quark mass at spacelike renormalization point μ.

These equations are consistent and coupled with an effective coupling function for which we employ the infrared constant Ansatz [26,27],

$$\frac{\mathcal{G}(s)}{s} = \frac{8\pi^2}{\omega^4}De^{-s/\omega^2} + \frac{8\pi^2\gamma_m\mathcal{F}(s)}{\ln[\tau + (1+s/\Lambda_{QCD}^2)^2]}. \tag{20}$$

The first term characterized by the parameters ω and D determines the infrared and intermediate-momentum part of the interaction. The second term describes the ultraviolet part and produces the correct one-loop perturbative QCD limit. $\mathcal{F}(s) = [1 - exp(-s/[4m_t^2])]/s$, where $m_t = 0.5$ GeV, $\tau = e^2 - 1$, $\gamma_m = 12/(33 - 2N_f)$ (usually) with $N_f = 4$, and $\Lambda_{\rm QCD}^{N_f=4} = 0.234$ GeV. The equations are renormalized at the scale $\mu = 2.0$ GeV.

The leading twist PDA of pseudoscalar meson can be defined by

$$f_{0^-}\varphi(x) = Z_2\int_{dk}\delta(n\cdot k - xn\cdot P)\mathrm{tr}[\gamma_5\gamma\cdot n\chi_{0^-}(k-\frac{P}{2};P)], \tag{21}$$

where f_{0^-} is the lepton decay constant with 0^- the angular momentum $J = 0$ and parity $C = -$, and the BS wave function χ_{0^-} we have mentioned can be defined easily in the rainbow-ladder truncation which is $\chi_{0^-} = S\Gamma_{0^-}S$. Two Dirac covariant components of the wave function contribute to the leading twist PDA, i.e.,

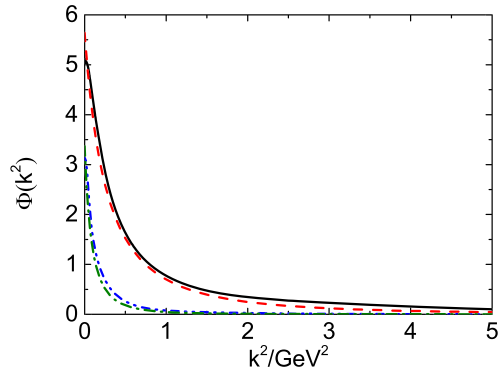

Fig. 2. The computed η_c meson's wave function (*solid curve*) and π meson's (*dot-dashed curve*) compared with the input ones (*dashed curve*) and (*dot–dot-dashed curve*), respectively.

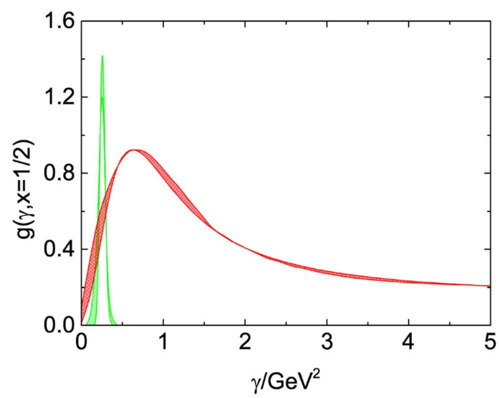

Fig. 3. Obtained weight function of π-meson (*slashed*) and that of η_c-meson (*squared*).

$$\chi_{0^-}(k;P) = \gamma\cdot P\chi^1_{0^-}(k;P) + \gamma\cdot k\chi^2_{0^-}(k;P) + \cdots, \tag{22}$$

where $\chi^{1,2}_{0^-}(k;P)$ depend on $(k^2; k\cdot P)$ and can be determined numerically through DSEs. We here define the wave function $\Phi(k,P)$ in Eq. (1) as $\Phi(k,P) = \Phi(k_i, \cos(\theta_j)) = \mathrm{tr}[\gamma_5\gamma\cdot n\chi_{0^-}(k-\frac{P}{2};P)]$ in which all elements have been obtained numerically through the DSE and BSE, and then the wave function data are created with θ_j fixed and k_i, $i = 1, ...N_{data}$. Here we still choose the typical value for N_{data} as $N_{data} = 40$.

If this wave function can be represented in the Nakanishi form in Eq. (1), the corresponding weight function can then be extracted by the MEM from the above wave function data, and the leading twist PDA can be then obtained via Eq. (3).

