



Magnetic moments of the lowest-lying singly heavy baryons

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ABSTRACT

A light baryon is viewed as N_c valence quarks bound by meson mean fields in the large N_c limit. In much the same way a singly heavy baryon is regarded as $N_c - 1$ valence quarks bound by the same mean fields, which makes it possible to use the properties of light baryons to investigate those of the heavy baryons. A heavy quark being regarded as a static color source in the limit of the infinitely heavy quark mass, the magnetic moments of the heavy baryon are determined entirely by the chiral soliton consisting of a light-quark pair. The magnetic moments of the baryon sextet are obtained by using the parameters fixed in the light-baryon sector. In this mean-field approach, the numerical results of the magnetic moments of the baryon sextet with spin $3/2$ are just $3/2$ larger than those with spin $1/2$. The magnetic moments of the bottom baryons are the same as those of the corresponding charmed baryons.

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1. Introduction

Very recently, Ref. [1] showed that when the number of colors (N_c) goes to infinity singly heavy baryons can be described as $N_c - 1$ valence quarks bound by the meson mean fields that also have portrayed light baryons as N_c valence quarks bound by the same mean fields [2,3], being motivated by Diakonov [4]. The masses of the lowest-lying singly heavy baryons were well reproduced in both the charmed and bottom sectors, and the mass of the Ω_b was predicted within this framework. Using the method developed in Ref. [1], we were able to interpret two narrow Ω_c resonances as exotic baryons belonging to the anti-decapentaplet ($\overline{15}$) [5] among five Ω_c s found by the LHCb Collaboration [6]. This mean-field approach is called the chiral quark-soliton model (χ QSM) [7] (for a review, see Refs. [8,9] and references therein). Very recently, the model has also described successfully strong decays of heavy baryons [10] including those of the newly found two narrow Ω_c s.

The magnetic moments of the heavy baryons have been already investigated within various different approaches such as quark models [11–13], the MIT bag model [14], the quark potential model [15,16], the Skyrme models in bound-state approaches [17, 18], a relativistic quark model [19], lattice QCD [20–22], heavy-

baryon chiral perturbation theories [23,24], and QCD sum rules [25, 26], and so on. Since there are no experimental data available yet, it is of great interest to compare the results with those from several different approaches. Since the baryon anti-triplet ($\overline{3}$) consists of the light-quark pair with the total light-quark spin $J = 0$, the corresponding magnetic moment vanishes in the present mean-field approach with the infinitely heavy-quark mass limit considered. Thus, in the present work, we want to employ the χ QSM to compute the magnetic moments of the lowest-lying singly heavy baryons, in particular, the baryon sextet ($\mathbf{6}$) with both spin $J' = 1/2$ and $J' = 3/2$. The magnetic moments of the light baryons were already studied within the χ QSM [27,28]. A merit of this approach is that we can deal with light and heavy baryons on the same footing. All the dynamical parameters required for the present analysis were determined in Ref. [29] based on the experimental data on the magnetic moments of the baryon octet, we have no additional free parameter to handle for those of the heavy baryons. We obtain the results for the magnetic moments of the baryon sextet and compare them with those from other models and lattice QCD. The results turn out to be consistent with those from the other works, in particular, with those from Ref. [19]. Compared with the results from the lattice QCD [20–22], the present ones are consistently larger than them except for the Σ_c^{++} magnetic moment.

The structure of the present work is sketched as follows: In Section 2, we briefly review the general formalism of the χ QSM in

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order to compute the magnetic moments of the heavy baryons. In Section 3, we show how to carry out the calculation of the magnetic moments within the present framework, using the dynamical parameters fixed in the light baryon sector. In Section 4, we present the numerical results of the magnetic moments of the heavy baryons, examining the effects of flavor $SU(3)_f$ breaking. We summarize the present work in the last Section.

