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Spectroscopic parameters and decays of the resonance $Z_b(10610)$

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Abstract The resonance $Z_b(10610)$ is investigated as the diquark–antidiquark $Z_b = [bu][\overline{bd}]$ state with spin–parity $J^P = 1^+$. The mass and current coupling of the resonance $Z_b(10610)$ are evaluated using QCD two-point sum rule and taking into account the vacuum condensates up to ten dimensions. We study the vertices $Z_b\Upsilon(nS)\pi$ ($n = 1, 2, 3$) by applying the QCD light-cone sum rule to compute the corresponding strong couplings $g_{Z_b\Upsilon(nS)\pi}$ and widths of the decays $Z_b \to \Upsilon(nS)\pi$. We explore also the vertices $Z_b h_b(mP)\pi$ ($m = 1, 2$) and calculate the couplings $g_{Z_b h_b(mP)\pi}$ and the widths of the decay channels $Z_b \to h_b(mP)\pi$. To this end, we calculate the mass and decay constants of the $h_b(1P)$ and $h_b(2P)$ mesons. The results obtained are compared with experimental data of the Belle Collaboration.

1 Introduction

Discovery of the charged resonances which cannot be explained as $\bar{c}c$ or $\bar{b}b$ states has opened a new page in the physics of exotic multi-quark systems. The first tetraquarks of this family are $Z^{\pm}(4430)$ states which were observed by the Belle Collaboration in B meson decays $B \to K\psi'\pi^{\pm}$ as resonances in the $\psi'\pi^{\pm}$ invariant mass distributions [1]. The masses and widths of these states were repeatedly measured and refined. Recently, the LHCb Collaboration confirmed the existence of the $Z^-(4430)$ structure in the decay $B^0 \to K^+\psi'\pi^-$ and unambiguously determined that its spin–parity is $J^P = 1^+$ [2,3]. They also measured the mass and width of $Z^-(4430)$ resonance and updated the existing experimental data. Two charmonium-like resonances, $Z_1(4050)$ and $Z_2(4250)$, were discovered by the Belle Collaboration in the decay $\bar{B}^0 \to K^-\pi^+\chi_{c1}$, emerging as broad peaks in the $\chi_{c1}\pi$ invariant mass distribution [4].

Famous members of the charged tetraquark family $Z_c^{\pm}(3900)$ were observed by the BESIII Collaboration in the process $e^+e^- \to J/\psi\pi^+\pi^-$ as resonances with $J^P = 1^+$ in the $J/\psi\pi^{\pm}$ mass distribution [5]. The charged state $Z_c(4020)$ was also found by the BESIII Collaboration in two different processes, $e^+e^- \to h_c\pi^+\pi^-$ and $e^+e^- \to (D^{\star}\bar{D}^{\star})^{\pm}\pi^{\mp}$ (see Refs. [6,7]).

There is another charged state, namely the $Z_c(4200)$ resonance which was detected and announced by Belle [8]. All aforementioned resonances belong to the class of the charmonium-like tetraquarks, and contain a $\bar{c}c$ pair and light quarks (antiquarks). They were mainly interpreted as diquark–antidiquark systems or bound states of D and/or $D^{\star}$ mesons.

It is remarkable that the b-counterparts of the charmonium-like states, i.e. charged resonances composed of a $\bar{b}b$ pair and light quarks were found as well. Thus, the Belle Collaboration discovered the resonances $Z_b(10610)$ and $Z_b(10650)$ (hereafter, Z_b and Z_b', respectively) in the decays $\Upsilon(5S) \to \Upsilon(nS)\pi^+\pi^-$, $n = 1, 2, 3$ and $\Upsilon(5S) \to h_b(mP)\pi^+\pi^-$, $m = 1, 2$ [9,10]. These two states with favored spin–parity $J^P = 1^+$ appear as resonances in the $\Upsilon(nS)\pi^{\pm}$ and $h_b(mP)\pi^{\pm}$ mass distributions. The masses of the Z_b and Z_b' resonances are

$$m = (10607.2 \pm 2.0)\ \text{MeV},$$
$$m' = (10652.2 \pm 1.5)\ \text{MeV}, \quad (1)$$

respectively. The width of the Z_b state averaged over five decay channels equals $\Gamma = (18.4 \pm 2.4)$ MeV, whereas the average width of Z_b' is $\Gamma' = (11.5 \pm 2.2)$ MeV. Recently, the dominant decay channel of Z_b, namely the $Z_b \to B^+\bar{B}^{\star 0} + \bar{B}^0 B^{\star +}$ process, was also observed [11]. In this work fractions of different channels of Z_b and Z_b' resonances were reported as well. Further information on the

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experimental status of the Z_b and Z_b' states and other heavy exotic mesons and baryons can be found in Ref. [12].

The existence of hidden-bottom states, i.e. of the Z_b resonances, was foreseen before their experimental observation. Thus, in Ref. [13] the authors suggested to look for the diquark–antidiquark systems with $b\bar{b}u\bar{d}$ content as peaks in the invariant mass of the $\Upsilon(1S)\pi$ and $\Upsilon(2S)\pi$ systems. The existence of the molecular state $B^{\star}\bar{B}$ was predicted in Ref. [14].

After discovery of the Z_b resonances theoretical studies of the charged hidden-bottom states became more intensive and fruitful. In fact, articles devoted to the structures and decay channels of the Z_b states encompass all existing models and computational schemes suitable to study the multiquark systems. Thus, in Refs. [15,16] the spectroscopic and decay properties of Z_b and Z_b' were explored using the heavy quark symmetry by modeling them as $J = 1$ S-wave molecular $B^{\star}\bar{B}$–$B\bar{B}^{\star}$ states and $B^{\star}\bar{B}^{\star}$, respectively. The existence of similar states with quantum numbers $0^+,\ 1^+,\ 2^+$ was predicted as well. The diquark–antidiquark interpretation of the Z_b states was proposed in Refs. [17,18]. It was demonstrated that Belle results on the decays $\Upsilon(5S) \to \Upsilon(nS)\pi^+\pi^-$ and $\Upsilon(5S) \to h_b(mP)\pi^+\pi^-$ support Z_b resonances as diquark–antidiquark states. This analysis is based on a scheme for the spin–spin quark interactions inside diquarks originally suggested and successfully used to explore hidden-charm resonances [19].

The Z_b resonance was considered in Ref. [20] as a $B^{\star}\bar{B}$ molecular state, where its mass was computed in the context of the QCD sum rule method. The prediction for the mass $m_{B^{\star}\bar{B}} = 10.54 \pm 0.22$ GeV obtained there allowed the authors to conclude that Z_b could be a $B^{\star}\bar{B}$ molecular state. Similar conclusions were also drawn in the framework of the chiral quark model. Indeed, in Ref. [21] the $B\bar{B}^{\star}$ and $B^{\star}\bar{B}^{\star}$ bound states with $J^{PC} = 1^{+-}$ were studied in the chiral quark model, and found to be good candidates for the Z_b and Z_b' resonances. Moreover, the existence of molecular states $B\bar{B}^{\star}$ with $J^{PC} = 1^{++}$, and $B^{\star}\bar{B}^{\star}$ with $J^{PC} = 0^{++},\ 2^{++}$ was predicted. Explorations performed using the one boson-exchange model also led to the molecular interpretations of the Z_b and Z_b' resonances [22]. However, an analysis carried out in the framework of the Bethe–Salpeter approach demonstrated that two heavy mesons can form an isospin singlet bound state but cannot form an isotriplet compound. Hence, the Z_b resonance presumably is a diquark–antidiquark, but not a molecular state [23].

Both the diquark–antidiquark and the molecular pictures for the internal organization of Z_b and Z_b' within the QCD sum rules method were examined in Ref. [24]. In this work the authors constructed different interpolating currents with $I^G J^P = 1^+1^+$ to explore the Z_b and Z_b' states and evaluate their masses. Among alternative interpretations of the Z_b states it is worth to note Refs. [25,26], where the peaks observed by the Belle Collaboration were explained as cusp and coupling channel effects, respectively.

Theoretical studies that address problems of the Z_b states are numerous (see Refs. [27–36]). An analysis of these and other investigations can be found in Refs. [37,38].

As is seen, the theoretical status of the resonances Z_b and Z_b' remains controversial and deserves further and detailed explorations. In the present work we are going to calculate the spectroscopic parameters of $Z_b = [bu][\overline{bd}]$ state by assuming that it is a tetraquark state with a diquark–antidiquark structure and positive charge. We use QCD two-point sum rules to evaluate its mass and current coupling by taking into account vacuum condensates up to ten dimensions. We also investigate five observed decay channels of the Z_b resonance employing QCD sum rules on the light-cone. As a byproduct, we derive the mass and decay constant of the $h_b(mP),\ m = 1, 2$ mesons.