We have carried out calculations by choosing the same parameters $\omega = 0.5$ GeV and $D\omega = (0.87\,\mathrm{GeV})^3$ as those in Ref. [5] to produce the π-meson PDA; and $\omega = 0.8$ GeV and $D\omega = (0.7\,\mathrm{GeV})^3$ (the corresponding leptonic decay constant of η_c is about 0.28 GeV) to calculate the η_c-meson PDA. After setting this, the calculated M_{bs} for π and η_c are 0.138 GeV and 0.29 GeV, respectively. The regulated scales are chosen to be the quark mass scales which are 0.35 GeV and 1.6 GeV, respectively.

In Fig. 2, we exhibit the obtained wave functions for π and η_c meson with the extracted weight function compared with the input Bethe–Salpeter wave functions. We can see the results are compatible. The interesting thing is the behaviour of the extracted weight functions shown in Fig. 3. For pion, the weight function $g(\gamma, 1-2x)$ shows a δ function behaviour respective to γ, which suggests that the one-variable Nakanishi-like representation in previous work is very insightful. The previous work based on such ansatz for pion's PDA and form factors would be appropriate. However, such a simple representation might not be appropriate in

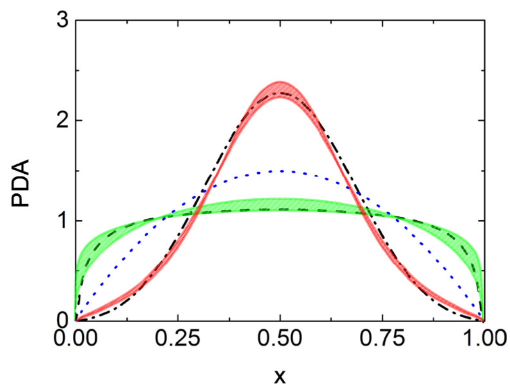

Fig. 4. Obtained PDA of π-meson and that of η_c-meson and the comparison with previous results (ϕ_π computed by the Nakanishi representation [5] (*dashed curve*) and ϕ_{η_c} calculated by brute-force procedure [7] (*dot-dashed curve*)). The asymptotic form is also exhibited with dotted curve.

heavy meson cases, as shown in the figure, the weight function is a smooth function that extends to a large scope. This behaviour means that the pion's BS wave function can be well described by the weight function at a fixed mass scale, and recalling the BS equations, we can find that such a behaviour might be owing to the behaviour of the interaction kernel we employed here, for which is infrared constant.

With the extracted weight function, we then compute the PDAs of π and η_c meson. It is apparent that with growth of quark masses from small to large values, then the pseudoscalar meson distribution amplitude changes from the flat shape to the $\delta(x-\frac{1}{2})$ function shape, and the transition boundary might lie just above strange quark mass [7,28]. The presently obtained results via MEM and the comparison with previous results are illustrated in Fig. 4. The error band is given by varying the MEM parameters $m_0 = 0.1 \sim 10$ and $\Lambda = 1 \sim 30$ GeV^2. Despite the slight uncertainties of the error band, the Fig. 4 shows apparently that the PDAs of the pseudoscalar mesons π and η_c presently obtained via the MEM and DSE approach of QCD match the previous results given in the same dynamical method in the valence region very well and the slight difference in the middle region of x is tolerable. It is noticeable that the previous results of PDAs are reconstructed from the Mellin moments via some specific formula, Gegenbauer polynomials for pion and the Gaussian behaviour for η_c. The most exciting observation for pion's PDA is that it confirms the concave behaviour as the prediction made in the previous work, and since the PDA is obtained here without any assumed structure, such a standpoint is quite conclusive. We also notice that at end points, for both cases, the extracted PDAs are slightly larger than the previous results. Such behaviour at end points occurs in every case we tried in MEM procedure, therefore, we cannot confirm it as the real behaviour of PDAs, and it is very probable that it's just an artificial phenomenon of the MEM procedure. With the current choice of N_{data} and assumed error σ our mock analysis has shown that the MEM can reproduce the weight function with an error of around 10%. The PDA can be well reconstructed, however, it is still difficult to extract further information, for example, the meson's light front wave function. The direct improvement can be done by enlarging the size N_{data} of input data and also choosing different values of θ_j, such work is in progress. However, such a good agreement for the general features of PDAs confirms the previous results on one hand and, on the other hand, indicates that the MEM is efficient to determine the PDAs.