2. General formalism

In the mean-field approach, a heavy baryon can be expressed by the correlation function of the $N_c - 1$ light-quark operators, while a heavy quark inside it is regarded as a static color source in the limit of the infinitely heavy quark mass ($m_Q \rightarrow \infty$). The heavy quark is required only to makes the heavy baryon a color singlet state. The electromagnetic current we now consider consists of both the light and heavy quark currents

$$J_\mu(x) = \bar{\psi}(x)\gamma_\mu\hat{Q}\psi(x) + e_Q\bar{Q}\gamma_\mu Q, \quad (1)$$

where \hat{Q} denotes the charge operator of the light quarks in flavor $SU(3)$ space, defined by

$$\hat{Q} = \begin{pmatrix} \frac{2}{3} & 0 & 0 \\ 0 & -\frac{1}{3} & 0 \\ 0 & 0 & -\frac{1}{3} \end{pmatrix} = \frac{1}{2} \left(\lambda_3 + \frac{1}{\sqrt{3}}\lambda_8 \right). \quad (2)$$

Here, λ_3 and λ_8 are the well-known flavor $SU(3)$ Gell-Mann matrices. The e_Q in the second part of the electromagnetic current in Eq. (1) stands for the heavy-quark charge, which is given as $e_c = 2/3$ for the charm quark or as $e_b = -1/3$ for the bottom quark. The magnetic moment of a heavy quark is proportional to the inverse of the corresponding mass, i.e. $\mu \sim (e_Q/m_Q)\sigma$, so that it should be very small in comparison with the light-quark contributions. It plays an essential role only in describing the baryon anti-triplet, which is understandable, because the light-quark pair constitutes a spin-zero state. However, its effect is rather small when it comes to the baryon sextet. In the lattice QCD [20–22], it is known that the contribution of the heavy quark to the magnetic moments of the baryon sextet is approximately one order smaller than the light-quark contributions. Ref. [19] also examined the heavy-quark contribution separately and found that its effect is in general tiny on the magnetic moments of the baryon sextet.

In principle, one could consider the heavy-quark effects on the magnetic moments as done in the quark models. It would give an overall constant contribution to the magnetic moments of the baryon sextet such as $-e_Q/6m_Q$ [19], which is parametrically very small. However, if one wants to consider the heavy-quark contribution within the present formalism consistently, one should go beyond the mean-field approximation. This is yet a difficult task, since we do not know proper nonperturbative interactions between the light and heavy quarks. Thus, we want to restrict ourselves to the light-quark contribution from the mean-field approximation, so we will ignore in the present work that from the heavy quark current in the limit of $m_Q \rightarrow \infty$.

Hence, we will deal with the first term of Eq. (1) when we compute the magnetic moments of heavy baryons by considering the following baryon matrix elements:

$$\langle B_Q | \bar{\psi}(x)\gamma_\mu\hat{Q}\psi(x) | B_Q \rangle. \quad (3)$$

Since we have ignored the heavy-quark contributions, we obtain the same results for both the charmed and bottom baryons. So, we will mainly focus on the magnetic moments of the charmed baryon sextet in the present work.

The general expressions for the magnetic moments of light baryons have been constructed already in previous works [27–30]. We will extend the formalism for those of heavy baryons in this work. Taking into account the rotational $1/N_c$ and linear m_s corrections, we are able to write the collective operator for the magnetic moments as

$$\hat{\mu} = \hat{\mu}^{(0)} + \hat{\mu}^{(1)}, \quad (4)$$

where $\hat{\mu}^{(0)}$ and $\hat{\mu}^{(1)}$ represent the leading and rotational $1/N_c$ contributions, and the linear m_s corrections respectively

$$\begin{aligned} \hat{\mu}^{(0)} &= w_1 D_{Q3}^{(8)} + w_2 d_{pq3} D_{Qp}^{(8)} \cdot \hat{J}_q + \frac{w_3}{\sqrt{3}} D_{Q8}^{(8)} \hat{J}_3, \\ \hat{\mu}^{(1)} &= \frac{w_4}{\sqrt{3}} d_{pq3} D_{Qp}^{(8)} D_{8q}^{(8)} + w_5 \left(D_{Q3}^{(8)} D_{88}^{(8)} + D_{Q8}^{(8)} D_{83}^{(8)} \right) \\ &\quad + w_6 \left(D_{Q3}^{(8)} D_{88}^{(8)} - D_{Q8}^{(8)} D_{83}^{(8)} \right). \end{aligned} \quad (5)$$