This work has the following structure: In Sect. 2 we calculate the mass and current coupling of the Z_b resonance. In Sect. 3 we analyze the decay channels $Z_b \to \Upsilon(nS)\pi,\ n = 1,\ 2,\ 3$, and we calculate their widths. Section 4 is devoted to an investigation of the decay modes $Z_b \to h_b(mP)\pi,\ m = 1,\ 2$, and it consists of two subsections. In the first subsection we calculate the mass and decay constant of the $h_b(1P)$ and $h_b(2P)$ mesons. To this end, we employ the two-point sum rule approach by including into the analysis condensates up to eight dimensions. In the next subsection using the parameters of the $h_b(mP)$ mesons we evaluate the widths of the decays under investigation. The last section is reserved for an analysis of the obtained results and a discussion of possible interpretations of the Z_b resonance.

2 Mass and current coupling of the Z_b state: QCD two-point sum rule predictions

In this section we derive QCD sum rules to calculate the mass and current coupling of the Z_b state by suggesting that it has a diquark–antidiquark structure with quantum numbers $I^G J^P = 1^+1^+$. To this end, we begin from the two-point correlation function

$$\Pi_{\mu\nu}(p) = i \int d^4x e^{ipx} \langle 0|\mathcal{T}\{J_{\mu}^{Z_b}(x) J_{\nu}^{Z_b\dagger}(0)\}|0\rangle, \qquad (2)$$

where $J_{\mu}^{Z_b}(x)$ is the interpolating current for the Z_b state with the required quark content and quantum numbers.

It is possible to construct various currents to interpolate the Z_b and Z_b' resonances. One of them is a $[ub]_{S=0}[\overline{db}]_{S=1}$–$[ub]_{S=1}[\overline{db}]_{S=0}$ type diquark–antidiquark current that is used to consider the Z_b state,

$$J_{\mu}^{Z_b}(x)=\frac{i\epsilon\tilde{\epsilon}}{\sqrt{2}}\left\{\left[u_a^T(x)C\gamma_5 b_b(x)\right]\left[\overline{d}_d(x)\gamma_\mu C\overline{b}_e^T(x)\right]\right.$$
$$\left.-\left[u_a^T(x)C\gamma_\mu b_b(x)\right]\left[\overline{d}_d(x)\gamma_5 C\overline{b}_e^T(x)\right]\right\}. \quad (3)$$

The current for Z_b' can be defined in the form

$$J_{\mu}^{Z_b'}(x)=\frac{\epsilon\tilde{\epsilon}}{\sqrt{2}}\varepsilon_{\mu\nu\alpha\beta}\left[u_a^T(x)C\gamma^\nu b_b(x)\right]$$
$$\times D^\alpha\left[\overline{d}_d(x)\gamma^\beta C\overline{b}_e^T(x)\right], \quad (4)$$

where $D^\alpha=\partial^\alpha-ig_sA^\alpha(x)$ [24]. In Eqs. (3) and (4) we have introduced the notation $\epsilon=\epsilon_{abc}$ and $\tilde{\epsilon}=\epsilon_{dec}$. In above expressions a,b,c,d and e are color indices, and C is the charge conjugation matrix.

By choosing different currents to interpolate the Z_b and Z_b' resonances one treats both of them as ground-state particles in the corresponding sum rules. We also follow this approach and use the current $J_\mu^{Z_b}(x)$ to calculate the mass and current coupling of the Z_b state. To find the QCD sum rules we first have to calculate the correlation function in terms of the physical degrees of freedom. To this end, we saturate $\Pi_{\mu\nu}(p)$ with a complete set of states with the quantum numbers of the Z_b resonance and perform in Eq. (2) an integration over x to get

$$\Pi_{\mu\nu}^{\rm Phys}(p)=\frac{\langle 0|J_\mu^{Z_b}|Z_b(p)\rangle\langle Z_b(p)|J_\nu^{Z_b\dagger}|0\rangle}{m_{Z_b}^2-p^2}+\ldots,$$

where m_{Z_b} is the mass of the Z_b state, and dots indicate the contributions of higher resonances and continuum states. We define the current coupling f_{Z_b} through the matrix element

$$\langle 0|J_\mu^{Z_b}|Z_b(p)\rangle=f_{Z_b}m_{Z_b}\varepsilon_\mu, \quad (5)$$

with ε_μ being the polarization vector of Z_b state. Then in terms of m_{Z_b} and f_{Z_b}, the correlation function can be written in the following form:

$$\Pi_{\mu\nu}^{\rm Phys}(p)=\frac{m_{Z_b}^2 f_{Z_b}^2}{m_{Z_b}^2-p^2}\left(-g_{\mu\nu}+\frac{p_\mu p_\nu}{m_{Z_b}^2}\right)+\ldots \quad (6)$$

The Borel transformation applied to Eq. (6) gives

$$\mathcal{B}\Pi_{\mu\nu}^{\rm Phys}(p)=m_{Z_b}^2 f_{Z_b}^2 e^{-m_{Z_b}^2/M^2}\left(-g_{\mu\nu}+\frac{p_\mu p_\nu}{m_{Z_b}^2}\right)+\ldots \quad (7)$$

At the next stage we derive the theoretical expression for the correlation function $\Pi_{\mu\nu}^{\rm QCD}(p)$ in terms of the quark–gluon degrees of freedom. It can be determined using the interpolating current $J_\mu^{Z_b}$ and quark propagators. After contracting in Eq. (2) the b-quark and light quark fields we get

$$\Pi_{\mu\nu}^{\rm QCD}(p)=-\frac{i}{2}\int d^4x e^{ipx}\epsilon\tilde{\epsilon}\epsilon'\tilde{\epsilon}'\left\{{\rm Tr}\left[\gamma_5\widetilde{S}_u^{aa'}(x)\right.\right.$$
$$\left.\times\gamma_5 S_b^{bb'}(x)\right]{\rm Tr}\left[\gamma_\mu\widetilde{S}_b^{e'e}(-x)\gamma_\nu S_d^{d'd}(-x)\right]$$
$$-{\rm Tr}\left[\gamma_\mu\widetilde{S}_b^{e'e}(-x)\gamma_5 S_d^{d'd}(-x)\right]{\rm Tr}\left[\gamma_\nu\widetilde{S}_u^{aa'}(x)\right.$$
$$\left.\times\gamma_5 S_b^{bb'}(x)\right]-{\rm Tr}\left[\gamma_5\widetilde{S}_u^{a'a}(x)\gamma_\mu S_b^{b'b}(x)\right]$$
$$\times{\rm Tr}\left[\gamma_5\widetilde{S}_b^{e'e}(-x)\gamma_\nu S_d^{d'd}(-x)\right]+{\rm Tr}\left[\gamma_\nu\widetilde{S}_u^{aa'}(x)\right.$$
$$\left.\left.\times\gamma_\mu S_b^{bb'}(x)\right]{\rm Tr}\left[\gamma_5\widetilde{S}_b^{e'e}(-x)\gamma_5 S_d^{d'd}(-x)\right]\right\}, \quad (8)$$

where

$$\widetilde{S}_{b(q)}^{ij}(x)=CS_{b(q)}^{ij{\rm T}}(x)C.$$

In the expressions above $S_q^{ab}(x)$ and $S_b^{ab}(x)$ are the light u-, and d-, and the heavy b-quark propagators, respectively. We choose the light quark propagator $S_q^{ab}(x)$ in the form

$$S_q^{ab}(x)=i\delta_{ab}\frac{\not{x}}{2\pi^2x^4}-\delta_{ab}\frac{m_q}{4\pi^2x^2}-\delta_{ab}\frac{\langle\overline{q}q\rangle}{12}$$
$$+i\delta_{ab}\frac{\not{x}m_q\langle\overline{q}q\rangle}{48}-\delta_{ab}\frac{x^2}{192}\langle\overline{q}g_s\sigma Gq\rangle$$
$$+i\delta_{ab}\frac{x^2\not{x}m_q}{1152}\langle\overline{q}g_s\sigma Gq\rangle$$
$$-i\frac{g_sG_{ab}^{\alpha\beta}}{32\pi^2x^2}\left[\not{x}\sigma_{\alpha\beta}+\sigma_{\alpha\beta}\not{x}\right]-i\delta_{ab}\frac{x^2\not{x}g_s^2\langle\overline{q}q\rangle^2}{7776}$$
$$-\delta_{ab}\frac{x^4\langle\overline{q}q\rangle\langle g_s^2G^2\rangle}{27648}+\ldots \quad (9)$$

