A RANKING METHOD FOR SELECTION OF η MESONS IN HIGH MULTIPLICITY EVENTS*

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The selection of η mesons with a high efficiency and a high purity can be important in the formation of statistically significant invariant mass spectra in the reconstruction of short-lived particles such as $\eta' \to \pi^+\pi^-\eta$. In this study, a cut-based standard method and a ranking method to reduce combinatorial background in the reconstruction of $\eta \to \gamma\gamma$ decays in high multiplicity hadronic events are presented. By using recorded ALEPH data and fully simulated events, the performances of the methods are compared. Results show that the ranking method yields significant improvements in the purity of the selected η meson relative to the standard method.

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1. Introduction

In particle collisions at high energies, one can have events containing a high multiplicity of hadronic particles. An example is the production of Z bosons from the decay channel $Z \to q\bar{q}$ at LEP, the e^-e^+ collider at $\sqrt{s} = 91.2$ GeV. Two or more jets of hadrons and other particles, on average 21 charged and 21 neutral particles per event, are produced by the hadronization of quarks and gluons in such decays [1].

Invariant mass spectra, built from selected final-state particles, can reveal the presence of short-lived 'mother' particles which present themselves as peaks on a combinatorial background. An example decay is $\eta'(958) \to$

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$\pi^+\pi^-\eta$. Here, the η' is reconstructed from measurements of the momenta of charged pions with good momentum resolution in the tracking chambers, and the η meson can be reconstructed from the two-photon channel $\eta(548) \to \gamma\gamma$ where the daughter photons are measured in the electromagnetic calorimeter typically with relatively poor energy resolution.

The selection of π^0 and η mesons in the two-photon decay mode can be a relatively simple task where candidates are selected from a mass window around the signal peaks. However, in environments where particle multiplicities are high, such as at LEP and LHC, further analysis and optimization can result in a higher selection purity and efficiency.

In this study, two methods for improving selection purity of η mesons are investigated in detail; a standard method and a ranking method. First, the standard method which is used to select signal candidates from a mass window around the η peak and to reject photons from neutral pion decays is described. Then, a probability density estimator, based on reconstructed kinematic parameters of the η mesons, for distinguishing backgrounds and signals, and the ranking method used for further improvement in the purity of reconstructed η mesons are presented respectively. Finally, using ALEPH Archived Data and Simulation [2], example applications of these two η-selection methods for improving signal significance of the decay channel $\eta' \to \pi^+\pi^-\eta$ are demonstrated at the end.

2. Event and track selection

In the event simulation, the decays $\pi^0 \to \gamma\gamma$, $\eta \to \gamma\gamma$, and $\eta' \to \pi^+\pi^-\eta$ are selected from e^-e^+ collision events representing simulations of hadronic Z decays at the LEP collider. The selected events are passed through the full simulation and the reconstruction program for the ALEPH detector [3] and so provide a realistic simulation of daughter momentum resolutions. Totally, 4,923,816 reconstructed events are used in the simulation studies.

Using the hadronic event selection criteria described in [4], a total of 3,239,746 hadronic Z decays around $\sqrt{s} = 91.2$ GeV recorded by ALEPH at LEP in the period between 1991 and 1995 are selected for the real data analysis.

In the physics analysis, unconverted photons with an energy greater than 0.8 GeV are selected. The reconstructed charged particles are required to have a polar angle in the range of $20° < \theta < 160°$ and a transverse momentum of at least 0.5 GeV.

3. Standard method

π^0 and η candidates are reconstructed by combining pairs of photons. The branching ratios of the decays $\pi^0 \to \gamma\gamma$ and $\eta \to \gamma\gamma$ are about 98.8%

and 39.4%, respectively [1]. The π^0 signal around 0.135 GeV and η signal around 0.548 GeV can be seen in two-photon invariant mass spectra, as shown in Fig. 1 (a). Due to its smaller mass and higher branching ratio, the π^0 multiplicity is much greater than that of the η; the π^0 and its combinatorial background, therefore, dominate the two-photon mass spectra making the η signal difficult to distinguish from the large combinatorial background.