The indices of symmetric tensor d_{pq3} run over $p = 4, \dots, 7$. \hat{J}_q and \hat{J}_p denote the third and the p th components of the spin operator acting on the soliton. $D_{ab}^{(v)}(R)$ stand for the $SU(3)$ Wigner matrices in the representation v , which arise from the quantization of the soliton. $D_{Q3}^{(8)}$ is defined by the combination of the $SU(3)$ Wigner D functions

$$D_{Q3}^{(8)} = \frac{1}{2} \left(D_{33}^{(8)} + \frac{1}{\sqrt{3}} D_{83}^{(8)} \right), \quad (6)$$

which is obtained from the $SU(3)$ rotation of the electromagnetic octet current. The coefficients w_i in Eq. (5) encode a concrete dynamics of the chiral soliton and are independent of baryons involved. In fact, w_1 includes the leading-order contribution, a part of the rotational $1/N_c$ corrections, and linear m_s corrections, whereas w_2 and w_3 represent the rest of the rotational $1/N_c$ corrections. The m_s dependent term in w_1 is not explicitly involved in the breaking of flavor $SU(3)$ symmetry. Thus, we will treat w_1 as if it had contained the $SU(3)$ symmetric part, when the magnetic moments are computed. On the other hand, w_4 , w_5 , and w_6 are indeed the $SU(3)$ symmetry breaking terms. Yet another m_s corrections will come from the collective wave functions, which we will discuss soon. In principle, w_i can be computed within a specific chiral solitonic model such as the χ QSM [27,30].

We want to emphasize that the structure of Eq. (5) is rather *model-independent* and is deeply rooted in the hedgehog Ansatz or hedgehog symmetry. Since we consider the embedding of the $SU(2)$ soliton into $SU(3)$ [3], which keeps the hedgehog symmetry preserved, we have $SU(2)_T \times U(1)_Y$ symmetry. So, the structure of the collective operator is determined by the $SU(2)_T \times U(1)_Y$ invariant tensors

$$\begin{aligned} d_{abc} &= \frac{1}{4} \text{tr}(\lambda_a \{\lambda_b, \lambda_c\}), \quad S_{ab3} = \sqrt{\frac{1}{3}} (\delta_{a3} \delta_{b8} + \delta_{b3} \delta_{a8}), \\ F_{ab3} &= \sqrt{\frac{1}{3}} (\delta_{a3} \delta_{b8} - \delta_{b3} \delta_{a8}). \end{aligned} \quad (7)$$

In this respect, we will determine w_i by using the experimental data on the magnetic moments of the baryon octet as done in Refs. [28,29,31], instead of relying on a specific model. We will briefly show how to fix w_i , using the experimental data in the next Section.

To obtain the magnetic moments of the heavy baryons, the operator $\hat{\mu}$ in Eq. (4) needs to be sandwiched between heavy baryon states. Since we consider the linear m_s corrections perturbatively,

the collective wave functions for the soliton consisting of the light-quark pair are no longer pure states. The collective Hamiltonian for flavor SU(3) symmetry breaking [1], which is expressed as

$$H_{\text{sb}} = \alpha D_{88}^{(8)} + \beta \hat{Y} + \frac{\gamma}{\sqrt{3}} \sum_{i=1}^3 D_{8i}^{(8)} \hat{J}_i, \quad (8)$$

brings about the mixing of the baryon wave functions with those in higher SU(3) representations. The parameters α , β , and γ for heavy baryons are written as

$$\alpha = \left(-\frac{\bar{\Sigma}_{\pi N}}{3m_0} + \frac{K_2}{I_2} \bar{Y} \right) m_s, \quad \beta = -\frac{K_2}{I_2} m_s, \\ \gamma = 2 \left(\frac{K_1}{I_1} - \frac{K_2}{I_2} \right) m_s. \quad (9)$$

Note that the three parameters α , β , and γ are expressed in terms of the moments of inertia $I_{1,2}$ and $K_{1,2}$. The valence parts of them are different from those in the light baryon sector by the color factor $N_c - 1$ in place of N_c . The expression of $\bar{\Sigma}_{\pi N}$ is similar to the πN sigma term again except for the N_c factor: $\bar{\Sigma}_{\pi N} = (N_c - 1)N_c^{-1} \Sigma_{\pi N}$. As mentioned previously, a singly heavy baryon consists of $N_c - 1$ light valence quarks, so the constraint imposed on the right hypercharge should be changed from $\bar{Y} = -Y' = N_c/3$ to $\bar{Y} = (N_c - 1)/3$.