For the b-quark propagator $S_b^{ab}(x)$ we employ the expression

$$S_b^{ab}(x)=i\int\frac{d^4k}{(2\pi)^4}e^{-ikx}\left\{\frac{\delta_{ab}\left(\not{k}+m_b\right)}{k^2-m_b^2}\right.$$
$$-\frac{g_sG_{ab}^{\alpha\beta}}{4}\frac{\sigma_{\alpha\beta}\left(\not{k}+m_b\right)+\left(\not{k}+m_b\right)\sigma_{\alpha\beta}}{(k^2-m_b^2)^2}$$
$$+\frac{g_s^2G^2}{12}\delta_{ab}m_b\frac{k^2+m_b\not{k}}{(k^2-m_b^2)^4}+\frac{g_s^3G^3}{48}\delta_{ab}\frac{\left(\not{k}+m_b\right)}{(k^2-m_b^2)^6}$$
$$\left.\times\left[\not{k}\left(k^2-3m_b^2\right)+2m_b\left(2k^2-m_b^2\right)\right]\left(\not{k}+m_b\right)+\ldots\right\}. \quad (10)$$

In Eqs. (9) and (10) we use the notation

$$G_{ab}^{\alpha\beta}=G_A^{\alpha\beta}t_{ab}^A,\quad G^2=G_{\alpha\beta}^AG_{\alpha\beta}^A,$$
$$G^3=f^{ABC}G_{\mu\nu}^AG_{\nu\delta}^BG_{\delta\mu}^C, \quad (11)$$

where $A,B,C=1,2\ldots 8$. In Eq. (11) $t^A=\lambda^A/2$, λ^A are the Gell-Mann matrices, and the gluon field strength tensor $G_{\alpha\beta}^A\equiv G_{\alpha\beta}^A(0)$ is fixed at $x=0$.

The QCD sum rule can be obtained by choosing the same Lorentz structures in both $\Pi_{\mu\nu}^{\rm Phys}(p)$ and $\Pi_{\mu\nu}^{\rm QCD}(p)$. We work with terms $\sim g_{\mu\nu}$, which do not contain the effects of spin-0 particles. The invariant amplitude $\Pi^{\rm QCD}(p^2)$ corresponding to this structure can be written down as the dispersion integral

$$\Pi^{\rm QCD}(p^2)=\int_{4m_b^2}^{\infty}\frac{\rho^{\rm QCD}(s)}{s-p^2}ds+\ldots, \tag{12}$$

where $\rho^{\rm QCD}(s)$ is the corresponding spectral density. It is a key ingredient of sum rules for $m_{Z_b}^2$ and $f_{Z_b}^2$ and can be obtained using the imaginary part of the invariant amplitude $\Pi^{\rm QCD}(p^2)$. Methods of such calculations are well known and presented numerously in the literature. Therefore, we omit further details, emphasizing only that $\rho^{\rm QCD}(s)$ in the present work is calculated by including into the analysis quark, gluon and mixed condensates up to ten dimensions.

After applying the Borel transformation on the variable p^2 to $\Pi^{\rm QCD}(p^2)$, equating the expression obtained to $\mathcal{B}\Pi^{\rm Phys}(p)$, and subtracting the continuum contribution, we obtain the required sum rules. Thus, the mass of the Z_b state can be evaluated from the sum rule

$$m_{Z_b}^2=\frac{\int_{4m_b^2}^{s_0}ds s\rho^{\rm QCD}(s)e^{-s/M^2}}{\int_{4m_b^2}^{s_0}ds\rho^{\rm QCD}(s)e^{-s/M^2}}, \tag{13}$$

whereas for the current coupling f_{Z_b} we employ the formula

$$f_{Z_b}^2=\frac{1}{m_{Z_b}^2}\int_{4m_b^2}^{s_0}ds\rho^{\rm QCD}(s)e^{(m_{Z_b}^2-s)/M^2}. \tag{14}$$

The sum rules for m_{Z_b} and f_{Z_b} depend on different vacuum condensates stemming from the quark propagators, on the mass of the b-quark, and on the Borel variable M^2 and continuum threshold s_0, which are auxiliary parameters of the numerical computations. The vacuum condensates are parameters that do not depend on the problem under consideration: their numerical values extracted once from some processes are applicable in all sum rule computations. For quark and mixed condensates in the present work we employ $\langle\bar{q}q\rangle=-(0.24\pm0.01)^3$ GeV^3, $\langle\bar{q}g_s\sigma Gq\rangle=m_0^2\langle\bar{q}q\rangle$, where $m_0^2=(0.8\pm0.1)$ GeV^2, whereas for the gluon condensates we utilize $\langle\alpha_s G^2/\pi\rangle=(0.012\pm0.004)$ GeV^4, $\langle g_s^3G^3\rangle=(0.57\pm0.29)$ GeV^6. The mass of the $b-$quark can be found in Ref. [46]: it is equal to $m_b=4.18^{+0.04}_{-0.03}$ GeV.

The choice of the Borel parameter M^2 and continuum threshold s_0 should obey some restrictions of sum rule calculations. Thus, the limits within of which M^2 can be varied (working window) are determined from convergence of the operator product expansion and dominance of the pole contribution. In the working window of the threshold parameter s_0 the dependence of the quantities on M^2 should be minimal. In real calculations, however, the quantities of interest depend on the parameters M^2 and s_0, which affects the accuracy of the extracted numerical values. Theoretical errors in sum rule calculations may amount to 30% of the predictions obtained, and a considerable part of these ambiguities are connected namely with the choice of M^2 and s_0.

An analysis performed in accordance with these requirements allows us to fix the working windows for M^2 and s_0:

$$M^2=9-12\ {\rm GeV}^2,\ s_0=123-127\ {\rm GeV}^2. \tag{15}$$

In Figs. 1 and 2 we demonstrate the results of the numerical computations of the mass m_{Z_b} and the current coupling f_{Z_b} as functions of the parameters M^2 and s_0. As is seen, m_{Z_b} and f_{Z_b} are rather stable within the working windows of the auxiliary parameters, but there is still a dependence on them in the plotted figures. Our results for m_{Z_b} and f_{Z_b} read

$$m_{Z_b}=10581^{+142}_{-164}\ {\rm MeV},\ \ f_{Z_b}=(2.79^{+0.55}_{-0.65})\times10^{-2}\ {\rm GeV}^4. \tag{16}$$

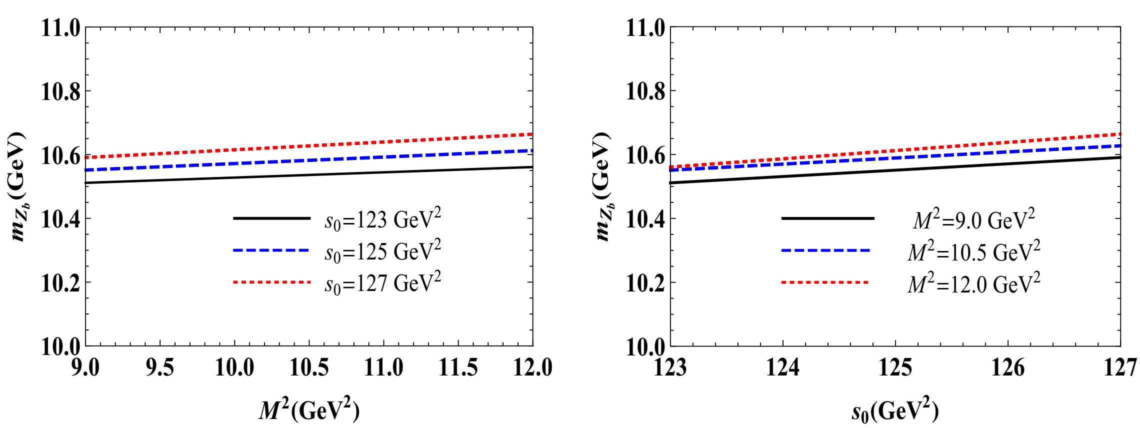

Fig. 1 The mass of the Z_b state vs. Borel parameter M^2 at fixed s_0 (left panel), and continuum threshold s_0 at fixed M^2 (right panel)