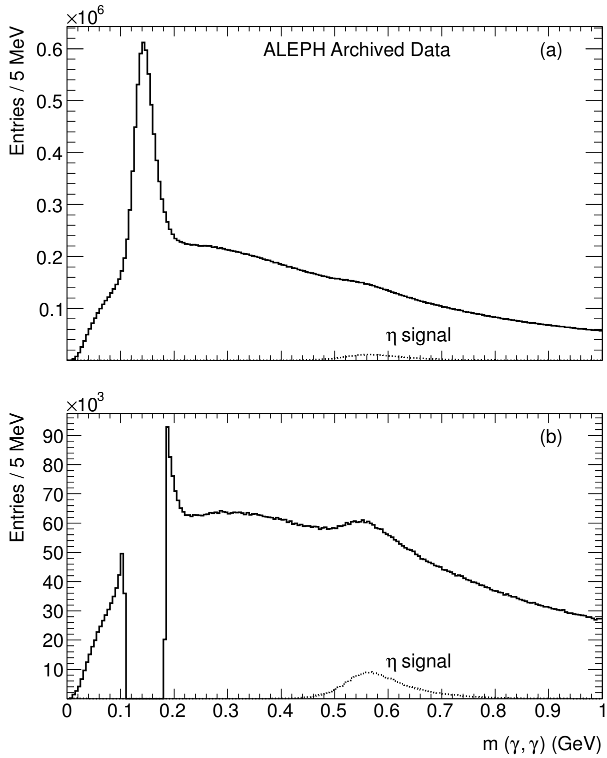

Fig. 1. Example of a two-photon invariant mass distribution (a) before and (b) after $\pm 2\sigma_{\rm r}$ rejection around the π^0 signal in simulation. The η signal is also shown by a dotted line under the full mass spectra. It is clear that the neutral pion rejection improves η-signal significance by reducing combinatorial background under the η signal in the case of (a).

π^0 and η candidates are directly selected from a mass window around the signal peaks. The selection of these particles with a high purity and high efficiency is important. In this study, we define the purity (P) and efficiency (E) as follows:

$$P = S/(S+B)\,, \tag{1}$$

$$E = S/S_0\,, \tag{2}$$

where S is the number of signal entries, B is the number of background entries within the given mass window and S_0 is the total number of signal entries within $\pm 6\sigma$ mass window. Here, σ is the mass resolution which, in this study, is defined as one standard deviation of the signal mass distribution in the simulation.

Note that, in the photon matching, a search for the best match between the generated and reconstructed photon candidates is performed based on the angular distance between reconstructed and truth photons. A pair with the smallest angular distance is considered to be the best match. To define the signal, S, two matched photons are combined if they originate from the same parent (η or π^0).

The poor purity of the η signal can be improved by rejecting photons that appear to originate from a π^0 decay (π^0 rejection). This is achieved by eliminating the photon pairs with an invariant mass within $\pm p\sigma_{\mathrm{r}}$ rejection mass window around the π^0 peak, where σ_{r} is the π^0 mass resolution and p is a real number. Figure 1 (b) shows the invariant mass spectra after $\pm 2\sigma_{\mathrm{r}}$ rejection around π^0 signal. The η peak, though significantly diminished, is much clearer due to the greatly reduced background after π^0 rejection.

After neutral pion rejection, η candidates can be selected from a mass window of size $\pm q\sigma_{\mathrm{s}}$ around the η peak. Here, σ_{s} is the η signal mass resolution, and q is a scale that directly effects the selection purity and efficiency. A narrow window (small q) will increase purity by selecting less background but reduce efficiency as it selects less signal. A narrow selection window will also tend to increase systematic errors when the mass spectra of data and simulation are not in a good agreement. The optimal value of q is, therefore, a balance between purity and efficiency. We define the optimization condition such that the product $E \times P$ is maximum[1].

This procedure was applied in Ref. [5] in order to extract the production rates of η and η' in hadronic Z decays by the ALEPH Collaboration using data collected during the 1990 and 1995 running period of LEP. They preferred the mass window scales of about $p = 1.5$ and $q = 1.5$ resulting in $P = 9.6\%$, $E = 71.0\%$ and $E \times P = 6.8\%$. However, in this study, the best scales are found to be $p = 2.0$ and $q = 1.5$, and the corresponding values for purity and efficiency are $P = 10.2\%$ and $E = 67.4\%$, respectively, giving the optimal product $E \times P = 6.9\%$. Although the statistical performance is very similar in the two cases, the wider selection window used in this study would favour lower systematic errors. The effect of varying p and q values will be discussed in Sec. 5.

In this study, the π^0 rejection and η-selection procedure defined above is called the 'standard method' which forms the starting point to study another method for improving the purity of η mesons.