Then the wave functions for the baryon anti-triplet ($J = 0$) and the sextet ($J = 1$) are obtained respectively as [32]

$$|B_{\bar{3}_0}\rangle = |\bar{\mathbf{3}}_0, B\rangle + p_{\bar{15}}^B |\bar{\mathbf{15}}_0, B\rangle, \\ |B_{\mathbf{6}_1}\rangle = |\mathbf{6}_1, B\rangle + q_{\bar{15}}^B |\bar{\mathbf{15}}_1, B\rangle + q_{\bar{24}}^B |\bar{\mathbf{24}}_1, B\rangle, \quad (10)$$

with the mixing coefficients

$$p_{\bar{15}}^B = p_{\bar{15}} \begin{bmatrix} -\sqrt{15}/10 \\ -3\sqrt{5}/20 \end{bmatrix}, \quad q_{\bar{15}}^B = q_{\bar{15}} \begin{bmatrix} \sqrt{5}/5 \\ \sqrt{30}/20 \\ 0 \end{bmatrix}, \\ q_{\bar{24}}^B = q_{\bar{24}} \begin{bmatrix} -\sqrt{10}/10 \\ -\sqrt{15}/10 \\ -\sqrt{15}/10 \end{bmatrix}, \quad (11)$$

respectively, in the basis $[\Lambda_Q, \Xi_Q]$ for the anti-triplet and $[\Sigma_Q (\Sigma_Q^*), \Xi'_Q (\Xi_Q^*), \Omega_Q (\Omega_Q^*)]$ for the sextets. The parameters $p_{\bar{15}}$, $q_{\bar{15}}$, and $q_{\bar{24}}$ are given by

$$p_{\bar{15}} = \frac{3}{4\sqrt{3}} \alpha \bar{I}_2, \quad q_{\bar{15}} = -\frac{1}{\sqrt{2}} \left(\alpha + \frac{2}{3} \gamma \right) \bar{I}_2, \\ q_{\bar{24}} = \frac{4}{5\sqrt{10}} \left(\alpha - \frac{1}{3} \gamma \right) \bar{I}_2, \quad (12)$$

where $\bar{I}_2 = (N_c - 1)N_c^{-1} I_2$.

To carry out actual computation, we need to know the explicit expression of the wave functions formally given in Eq. (10). The wave function of a state with flavor $F = (Y, T, T_3)$ and spin $S = (Y' = -2/3, J, J_3)$ in the representation ν is obtained in terms of a tensor with two indices, i.e. $\psi_{(\nu; F), (\bar{\nu}; \bar{S})}$, one running over the states F in the representation ν and the other one over the states \bar{S} in the representation $\bar{\nu}$. Here, $\bar{\nu}$ stands for the complex conjugate of the representation ν , and the complex conjugate of S is written as $\bar{S} = (2/3, J, J_3)$. N_c being taken to be 3, the spin with hypercharge is given by $\bar{S} = (2/3, J, J_3)$. Thus, the collective wave

function for the soliton with a light-quark pair is expressed as

$$\psi_{(\nu; F), (\bar{\nu}; \bar{S})}(\mathbf{R}) = \sqrt{\dim(\nu)} (-1)^{Q_S} [D_F^{(\nu)}(\mathbf{R})]^*, \quad (13)$$

where $\dim(\nu)$ denotes the dimension of the representation ν and Q_S a charge corresponding to the baryon state S , i.e. $Q_S = J_3 + Y'/2$.