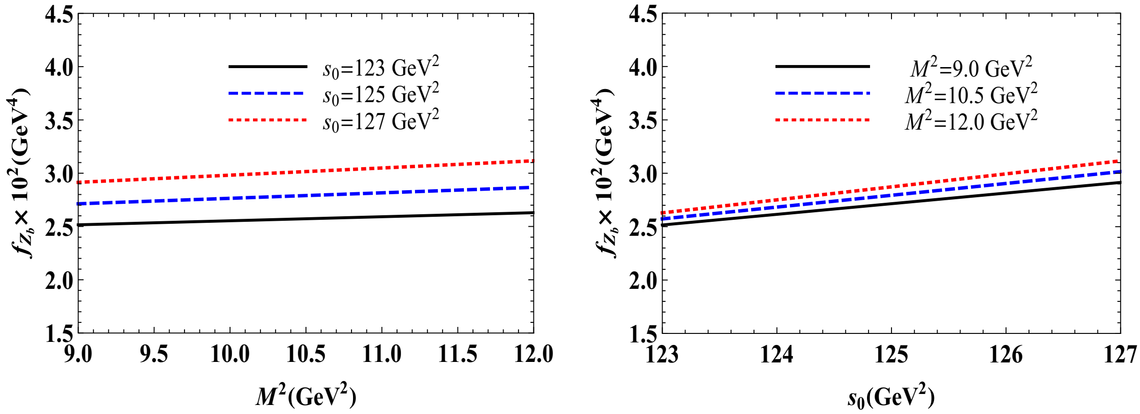

Fig. 2 The dependence of the current coupling f_{Z_b} of the Z_b resonance on the Borel parameter at chosen values of s_0 (left panel), and on the s_0 at fixed M^2 (right panel)

Within theoretical errors m_{Z_b} is in agreement with experimental measurements of the Belle Collaboration (1). The mass and current coupling Z_b given by Eq. (16) will be used as input parameters in the next sections to find the widths of the decays $Z_b \to \Upsilon(nS)\pi$ and $Z_b \to h_b(mP)\pi$.

3 Decay channels $Z_b \to \Upsilon(nS)\pi,\ n=1,\ 2,\ 3.$

This section is devoted to the calculation of the width of the $Z_b \to \Upsilon(nS)\pi,\ \ n=1,\,2,\,3,$ decays. To this end we determine the strong couplings $g_{Z_b\Upsilon_n\pi},\ \ n=1,2,3$ (in the formulas we utilize $\Upsilon_n \equiv \Upsilon(nS)$) using QCD sum rules on the light-cone in conjunction with the ideas of the soft-meson approximation.

We start from an analysis of the vertices $Z_b\Upsilon_n\pi$ aiming to calculate $g_{Z_b\Upsilon_n\pi}$, and therefore we consider the correlation function

$$\Pi_{\mu\nu}(p,q)=i\int d^4x e^{ipx}\left\langle \pi(q)\left|\mathcal{T}\{J_\mu^{\Upsilon}(x)J_\nu^{Z_b\dagger}(0)\}\right|0\right\rangle, \tag{17}$$

where

$$J_\mu^{\Upsilon}(x)=\overline{b}_i(x)\gamma_\mu b_i(x) \tag{18}$$

is the interpolating current for mesons $\Upsilon(nS)$. Here $p,\ q$ and $p'=p+q$ are the momenta of $\Upsilon(nS)$, π and Z_b, respectively.

To derive the sum rules for the couplings $g_{Z_b\Upsilon_n\pi}$, we calculate $\Pi_{\mu\nu}(p,q)$ in terms of the physical degrees of freedom. It is not difficult to obtain

$$\Pi_{\mu\nu}^{\rm Phys}(p,q)=\sum_{n=1}^{3}\frac{\left\langle 0\left|J_\mu^{\Upsilon}\right|\Upsilon_n(p)\right\rangle}{p^2-m_{\Upsilon(nS)}^2}\langle\Upsilon_n(p)\pi(q)|Z_b(p')\rangle \times\frac{\left\langle Z_b(p')|J_\nu^{Z_b\dagger}|0\right\rangle}{p'^2-m_{Z_b}^2}+\ldots, \tag{19}$$

where the dots denote the contributions of the higher resonances and continuum states.

We introduce the matrix elements

$$\begin{aligned}&\langle 0|J_\mu^{\Upsilon}|\Upsilon_n(p)\rangle=f_{\Upsilon_n}m_{\Upsilon_n}\varepsilon_\mu,\\&\langle Z_b(p')|J_\nu^{Z_b\dagger}|0\rangle=f_{Z_b}m_{Z_b}\varepsilon_\nu'^{*},\\&\langle\Upsilon_n(p)\pi(q)|Z_b(p')\rangle=g_{Z_b\Upsilon_n\pi}\left[(p\cdot p')\right.\\&\left.\quad\times(\varepsilon^*\cdot\varepsilon')-(p\cdot\varepsilon')(p'\cdot\varepsilon^*)\right],\end{aligned} \tag{20}$$

where $f_{\Upsilon_n},\ m_{\Upsilon_n},\ \varepsilon_\mu$ are the decay constant, mass and polarization vector of the $\Upsilon(nS)$ meson, and ε_ν' is the polarization vector of the Z_b state.

Having used Eq. (20) we rewrite the correlation function in the form

$$\Pi_{\mu\nu}^{\rm Phys}(p,q)=\sum_{n=1}^{3}\frac{g_{Z_b\Upsilon_n\pi}f_{\Upsilon_n}f_{Z_b}m_{Z_b}m_{\Upsilon_n}}{\left(p'^2-m_{Z_b}^2\right)\left(p^2-m_{\Upsilon_n}^2\right)}\times\left(\frac{m_{Z_b}^2+m_{\Upsilon_n}^2}{2}g_{\mu\nu}-p'_\mu p_\nu\right)+\ldots \tag{21}$$

For calculation of the strong couplings we choose to work with the structure $\sim g_{\mu\nu}$. To this end, we have to isolate the invariant function $\Pi^{\rm Phys}(p^2,p'^2)$ corresponding to this structure and find its double Borel transformation. But it is well known that in the case of vertices involving a tetraquark and two conventional mesons one has to set $q=0$ [39]. This is connected with the fact that the interpolating current for the tetraquark is composed of four quark fields and, after contracting two of them in the correlation function $\Pi_{\mu\nu}(p,q)$

with the relevant quark fields from the heavy meson's current, we encounter a situation where the remaining quarks are located at the same space-time point. These quarks fields, sandwiched between a light meson and vacuum instead of generating light meson's distribution amplitudes, create the local matrix elements. Then, in accordance with the four-momentum conservation at such vertices, we have to set $q = 0$. In QCD light-cone sum rules the limit $q \to 0$, when the light-cone expansion reduces to a short-distant expansion over local matrix elements, is known as the "soft-meson approximation". The mathematical methods to handle the soft-meson limit were elaborated in Refs. [40,41] and were successfully applied to tetraquark vertices in our previous articles [42–45]. In the soft limit $p' = p$ and the relevant invariant amplitudes in the correlation function depend only on one variable: p^2. In the present work we use this approach, which implies calculation of the correlation function with equal initial and final momenta, $p' = p$, and dealing with the double pole terms obtained.

In fact, in the limit $p = p'$ we replace in Eq. (21)

$$\frac{1}{\left(p'^2 - m_{Z_b}^2\right)\left(p^2 - m_{\Upsilon_n}^2\right)}$$

by double pole factors

$$\frac{1}{\left(p^2 - m_n^2\right)^2},$$

where $m_n^2 = (m_{Z_b}^2 + m_{\Upsilon_n}^2)/2$, and we carry out the Borel transformation over p^2. Then for the Borel transformation of $\Pi^{\text{Phys}}(p^2)$ we get

$$\mathcal{B}\Pi^{\text{Phys}}(p^2) = \sum_{n=1}^{3} g_{Z_b\Upsilon_n\pi} f_{\Upsilon_n} f_{Z_b} m_{Z_b} m_{\Upsilon_n} \times m_n^2 \frac{e^{-m_n^2/M^2}}{M^2} + \ldots \tag{22}$$

Now one has to derive the correlation function in terms of the quark–gluon degrees of freedom and find the QCD side of the sum rules. Contracting of the heavy quark fields in Eq. (17) yields

$$\Pi_{\mu\nu}^{\text{QCD}}(p,q) = \int d^4x e^{ipx} \frac{\epsilon\widetilde{\epsilon}}{\sqrt{2}} \Big[\gamma_5 \widetilde{S}_b^{ib}(x)\gamma_\mu \times \widetilde{S}_b^{ei}(-x)\gamma_\nu + \gamma_\nu \widetilde{S}_b^{ib}(x)\gamma_\mu \widetilde{S}_b^{ei}(-x)\gamma_5\Big]_{\alpha\beta} \times \langle\pi(q)|\overline{u}_\alpha^a(0) d_\beta^d(0)|0\rangle, \tag{23}$$

where α and β are the spinor indices. We continue and use the expansion

$$\overline{u}_\alpha^a d_\beta^d \to \frac{1}{4}\Gamma_{\beta\alpha}^j \left(\overline{u}^a \Gamma^j d^d\right), \tag{24}$$

where Γ^j is the full set of Dirac matrices

$$\Gamma^j = \mathbf{1},\ \gamma_5,\ \gamma_\lambda,\ i\gamma_5\gamma_\lambda,\ \sigma_{\lambda\rho}/\sqrt{2}.$$