[1] The product $E \times P$ is closely related to signal-to-noise ratio which is defined as $S/N = S/\sqrt{S+B}$. Thus, $E \times P = (S/N)^2/S_0$.

4. η estimator

An estimator obtained from kinematic properties of the η signal can be defined to discriminate between the η signal and background. In this study, three discriminating variables are used as the η estimator; the opening angle between photon pairs (θ_{12}), invariant mass of the pairs (M_{12}) and total energy of the pairs (E_{12}).

The top row of Fig. 2 shows the distribution of these variables in simulation for $p = 0$ and $q = 4$. The signal (solid line) and background (dotted line) components are shown separately. In the bottom row, for each variable, a purity histogram is obtained from the ratio of the signal to the sum of signal and background histograms. Each purity histogram is then fitted with a suitable function. It is clear that the signal tends to have smaller values of opening angles, closer mass values to the nominal mass (0.548 GeV) and larger energy values than the background.

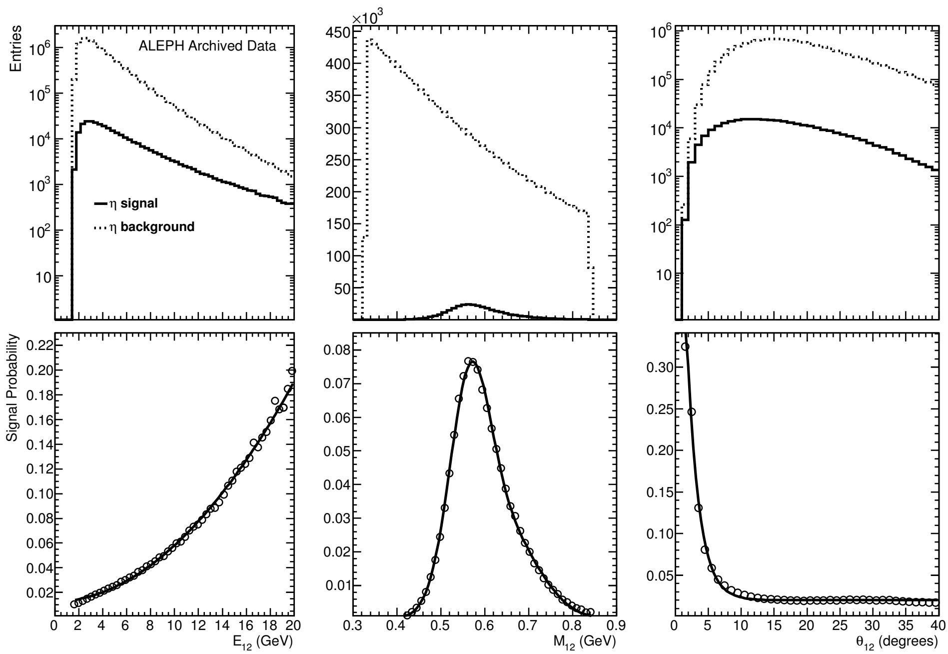

Fig. 2. Distributions of discriminating variables used in the study (top row) and their corresponding purity distributions (bottom row) indicated by open circles in simulation. For each variable, the purity histogram is obtained from the ratio of the signal (solid line) to the sum of signal and background (dotted line) histograms. Each purity histogram is fitted with a suitable function such that χ^2/n.d.f. < 1.

A probability density estimator (PDE) for η candidates can be built from the product of these three purity (or probability) values evaluated from the best fit functions. Hence the PDE function is given by

$$e_{\rm PDE}(\theta_{12}, M_{12}, E_{12}) = f_1(\theta_{12}) f_2(M_{12}) f_3(E_{12})\,, \tag{3}$$

where the explicit forms of the fit functions are respectively as follows:

$$f_1(\theta_{12}) = a_0 + a_1 e^{a_2\theta_{12}}\,, \tag{4}$$
$$f_2(M_{12}) = a_3 e^{-((M_{12}-a_4)/a_5)^2/2} + a_6 e^{-((M_{12}-a_7)/a_8)^2/2}\,, \tag{5}$$
$$f_3(E_{12}) = a_9 + a_{10}E_{12} + a_{11}E_{12}^2\,. \tag{6}$$

Here, a_i are free parameters obtained from the fitting procedure.