4. Summary and remarks

In this Letter we propose a practical algorithm to determine the PDAs of mesons in the framework of meson's Bethe–Salpeter equation and Dyson–Schwinger equation approach of QCD. The key point of our new algorithm is to implement the MEM to extract the weight function of the Nakanishi representation of the meson's Bethe–Salpeter wave function. The merit of the MEM is that one neither needs to rely on the limit knowledge of the Chebyshev moments of the meson's Bethe–Salpeter amplitude to parameterize the Nakanishi weight function (like previous π case) by special form, nor has to be restricted by the limit number of Mellin moments (like previous η_c case) to suppose some special forms of PDA. The potential advantage of MEM can be applied to find the light-front wave function of meson when one has the Bethe–Salpeter wave function in hand, which we will leave for future work. The MEM procedure might get in some trouble if the extracted weight function is not positive definite that might be the case of PDA for scalar meson's ground state and excited state, and it will be solved by splitting the weight function into an odd part $zg_1(\gamma, 1-2x)$ and an even part $g_2(\gamma, 1-2x)$. The difficulty of this procedure is when the Bethe–Salpeter wave function is not monotonous, the error will become very large. This problem indicates that more complex structures are needed in addition to the structure in Eq. (1). However, the equivalence of the three methods mentioned above allows us to choose appropriate one to analyze the PDA case-by-case.

Acknowledgments

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References

[1] G.P. Lepage, S.J. Brodsky, Phys. Rev. D 22 (1980) 2157.
[2] A.V. Efremov, A.V. Radyushkin, Phys. Lett. B 94 (1980) 245–250.
[3] J. Carbonell, V.A. Karmanov, Eur. Phys. J. A 46 (2010) 387.
[4] N. Nakanishi, Phys. Rev. 130 (1963) 1230.
[5] L. Chang, I.C. Clöet, J.J. Cobos-Martines, C.D. Roberts, S.M. Schmidt, P.C. Tandy, Phys. Rev. Lett. 110 (2013) 132001.
[6] L. Chang, I.C. Clöet, C.D. Roberts, S.M. Schmidt, P.C. Tandy, Phys. Rev. Lett. 111 (2013) 141802.
[7] M.H. Ding, F. Gao, L. Chang, Y.X. Liu, C.D. Roberts, Phys. Lett. B 753 (2016) 330.
[8] T. Frederico, J. Carbonell, V. Gigante, V.A. Karmanov, Few-Body Syst. 57 (2016) 549.
[9] R.K. Bryan, Eur. Biophys. J. 18 (1990) 165.
[10] M. Asakawa, T. Hatsuda, Y. Nakahara, Prog. Part. Nucl. Phys. 46 (1990) 459.
[11] D. Nickel, Ann. Phys. 322 (2007) 1949.
[12] H. Jeffreys, Theory of Probability, third edition, Oxford University Press, Oxford, UK, 1998.
[13] C.E. Shannon, Bell Syst. Tech. J. 27 (1948) 379.
[14] E.T. Jaynes, Phys. Rev. 106 (1957) 620.
[15] E. Jaynes, Phys. Rev. 108 (1957) 171.
[16] J.A. Mueller, C.S. Fischer, D. Nickel, Eur. Phys. J. C 70 (2010) 1037.
[17] S.X. Qin, L. Chang, Y.X. Liu, C.D. Roberts, Phys. Rev. D 84 (2011) 014017.
[18] S.X. Qin, D.H. Rischke, Phys. Rev. D 88 (2013) 056007.
[19] F. Gao, S.X. Qin, Y.X. Liu, C.D. Roberts, S.M. Schmidt, Phys. Rev. D 89 (2014) 076009.
[20] C.D. Roberts, A.G. Williams, Prog. Part. Nucl. Phys. 33 (1994) 477.
[21] C.D. Roberts, S.M. Schmidt, Prog. Part. Nucl. Phys. 45 (2000) S1.
[22] P. Maris, C.D. Roberts, Int. J. Mod. Phys. E 12 (2003) 297.
[23] A. Bashir, L. Chang, I.C. Cloet, B. El-Bennich, Y.X. Liu, C.D. Roberts, P.C. Tandy, Commun. Theor. Phys. 58 (2012) 79.
[24] I.C. Cloet, C.D. Roberts, Prog. Part. Nucl. Phys. 77 (2014) 1.
[25] R.E. Cutkosky, M. Leon, Phys. Rev. 135 (1964) B1445; N. Nakanishi, Phys. Rev. 138 (1965) B1182.
[26] L. Chang, C.D. Roberts, Phys. Rev. Lett. 103 (2009) 081601.
[27] S.X. Qin, L. Chang, Y.X. Liu, C.D. Roberts, D.J. Wilson, Phys. Rev. C 84 (2011) 042202.
[28] I.V. Anikin, A.E. Dorokov, L. Tomio, Phys. Lett. B 475 (2000) 361.