Replacing $\overline{u}_\alpha^a d_\beta^d$ in Eq. (23) by this expansion and performing summations over color indices it is not difficult to determine local matrix elements of the pion which contribute to $\Pi_{\mu\nu}^{\text{QCD}}(p,q)$ (see Ref. [39] for details). It turns out that in the soft limit the pion's local matrix element which contributes to $\text{Im}\Pi_{\mu\nu}^{\text{QCD}}(p, q = 0)$ is

$$\langle 0|\overline{d}(0) i\gamma_5 u(0)|\pi(q)\rangle = f_\pi \mu_\pi, \tag{25}$$

where

$$\mu_\pi = \frac{m_\pi^2}{m_u + m_d}.$$

After fixing in $\text{Im}\Pi_{\mu\nu}^{\text{QCD}}(p, q = 0)$ the structure $\sim g_{\mu\nu}$ it is straightforward to extract $\rho_\Upsilon^{\text{QCD}}(s)$ as a sum of the perturbative and nonperturbative components:

$$\rho_\Upsilon^{\text{QCD}}(s) = \frac{f_\pi \mu_\pi}{12\sqrt{2}} \left[\rho^{\text{pert.}}(s) + \rho^{\text{n.-pert.}}(s)\right]. \tag{26}$$

The $\rho_\Upsilon^{\text{QCD}}(s)$ can be obtained after the replacement $m_c \to m_b$ from the spectral density of $Z_c \to J/\psi\pi$ decay calculated in Refs. [39,45]. Its perturbative component $\rho^{\text{pert.}}(s)$ has a simple form and reads

$$\rho^{\text{pert.}}(s) = \frac{(s + 2m_b^2)\sqrt{s(s - 4m_b^2)}}{\pi^2 s}. \tag{27}$$

The nonperturbative contribution $\rho^{\text{n.-pert.}}(s)$ depends on the vacuum expectation values of the gluon operators and contains terms of four, six and eight dimensions. Its explicit expression was presented in the appendix of Ref. [45].

The continuum subtraction in the case under consideration can be done using quark–hadron duality, which leads to the desired sum rule for the strong couplings. We get

$$\sum_{n=1}^{3} g_{Z_b\Upsilon_n\pi} f_{\Upsilon_n} f_{Z_b} m_{Z_b} m_{\Upsilon_n} m_n^2 \frac{\mathrm{e}^{-m_n^2/M^2}}{M^2} = \int_{4m_b^2}^{s_0} \mathrm{d}s e^{-s/M^2} \rho_\Upsilon^{\text{QCD}}(s). \tag{28}$$

Here some comments are in order on the expression obtained, Eq. (28). It is well known that the soft limit considerably simplifies the QCD side of light-cone sum rule expressions [40]. At the same time, in the limit $q \to 0$ the phenomenological side of the sum rules gains contributions which are not suppressed relative to a main term. In our case the main term corresponds to vertex $Z_b\Upsilon(1S)\pi$, where the tetraquark and mesons are ground-state particles. Additional contributions emerge due to vertices $Z_b\Upsilon\pi$ where some of particles (or all of them) are on their excited states. In Eq. (28) terms corresponding to vertices $Z_b\Upsilon(2S)\pi$ and $Z_b\Upsilon(3S)\pi$ belong to this class of contributions. When we are interested in extraction of parameters of a vertex built of only ground-state particles, these additional contributions are undesired contaminations which may affect the accuracy of the calculations. A technique to eliminate them from sum rules is also well known [40,41]. To this end, in accordance with elaborated recipes one has to act by the operator

$$\mathcal{P}(M^2, m_n^2) = \left(1 - M^2\frac{d}{dM^2}\right) M^2 e^{m_n^2/M^2} \tag{29}$$

to Eq. (28). In the present work we are going to evaluate three strong couplings $g_{Z_b\Upsilon_n\pi}$ and therefore we use the original form of the sum rule given by Eq. (28). But it provides only one equality for three unknown quantities. In order to get two additional equations we act by operators $d/d(-1/M^2)$ and $d^2/d(-1/M^2)^2$ to both sides of Eq. (28) and solve obtained equations to find $g_{Z_b\Upsilon_n\pi}$.

The widths of the decays $Z_b \to \Upsilon(nS)\pi,\ n = 1, 2, 3$ can be calculated applying the standard methods and have the same form as in the case of the decay $Z_c \to J/\psi\pi$. After evident replacements in the corresponding formula we get

$$\Gamma\left(Z_b \to \Upsilon_n\pi\right) = \frac{g^2_{Z_b\Upsilon_n\pi} m^2_{\Upsilon_n}}{24\pi}\lambda\left(m_{Z_b},\ m_{\Upsilon_n}, m_\pi\right) \times\left[3 + \frac{2\lambda^2\left(m_{Z_b},\ m_{\Upsilon_n}, m_\pi\right)}{m^2_{\Upsilon_n}}\right], \tag{30}$$

where

$$\lambda(a,\ b,\ c) = \frac{\sqrt{a^4 + b^4 + c^4 - 2\left(a^2b^2 + a^2c^2 + b^2c^2\right)}}{2a}.$$

The key component in Eq. (30) is the strong coupling $g_{Z_b\Upsilon_n\pi}$. Relevant sum rules contain spectroscopic parameters of the tetraquark Z_b and the mesons $\Upsilon(nS)$ and π. The mass and current coupling of the Z_b resonance have been calculated in the previous section. For numerical computations we take masses m_{Υ_n} and decay constants f_{Υ_n} of the mesons $\Upsilon(nS)$ from Ref. [46]. The relevant information is shown in Table 1.

In our calculations the Borel parameter M^2 and continuum threshold s_0 are varied within the regions

$$M^2 = 10 - 13\ \text{GeV}^2,\ s_0 = 124 - 128\ \text{GeV}^2, \tag{31}$$

which are almost identical to similar working windows in the mass and current coupling calculations being slightly shifted towards larger values.

Table 1 Spectroscopic parameters of the mesons Υ_{nS} and π

Parameters	Values (in (MeV)
m_{Υ_1}	9460.30 ± 0.26
f_{Υ_1}	708 ± 8
m_{Υ_2}	10023.26 ± 0.31
f_{Υ_2}	482 ± 10
m_{Υ_3}	10355.2 ± 0.5
f_{Υ_3}	346 ± 50
m_π	139.57061 ± 0.00024
f_π	131.5

For the couplings $g_{Z_b\Upsilon_n\pi}$ we obtain (in GeV^{-1}):

$$g_{Z_b\Upsilon_1\pi} = 0.019 \pm 0.005,\quad g_{Z_b\Upsilon_2\pi} = 0.090 \pm 0.031,$$
$$g_{Z_b\Upsilon_3\pi} = 0.104 \pm 0.031. \tag{32}$$

For the widths of the decays $Z_b \to \Upsilon(nS)\pi$ these couplings lead to the predictions

$$\Gamma(Z_b \to \Upsilon(1S)\pi) = 1.36 \pm 0.43\ \text{MeV},$$
$$\Gamma(Z_b \to \Upsilon(2S)\pi) = 17.18 \pm 5.01\ \text{MeV},$$
$$\Gamma(Z_b \to \Upsilon(3S)\pi) = 8.27 \pm 2.69\ \text{MeV}. \tag{33}$$

The predictions obtained for the widths of the decays $\Gamma(Z_b \to \Upsilon(nS)\pi)$ are the final results of this section and will be used for comparison with the experimental data.

4 $Z_b \to h_b(1P)\pi$ and $Z_b \to h_b(2P)\pi$ decays

The second class of decays which we consider contains two processes $Z_b \to h_b(mP)\pi,\ m = 1, 2$. We follow the same prescriptions as in the case of $Z_b \to \Upsilon(nP)\pi$ decays and derive sum rules for the strong couplings $g_{Z_bh_b\pi}$ and $g_{Z_bh_b'\pi}$ (hereafter we employ the short-hand notations $h_b \equiv h_b(1P)$ and $h_b' \equiv h_b(2P)$). From the analysis performed in the previous section it is clear that the corresponding sum rules will depend on numerous input parameters including mass and decay constant of the mesons $h_b(1P)$ and $h_b(2P)$. Information on the spectroscopic parameters of $h_b(1P)$ is available in the literature. Indeed, in the context of the QCD sum rule method mass and decay constant of $h(1P)$ were calculated in Ref. [47]. But the decay constant $f_{h_b'}$ of the meson $h_b(2P)$ was not evaluated; therefore, in the present work, we have first to find the parameters $m_{h_b'}$ and $f_{h_b'}$, and we shall turn after that to our main task.