Figure 3 shows the distribution of the PDE values for the signal (solid line) and the background (dotted line). As expected, η candidates that are formed from η decays (the signal) tend to have larger PDE values than η candidates formed from the combinatorial background. Equation (3) can, therefore, be used to attempt to distinguish between correct and wrong combinations of photon pairs.

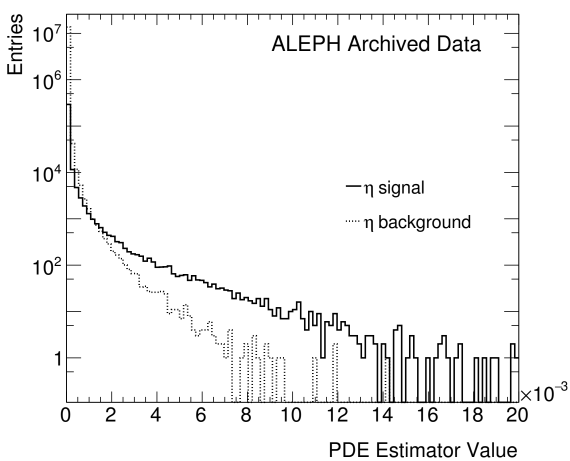

Fig. 3. Distributions of estimator values obtained from the PDE method for discriminating variables, Eq. (3), for the η signal (solid line) and the background (dotted line) components. Candidates are selected from a large window ($q = 4$) with no pion rejection ($p = 0$).

Advanced multivariate classifiers such as boosted decision trees and neural networks have become increasingly popular in particle physics due to their superior discrimination capabilities and growth of available computing power in recent years. In order to compare with the PDE, we use the boosted decision trees (BDT) method with 600 trees[2] in the Toolkit for MultiVariate data Analysis with ROOT (TMVA) [6]. The distribution of estimator values obtained from the BDT method is shown in Fig. 4. η signals, on average, have larger estimator values than background as in the PDE method.

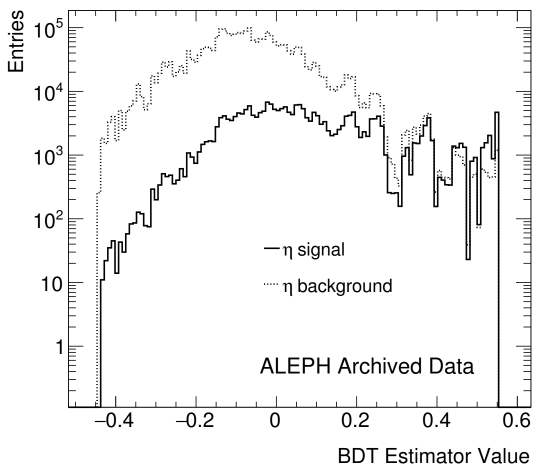

Fig. 4. Distributions of estimator values obtained from the BDT method for discriminating variables. Candidates are selected from a large window ($q = 4$) with no pion rejection ($p = 0$).

Note that the compact analytical form of the estimator function for this case, $e_{\text{BDT}}(\theta_{12}, M_{12}, E_{12})$, cannot be written explicitly as in Eq. (3). Instead, it is evaluated using the parameterized output file obtained from BDT training. It is also clear that BDT (and neural networks) can capture dependencies between the variables, while the PDE method implicitly assumes that the variables are independent.

5. Ranking method

Further improvement in η purity is achieved by applying a ranking method to the remaining candidates after the initial mass window cuts. The ranking method with a relatively crude estimator and its performance for the neutral pion selection are described in detail elsewhere [7][3]. In this paper, the method is improved by using more sophisticated estimators and applied to the $\eta \to \gamma\gamma$ channel for the first time. A summary of the method is as follows:

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[2] 600 trees are found to be optimum to reduce background candidates maximally while saving more signal.

[3] In Ref. [7], only simulated events were used to generate hadronic decays of the Z boson at various center-of-mass energies.

1. An estimator value is assigned for each η candidate in an event according to the values from the estimator function defined in Sec. 4.

2. Candidates are then ranked in decreasing order of estimator (true ηs are most likely to be nearer the top of the list).

3. Candidates are removed from the list if their estimator values are less than a predefined value of an estimator cut, $e_{\rm cut}$ (for this study, the optimal values corresponding to maximum $E \times P$ are found to be $e_{\rm cut} = 2.5 \times 10^{-5}$ for the PDE method and $e_{\rm cut} = 0.0$ for the BDT method).