4.1 Spectroscopic parameters of the mesons $h_b(1P)$ and $h_b(2P)$

The meson $h(1P)$ is the spin-singlet P-wave bottomonium with quantum numbers $J^{PC} = 1^{+-}$, whereas $h(2P)$ is its first radial excitation. The parameters of the $h_b(1P)$ and $h_b(2P)$ mesons in the framework of QCD two-point sum rule method can be extracted from the correlation function

$$\Pi_{\mu\nu\alpha\beta}(p) = i\int d^4x e^{ipx}\left\langle 0\left|\mathcal{T}\left\{J^h_{\mu\nu}(x)J^{h\dagger}_{\alpha\beta}(0)\right\}\right|0\right\rangle, \quad (34)$$

where the interpolating current for $h_b(mP)$ mesons is chosen as

$$J^h_{\mu\nu}(x) = \overline{b}^i(x)\sigma_{\mu\nu}\gamma_5 b^i(x). \quad (35)$$

It couples both to $h_b(1P)$ and $h_b(2P)$, and is convenient for the analysis of $J^{PC} = 1^{+-}$ mesons (see Ref. [47]).

In order to find the required sum rules we use the "ground-state+first radial excitation+continuum" scheme. Then the physical side of the sum rule,

$$\Pi^{\rm Phys}_{\mu\nu\alpha\beta}(p) = \frac{\left\langle 0\left|J^h_{\mu\nu}|h_b(p)\rangle\langle h_b(p)|J^{h\dagger}_{\alpha\beta}(0)\right|0\right\rangle}{m^2_{h_b}-p^2} + \frac{\left\langle 0|J^h_{\mu\nu}\left|h'_b(p)\rangle\langle h'_b(p)|J^{h\dagger}_{\alpha\beta}(0)\right|0\right\rangle}{m^2_{h'_b}-p^2} + \ldots, \quad (36)$$

contains two terms of interest and also a contribution of higher resonances and continuum states, denoted by dots. We continue by introducing the matrix elements

$$\langle 0|J^h_{\mu\nu}|h^{(\prime)}_b(p)\rangle = f_{h^{(\prime)}_b}(\varepsilon^{(\prime)}_\mu p_\nu - \varepsilon^{(\prime)}_\nu p_\mu), \quad (37)$$

and we recast the correlation function $\Pi^{\rm Phys}_{\mu\nu\alpha\beta}(p)$ into the form

$$\Pi^{\rm Phys}_{\mu\nu\alpha\beta}(p) = \frac{f^2_{h_b}}{m^2_{h_b}-p^2}\left[\widetilde{g}_{\mu\alpha}p_\nu p_\beta - \widetilde{g}_{\mu\beta}p_\nu p_\alpha - \widetilde{g}_{\nu\alpha}p_\mu p_\beta + \widetilde{g}_{\nu\beta}p_\mu p_\alpha\right] + \frac{f^2_{h'_b}}{m^2_{h'_b}-p^2}\left[\widetilde{g}'_{\mu\alpha}p_\nu p_\beta - \widetilde{g}'_{\mu\beta}p_\nu p_\alpha - \widetilde{g}'_{\nu\alpha}p_\mu p_\beta + \widetilde{g}'_{\nu\beta}p_\mu p_\alpha\right], \quad (38)$$

where

$$\widetilde{g}^{(\prime)}_{\mu\alpha} = -g_{\mu\alpha} + \frac{p_\mu p_\alpha}{m^2_{h^{(\prime)}_b}}.$$

The Borel transformation of $\Pi^{\rm Phys}_{\mu\nu\alpha\beta}(p)$ can be obtained by simple replacements in Eq. (38):

$$\mathcal{B}\frac{f^2_{h^{(\prime)}_b}}{m^2_{h^{(\prime)}_b}-p^2} = f^2_{h^{(\prime)}_b}e^{-m^2_{h^{(\prime)}_b}/M^2}.$$

The expression obtained in this way contains numerous Lorentz structures which, in general, may be employed to derive the sum rules for masses and decay constants: We choose a structure $\sim \widetilde{g}_{\mu\alpha}p_\nu p_\beta$ to extract sum rules. The term with the same structure should be isolated in the Borel transformation of $\Pi^{\rm QCD}_{\mu\nu\alpha\beta}(p)$, i.e. in the expression of the correlation function calculated using the quark–gluon degrees of freedom.

After simple computations for $\Pi^{\rm QCD}_{\mu\nu\alpha\beta}(p)$ we get

$$\Pi^{\rm QCD}_{\mu\nu\alpha\beta}(p) = i\int d^4x e^{ipx}{\rm Tr}\left[\gamma_5\sigma_{\alpha\beta}S^{ji}_b(-x) \times\sigma_{\mu\nu}\gamma_5 S^{ij}_b(x)\right]. \quad (39)$$

The following operations are standard manipulations; they imply the Borel transforming of $\Pi^{\rm QCD}_{\mu\nu\alpha\beta}(p)$, equating the structures $\sim \widetilde{g}_{\mu\alpha}p_\nu p_\beta$ in both the physical and the QCD sides of the equality obtained, and subtracting the continuum contribution. We obtain the second sum rule by acting on the first one by $d/d(-1/M^2)$. These two sum rules allow us to evaluate the masses and decay constants of the $h_b(1P)$ and $h_b(2P)$ mesons. At the first stage we employ the "ground-state +continuum" scheme, which is commonly used in sum rule computations. This means that we include the excited $h_b(2P)$ meson into the "higher resonances and continuum" part of sum rules and fix working windows for M^2 and s_0. From these sum rules we extract spectroscopic parameters of the $h_b(1P)$ meson m_{h_b} and f_{h_b}. At the next step we employ the same sum rules with $s_0^* > s_0$ to include the contribution arising from $h_b(2P)$, and we treat m_{h_b} and f_{h_b} evaluated at the first stage as fixed parameters.

Numerical analysis restricts the variation of the parameters M^2 and s_0 within the regions

$$M^2 = 10-12~{\rm GeV}^2,\ s_0 = 103-105~{\rm GeV}^2,$$

and we find

$$m_{h_b} = 9886^{+81}_{-78}~{\rm MeV},\ f_{h_b} = 325^{+61}_{-57}~{\rm MeV}. \quad (40)$$

At the next step we use

$$s_0^* = 109-111~{\rm GeV}^2$$

and get

$$m_{h'_b} = 10331^{+108}_{-117}~{\rm MeV},\ f_{h'_b} = 286^{+58}_{-53}~{\rm MeV}. \quad (41)$$

The parameters of the $h_b(2P)$ meson are among essentially new results of the present work; therefore, in Figs. 3 and 4 we demonstrate $m_{h_b(2P)}$ and $f_{h_b(2P)}$ as functions of the Borel parameter M^2 and the continuum threshold s_0.

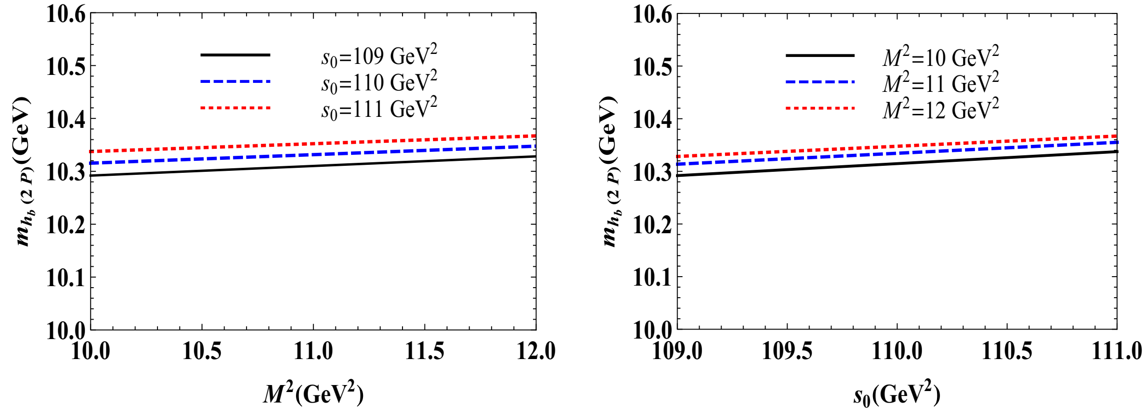

Fig. 3 The mass of the meson $h_b(2P)$ as a function of the Borel parameter M^2 at fixed s_0 (left panel), and as a function of the continuum threshold s_0 at fixed M^2 (right panel)