4. A scan is then made through the list for pairs of η candidates which share photons. If there exists such a pair, the candidate with the smallest estimator value is removed from the list.

An example application of the ranking method in a simulated event containing ten η candidates selected for $p = 2$ and $q = 4$ is shown below. η candidates are labeled with integers from 1 to 10 and photons are represented by integers from 21 to 29.

Candidates before ranking:

η-signal candidate	Estimator value	Photon #1	Photon #2	Truth info.
1	1.313e−03	22	24	True
2	4.148e−04	26	28	False
3	3.381e−04	22	25	False
4	1.276e−04	21	28	False
5	1.252e−04	23	25	False
6	3.726e−05	23	26	True
7	2.535e−05	23	29	False
8	1.831e−05	21	23	False
9	8.132e−06	24	25	False
10	5.654e−06	22	29	False

According to the ranking algorithm above, the last three candidates in the list must be removed since their estimator values are less than $e_{\rm cut} = 2.5 \times 10^{-5}$. η candidate 1 removes candidate 3 from the list as they both share photon 22. Similarly, candidate 2 removes candidate 4, and candidate 5 removes candidates 6 and 7. Although the $6^{\rm th}$ candidate is a *signal*, it is removed from the list since it shares the photon 23 with a background (candidate 5) which has, by chance, a larger estimator value. As a result, the list of selected candidates after ranking is as follows:

η-signal candidate	Estimator value	Photon #1	Photon #2	Truth info.
1	1.313e−03	22	24	True
2	4.148e−04	26	28	False
5	1.252e−04	23	25	False

After ranking, one out of two true candidates is selected, whereas six out of eight background candidates are rejected. On average, ranking improves the selection purity of η mesons with a reduction in selection efficiency.

6. Performance

To investigate the performance of the ranking method and the standard method, the width of the π^0 rejection window (controlled by the scale p) and the width of the η selection mass window (controlled by the scale q) are varied. The selection efficiency and purity, and their product, are then used to compare the performance of the two methods. Note that the selection efficiency of the η signal is calculated for $p = 0$ and $q = 6$.

Detailed results of the study is shown in Table I. For both methods, improvements in purity and the product of purity and efficiency are evident with respect to $p = 0$. However, the ranking method using either PDE or BDT significantly improves the selection purity of η signals compared to the standard method for all values of p and q. The maximum products are obtained as $E \times P = 7.33\%$ and $E \times P = 8.04\%$ in the ranking method with the PDE method and the BDT method, respectively, for $p = q = 2$.

Figure 5 shows the efficiency, the purity and the product values as a function of q for the fixed value of $p = 2$. The ranking method with PDE (full circles) and with BDT (stars) results in higher purity and products values with loss of efficiency relative to the standard method (open circles) at each point. It is clear that ranking with PDE exhibits a close performance to that of ranking with BDT which appears to exhibit the best performance.

7. Case study

η-selection methods described in the previous sections are applied to the decay channel $\eta' \to \pi^+\pi^-\eta$, (BR $= 42.9 \pm 0.7\%$) [1]. In ALEPH, charged pions are measured in the tracking chamber with a good momentum resolution. However, the reconstructed momentum resolution of $\eta \to \gamma\gamma$ decays is very poor since the energy resolution of the electromagnetic calorimeter for unconverted photons is worse. In order to improve the momentum resolution of η candidates, the reconstructed mass of photon pairs is constrained to the nominal η mass using a fast method described in [8].

TABLE I

Effect of varying the mass window scales p and q on the selection efficiency and purity of η candidates using the standard method (STD) and the ranking method with PDE and with BDT. The number of η signal (S) and background (B) candidates are given as well.