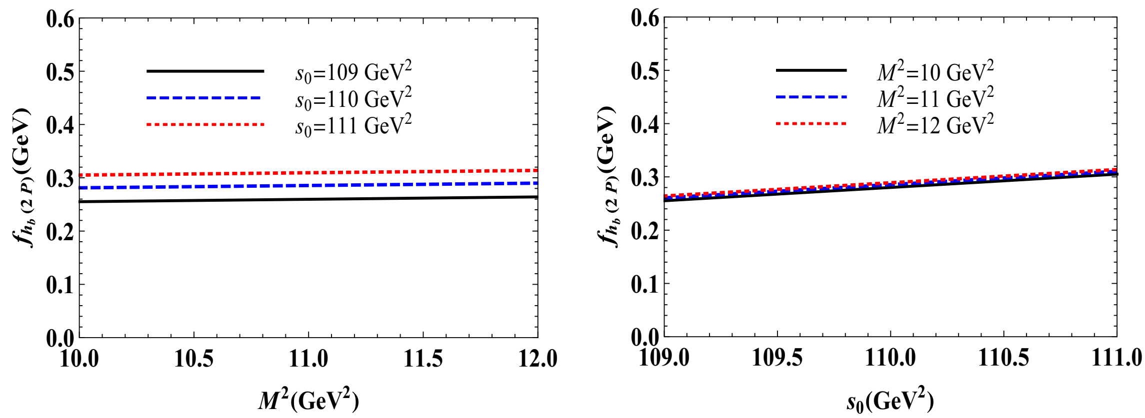

Fig. 4 The dependence of the decay constant $f_{h_b(2P)}$ on the Borel parameter at chosen values of s_0 (left panel), and on s_0 at fixed M^2 (right panel)

Comparing our results with experimental information on the masses of the $h_b(mP)$ mesons [46]

$$m_{h_b} = 9899.3 \pm 0.8 \text{ MeV},$$
$$m_{h_b'} = 10259.8 \pm 1.2 \text{ MeV},$$

we see a reasonable agreement between them.

4.2 Widths of decays $Z_b \to h_b(1P)\pi$ and $Z_b \to h_b(2P)\pi$

Analysis of the vertices $Z_b h_b(mP)\pi$ does not differ from the analogous investigation carried out in the previous section. We start here from the correlator

$$\Pi_{\mu\nu\lambda}(p,q) = i\int d^4x e^{ipx}\langle\pi(q)|\mathcal{T}\{J^h_{\mu\nu}(x)J^{Z_b\dagger}_\lambda(0)\}|0\rangle, \tag{42}$$

and for its phenomenological representation get

$$\Pi^{\text{Phys}}_{\mu\nu\lambda}(p,q) = \frac{\langle 0\,|J^h_{\mu\nu}|\,h_b\,(p)\rangle}{p^2 - m^2_{h_b}}\langle h_b\,(p)\,\pi(q)|Z_b(p')\rangle \times \frac{\left\langle Z_b(p')|J^{Z_b\dagger}_\lambda|0\right\rangle}{p'^2 - m^2_{Z_b}} + \frac{\langle 0|J^h_{\mu\nu}|h'_b\,(p)\rangle}{p^2 - m^2_{h'_b}} \times \langle h'_b\,(p)\,\pi(q)|Z_b(p')\rangle\frac{\left\langle Z_b(p')|J^{Z_b\dagger}_\lambda|0\right\rangle}{p'^2 - m^2_{Z_b}}\ldots. \tag{43}$$

$\Pi^{\text{Phys}}_{\mu\nu\lambda}(p,q)$ contains two terms of interest and contributions coming from higher resonances and continuum shown above as dots. Using matrix elements of the currents $J^h_{\mu\nu}$ and $J^{Z_b}_\lambda$ and introducing the vertex

$$\left\langle h^{(\prime)}_b\,(p)\,\pi(q)|Z_b(p')\right\rangle = g_{Z_b h^{(\prime)}_b \pi}\epsilon_{\alpha\beta\gamma\delta}\varepsilon^*_\alpha(p)\varepsilon'_\beta(p')p_\gamma p'_\delta, \tag{44}$$

we find

$$\Pi^{\rm Phys}_{\mu\nu\lambda}(p,q) = \frac{f_{Z_b} m_{Z_b}}{\left(p'^2 - m^2_{Z_b}\right)} \left[\frac{g_{Z_b h_b \pi} f_{h_b}}{\left(p^2 - m^2_{h_b}\right)} + \frac{g_{Z_b h'_b \pi} f_{h'_b}}{\left(p^2 - m^2_{h'_b}\right)} \right] \times \left(\epsilon_{\mu\lambda\gamma\delta} p_\gamma p'_\delta p_\nu - \epsilon_{\nu\lambda\gamma\delta} p_\gamma p'_\delta p_\mu\right) + \ldots \quad (45)$$

The same correlation function expressed in terms of quark propagators takes the following form:

$$\Pi^{\rm QCD}_{\mu\nu\lambda}(p,q) = \int d^4 x e^{ipx} \frac{\epsilon\widetilde{\epsilon}}{\sqrt{2}} \left[\gamma_5 \widetilde{S}^{ib}_b(x)\gamma_5\sigma_{\mu\nu} \times \widetilde{S}^{ei}_b(-x)\gamma_\lambda + \gamma_\lambda \widetilde{S}^{ib}_b(x)\gamma_5\sigma_{\mu\nu}\widetilde{S}^{ei}_b(-x)\gamma_5\right]_{\alpha\beta} \times \left\langle \pi(q)|\overline{u}^a_\alpha(0) d^d_\beta(0)|0\right\rangle. \quad (46)$$

Expanding $\overline{u}^a_\alpha d^d_\beta$ in accordance with Eq. (24) and substituting into Eq. (46) the local matrix elements of the pion we obtain $\Pi^{\rm QCD}_{\mu\nu\lambda}(p,q)$, which can be matched to $\Pi^{\rm Phys}_{\mu\nu\lambda}(p,q)$ to fix the same tensor structures. In order to derive the sum rule we use the structures $\sim \epsilon_{\mu\lambda\gamma\delta} p_\gamma p'_\delta p_\nu$ from both sides of the equality. The pion matrix element that contributes to this structure is

$$0|\overline{d}(0)\gamma_5\gamma_\mu u(0)|\pi(q)\rangle = i f_\pi q_\mu.$$

In fact, it can be included into the chosen structure after replacement $q_\mu = p'_\mu - p_\mu$.

In the equality obtained we apply the soft limit $q \to 0$ $(p = p')$ and perform the Borel transformation on the variable p^2. This operation leads to a sum rule for the two strong couplings $g_{Z_b h_b \pi}$ and $g_{Z_b h'_b \pi}$. The second expression is obtained from the first one by applying the operator $d/d(-1/M^2)$.

The principal output of these calculations, i.e. the spectral density $\rho^{\rm QCD}_{\rm h}(s)$, reads

$$\rho^{\rm QCD}_{\rm h}(s) = \frac{f_\pi}{12\sqrt{2}} \left[\rho^{\rm pert.}(s) + \rho^{\rm n.-pert.}(s)\right], \quad (47)$$

where its perturbative part is given by the formula

$$\rho^{\rm pert.}(s) = \frac{(s+2m_b^2)\sqrt{s(s-4m_b^2)}}{\pi^2 s^2}. \quad (48)$$

The nonperturbative component of $\rho^{\rm QCD}_{\rm h}(s)$ includes contributions up to eight dimensions and has the form

$$\rho^{\rm n.-pert.}(s) = \left\langle \frac{\alpha_s G^2}{\pi}\right\rangle m_b^2 \int_0^1 f_1(z,s){\rm d}z + \left\langle g_s^3 G^3\right\rangle \int_0^1 f_2(z,s){\rm d}z - \left\langle \frac{\alpha_s G^2}{\pi}\right\rangle^2 m_b^4 \int_0^1 f_3(z,s){\rm d}z. \quad (49)$$

Here the functions $f_k(z,s)$ are

$$f_1(z,s) = \frac{1}{3}\frac{(1+3r)}{r^2}\delta^{(2)}(s-\Phi),$$
$$f_2(z,s) = \frac{1}{15\cdot 2^6}\frac{1}{r^4}\Big\{4r^2\left(3+17r+21r^2\right)\delta^{(2)}(s-\Phi) +2r\left[sr^2(4+13r)+3m_b^2(3+16r+18r^2)\right]\delta^{(3)}(s-\Phi) +\left[s^2r^4+6m_b^2 sr^2(1+3r) -7m_b^4\left(1+5r+5r^2\right)\right] \times\delta^{(4)}(s-\Phi)\Big\},$$
$$f_3(z,s) = \frac{1}{54}\frac{\pi^2}{r^2}\delta^{(5)}(s-\Phi),$$

where

$$r = z(z-1),\ \ \Phi = \frac{m_b^2}{z(1-z)}.$$

In the expressions above the Dirac delta function $\delta^{(n)}(s-\Phi)$ is defined in accordance with

$$\delta^{(n)}(s-\Phi) = \frac{d^n}{ds^n}\delta(s-\Phi). \quad (50)$$

The widths of the decays $Z_b \to h_b(1P)\pi$ and $Z_b \to h_b(2P)\pi$ are calculated using the formula

$$\Gamma(Z_b \to h_b(mP)\pi) = g^2_{Z_b h_b(mP)\pi} \frac{\lambda\left(m_{Z_b},\ m_{h(mP)}, m_\pi\right)^3}{12\pi}.$$

In our numerical computations we employ the parameters of the $h_b(mP)$ mesons obtained in the previous subsection. The working regions of the Borel parameter M^2 and continuum threshold s_0 are the same as in the analysis of the $Z_b \to \Upsilon(nS)\pi$ decays. Below we provide our results for the strong couplings (in units GeV^{-1})

$$g_{Z_b h_b \pi} = 0.94 \pm 0.27,\ \ g_{Z_b h'_b \pi} = 3.43 \pm 0.93. \quad (51)$$

In Fig. 5 we plot the coupling $g_{Z_b h'_b \pi}$ as a function of the Borel parameter and continuum threshold to show its dependence on these auxiliary parameters. It is easy to see that theoretical errors are within the limits accepted in sum rule calculations.

Using Eq. (51) it is not difficult we evaluate the widths of the decays:

$$\Gamma(Z_b \to h_b(1P)\pi) = 6.30 \pm 1.76\ {\rm MeV},$$
$$\Gamma(Z_b \to h_b(2P)\pi) = 7.35 \pm 2.13\ {\rm MeV}. \quad (52)$$

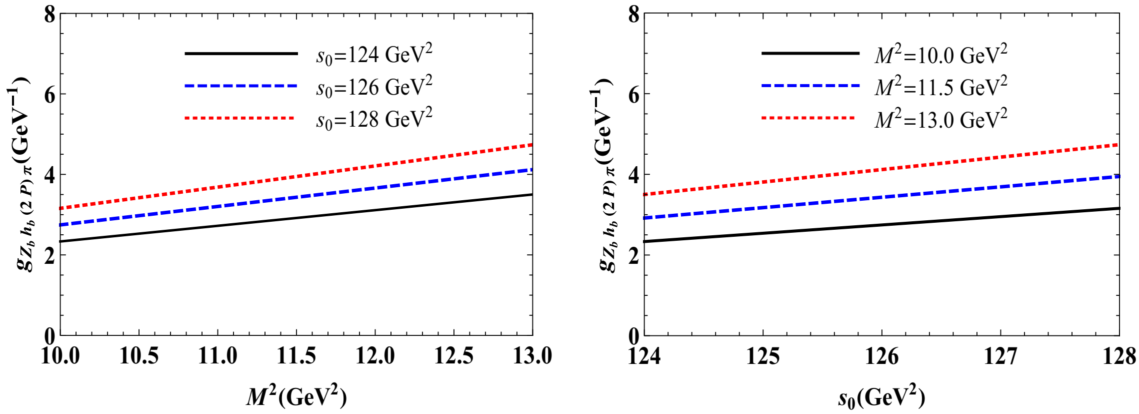

Fig. 5 The coupling $g_{Z_b h_b' \pi}$ vs. Borel parameter M^2 (left panel), and continuum threshold s_0 (right panel)

5 Analysis and concluding notes

The experimental data on the decay channels of the $Z_b(10610)$ resonance were studied and presented in a rather detailed form in Refs. [9–11]. Its full width was estimated as $\Gamma = 18.4 \pm 2.4$ MeV, an essential part of which, i.e. approximately 86% of Γ, is due to the decay $Z_b \to B^+\overline{B}^{*0} + B^{*+}\overline{B}^0$. The remaining part of the full width is formed by five decay channels investigated in the present work. It is clear that our results for the widths of the decays $Z_b \to \Upsilon(nS)\pi$ and $Z_b \to h_b(nP)\pi$ overshoot the experimental data. Therefore, in the light of the present studies we refrain from an interpretation of the $Z_b(10610)$ resonance as a pure diquark–antidiquark $[bu][\overline{bd}]$ state.

Nevertheless, theoretical predictions are encouraging for the ratios

$$\mathcal{R}(n) = \frac{\Gamma(Z_b \to \Upsilon(nS)\pi)}{\Gamma(Z_b \to \Upsilon(1S)\pi)}, \mathcal{R}(m) = \frac{\Gamma(Z_b \to h_b(mS)\pi)}{\Gamma(Z_b \to \Upsilon(1S)\pi)}, \tag{53}$$

where we normalize the widths of the different decay channels to $\Gamma(Z_b \to \Upsilon(1S)\pi)$. The ratio $\mathcal{R}$ can be extracted from the available experimental data and can be calculated from the decay widths obtained in the present work. In order to fix existing similarities and differences between theoretical and experimental information on $\mathcal{R}$ we provide two sets of corresponding values in Table 2. It is worth to note that we use the latest available experimental information from Ref. [46].

Table 2 Experimental values and theoretical predictions for $\mathcal{R}$

$\mathcal{R}$	$n = 2$	$n = 3$	$m = 1$	$m = 2$
Exp. [46]	$6.67^{+3.11}_{-2.37}$	$3.89^{+2.02}_{-1.55}$	$6.48^{+3.18}_{-2.45}$	$8.70^{+4.39}_{-3.41}$
This work	12.63 ± 5.43	6.08 ± 2.76	4.63 ± 1.95	5.40 ± 2.32

It is seen that theoretical predictions follow the pattern of the experimental data: we observe the same hierarchy of theoretical and experimental decay widths. At the time, numerical differences between them are noticeable. Nevertheless, as a result of the large errors in both sets, there are sizable overlap regions for each pair of $\mathcal{R}$s, which demonstrates not only qualitative agreement between them but also the quantitative compatibility of the two sets.

These observations may help one to understand the nature of the Z_b resonance. The Belle Collaboration discovered two Z_b and Z_b' resonances with very close masses. We have calculated the parameters of an axial-vector diquark–antidiquark state $[bu][\overline{bd}]$, and we interpreted it as Z_b. It is possible to model the second Z_b' resonance using an alternative interpolating current, as has been emphasized in Sect. 2, and we explore its properties. The current with the same quantum numbers but different color organization may also play a role of such alternative (see, for example, Ref. [44]). One of the possible scenarios implies that the observed resonances are admixtures of these tetraquarks, which may fit measured decay widths.

The diquark–antidiquark interpolating current used in the present work can be rewritten as a sum of molecular-type terms. In other words, some of the molecular-type currents effectively contribute to our predictions, and by enhancing these components (i.e. by adding them to the interpolating current with some coefficients) better agreement with the experimental data may be achieved. In other words, the resonances Z_b and Z_b' may "contain" both the diquark–antidiquark and the molecular components.

Finally, the Z_b and Z_b' states may have pure molecular structures. But pure molecular-type bound states of mesons are usually broader than diquark–antidiquarks with the same quantum numbers and quark contents. In any case, all these suggestions require additional and detailed investigations.

In the present study we have fulfilled only a part of this program. In the framework of QCD sum rule methods we have calculated the spectroscopic parameters of the Z_b state by modeling it as a diquark–antidiquark state, and we found the widths of five of its observed decay channels. We have also evaluated the mass and decay constant of te $h_b(2P)$ meson, which are necessary for analysis of the $Z_b \to h_b(2P)\pi$ decay. Calculation of the Z_b resonance's dominant decay channel may be performed, for example, using the QCD three-point sum rule approach, which is beyond the scope of the present work. The decays considered here involve the excited mesons $\Upsilon(nS)$ and $h(mP)$, the parameters of which require detailed analysis in the future. More precise measurements of Z_b and Z_b' partial decay widths can also help in making a choice between the scenarios outlined.

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