Method	p	q	S	B	E [%]	P [%]	$E \times P$
STD	0	1	222255	3321452	68.5	6.3	4.30
STD	0	2	301772	6690397	93.0	4.3	4.01
STD	0	3	318722	10147620	98.2	3.0	2.99
STD	0	4	322604	13707263	99.4	2.3	2.29
STD	1	1	196960	1801165	60.7	9.9	5.98
PDE	1	1	125242	604917	38.6	17.2	6.62
BDT	1	1	110752	423415	34.1	20.7	7.08
STD	1	2	267259	3611326	82.4	6.9	5.68
PDE	1	2	132501	677472	40.8	16.4	6.68
BDT	1	2	150675	815142	46.4	15.6	7.24
STD	1	3	282665	5434268	87.1	4.9	4.31
PDE	1	3	132430	685707	40.8	16.2	6.61
BDT	1	3	181834	1301457	56.0	12.3	6.87
STD	1	4	286337	7256465	88.2	3.8	3.35
PDE	1	4	132426	686326	40.8	16.2	6.60
BDT	1	4	197262	1642840	60.8	10.7	6.52
STD	2	1	176513	1279482	54.4	12.1	6.60
PDE	2	1	113410	432452	35.0	20.8	7.26
BDT	2	1	103029	318512	31.8	24.4	7.76
STD	2	2	239710	2558780	73.9	8.6	6.33
PDE	2	2	120335	488306	37.1	19.8	7.33
BDT	2	2	139364	605411	43.0	18.7	8.04
STD	2	3	253798	3837930	78.2	6.2	4.85
PDE	2	3	120277	495194	37.1	19.5	7.24
BDT	2	3	167943	966310	51.8	14.8	7.66
STD	2	4	257260	5101272	79.3	4.8	3.81
PDE	2	4	120274	495755	37.1	19.5	7.24
BDT	2	4	190039	1381700	58.6	12.1	7.08
STD	3	1	160462	1074061	49.5	13.0	6.43
PDE	3	1	103425	368805	31.9	21.9	6.98
BDT	3	1	95637	271265	29.5	26.1	7.68
STD	3	2	217820	2146755	67.1	9.2	6.18
PDE	3	2	109868	418822	33.9	20.8	7.04
Bdt	3	2	131311	534437	40.5	19.7	7.98
STD	3	3	230832	3216074	71.1	6.7	4.76
PDE	3	3	109814	425257	33.8	20.5	6.95
BDT	3	3	161957	898342	49.9	15.3	7.62
STD	3	4	234102	4269366	72.1	5.2	3.75
PDE	3	4	109811	425806	33.8	20.5	6.94
BDT	3	4	176812	1144304	54.5	13.4	7.29

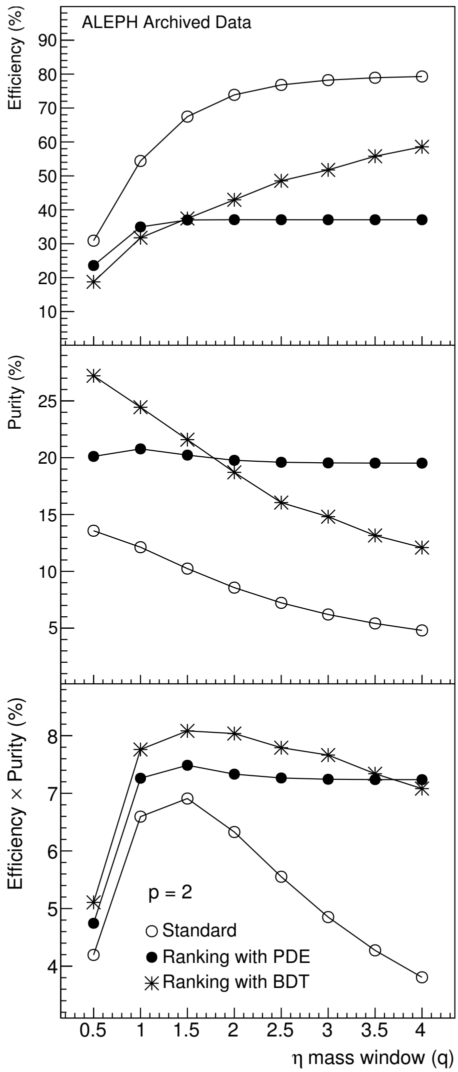

Fig. 5. Effect of varying q on η selection efficiency, purity and their product values when applying the standard method (open circles) and the ranking method with PDE (full circles) and with BDT (stars) for the fixed value of $p = 2$. The selection efficiency of η-signal candidates is calculated with respect to mass window cuts corresponding to $p = 0$ and $q = 6$. The error bars at each point are much smaller than the marker sizes.

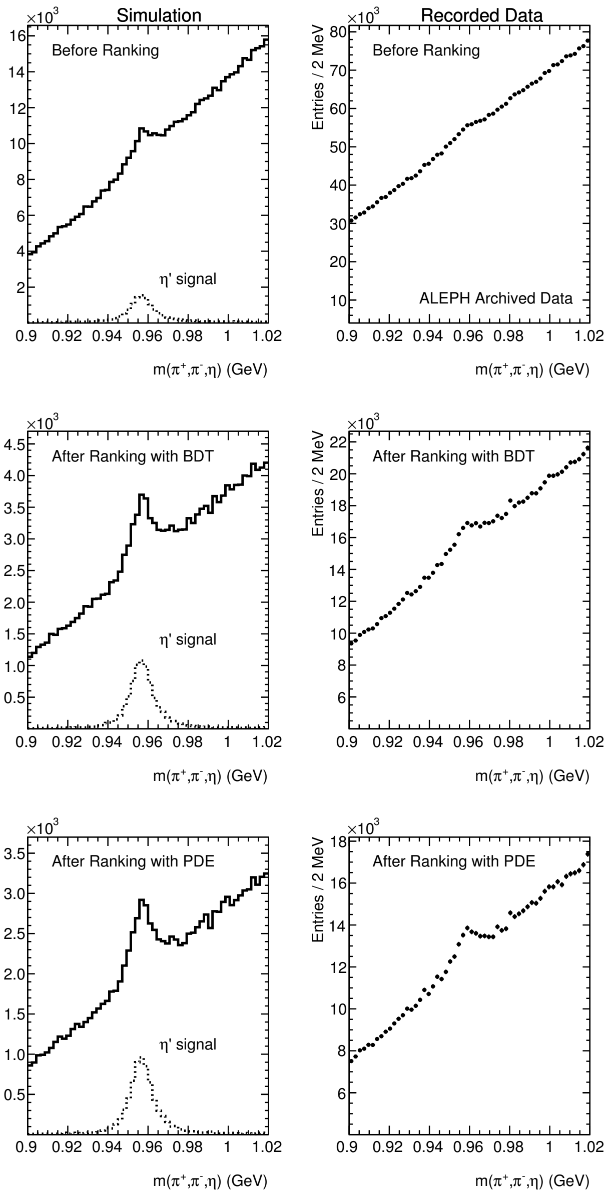

Fig. 6. Invariant mass spectra of π^+, π^- and η candidates selected from hadronic Z decays. An η' signal is seen around 0.958 GeV. Before ranking, the standard method is applied for $p = q = 2$. After ranking (with PDE or with BDT), the signal appears much clearer for both simulated (left column) and recorded ALEPH data (right column).

Figure 6 shows example mass spectra for simulated and recorded ALEPH data. Before ranking, the standard method is applied such that η candidates are selected for the case corresponding to[4] $p = q = 2$. Then, applying the ranking method (with PDE and BDT estimators) to remove some combinatorial background (for the same mass window cuts), results in a clearer η' signal around 0.958 GeV. Evidently, the ranking methods improve the statistical significance of the η' signal, and can be expected to improve the fit stability when fitting mass spectra whose background is very large.

As an alternative, we have also tried to use the PDE and BDT methods without ranking. For this case, estimator cuts are optimized separately for the analysis with and without ranking. The performance, however, is found to be significantly worse.

8. Summary and conclusion

Reconstruction of η mesons plays a dominant role in the resultant purity and momentum resolution of reconstructed mother particles. The purity of η signals, for the decay channel $\eta \to \gamma\gamma$, is very poor due to the large combinatorial background in high multiplicity hadronic events. In this study, two methods for the improvement of selection purity of η mesons are presented; the cut-based standard method and the ranking method. The ranking method results in significant improvements in η purity in high multiplicity events. However, it does not perform well if an event contains a small number of η candidates (such as one or two). In addition, the ranking method using a PDE exhibits a close performance to that of the ranking using a BDT which appears to exhibit the best performance.

The methods are applied in the reconstruction of the decay $\eta' \to \pi^+\pi^-\eta$. It is found that the selection of η candidates with a higher purity using the ranking method improves the significance of the η' signal relative to the standard selection method.

Finally, the authors suggest that the methods discussed here can also be employed to extract short lived particles, whose systematic errors are dominated by uncertainties arising from the fitting procedure, such as $J/\psi \to \pi^+\pi^-\eta$ and $D^0 \to \eta\eta$ in proton–proton collision events at the LHC where particle multiplicities are very high.

The authors wish to thank the ALEPH Collaboration for access to the archived data since the closure of the collaboration [2].

[4] Although optimum values are $p = 2$ and $q = 1.5$, we have used $p = q = 2$ to reduce possible systematic uncertainties originating from mass window cuts.

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