To construct the complete wave function for a heavy baryon, we need to couple the soliton wave function to the heavy quark such that the heavy baryon becomes a color singlet. Thus, the wave function for the heavy baryon should be written as

$$\Psi_{B_Q}^{(\mathcal{R})}(\mathbf{R}) = \sum_{J_3, J_{Q3}} C_{J, J_3, J_Q, J_{Q3}}^{J' J'_3} \chi_{J_{Q3}} \psi_{(\nu; Y, T, T_3), (\bar{\nu}; Y', J, J_3)}(\mathbf{R}) \quad (14)$$

where $\chi_{J_{Q3}}$ denote the Pauli spinors and $C_{J, J_3, J_Q, J_{Q3}}^{J' J'_3}$ the Clebsch-Gordan coefficients. Using the wave functions given in Eq. (14), we can derive the magnetic moments of the heavy baryons

$$\mu_B = \mu_B^{(0)} + \mu_B^{(\text{op})} + \mu_B^{(\text{wf})} \quad (15)$$

where $\mu_B^{(0)}$ represents the part of the magnetic moment in the chiral limit and $\mu_B^{(\text{op})}$ arises from $\hat{\mu}^{(1)}$ in Eq. (4), which contain w_4 , w_5 , and w_6 . $\mu_B^{(\text{wf})}$ comes from the interference between the $\mathcal{O}(m_s)$ and $\mathcal{O}(1)$ parts of the collective wave functions given in Eq. (10).

3. Magnetic moments of the baryon sextet

In the present approach, it is trivial to compute the magnetic moments of the baryon anti-triplet that consists of the soliton with spin $J = 0$. Since any scalar particle does not carry the magnetic moment, the magnetic moments of the baryon anti-triplet all turn out to vanish [19,23]. This implies that the magnetic moments of Λ_c^+ and Ξ_c should be tiny. So, we concentrate in this work on the magnetic moments of the baryon sextet.

We start with the magnetic moments in the chiral limit, for which we need to consider $\mu_B^{(0)}$ in Eq. (15). As mentioned already, w_1 contains both the leading-order and a part of the $1/N_c$ rotational corrections. Since we will use the experimental data to fit w_1 , it is difficult to decompose these two different terms as already discussed in Ref. [10]. Thus, we will follow the argument of Ref. [10] to fit the parameters required for the magnetic moments of the baryon sextet. Using the explicit expression of the magnetic moments given in Refs. [30,33], we can write w_1 , w_2 , and w_3 as

$$w_1 = M_0 - \frac{M_1^{(-)}}{I_1^{(+)}}, \quad w_2 = -2 \frac{M_2^{(-)}}{I_2^{(+)}}, \quad w_3 = -2 \frac{M_1^{(+)}}{I_1^{(+)}}, \quad (16)$$

where the explicit forms of M_0 , $M_1^{(\pm)}$, $M_2^{(-)}$ can be found in Refs. [30,33]. $I_1^{(+)}$ and $I_2^{(+)}$ denote the moments of inertia with the notation of Ref. [33] taken. It was shown in Ref. [33] that in the limit of the small soliton size the parameters in Eq. (16) can be expressed as

$$M_0 \rightarrow -2N_c K, \quad \frac{M_1^{(-)}}{I_1^{(+)}} \rightarrow \frac{4}{3} K, \quad \frac{M_1^{(+)}}{I_1^{(+)}} \rightarrow -\frac{2}{3} K, \\ \frac{M_2^{(-)}}{I_2^{(+)}} \rightarrow -\frac{4}{3} K. \quad (17)$$

These results with the small soliton size lead to the expressions of the magnetic moments in the nonrelativistic (NR) quark model.

Indeed, Eq. (16) reproduces the correct ratio of the proton and magnetic moments $\mu_p/\mu_n = -3/2$. So, the limit of the small soliton size is identical to the NR limit [33]. In the NR limit, we also obtain the relation $M_1^{(-)} = -2M_1^{(+)}$. To carry on the computation, we have to assume that this relation is also valid in the case of the realistic soliton size. Then, we are able to express the leading-order contribution M_0 in terms of w_1 and w_3

$$M_0 = w_1 + w_3. \quad (18)$$

Since a heavy baryon consists of $N_c - 1$ valence quarks, the original M_0 should be modified by introducing $(N_c - 1)/N_c$ as done similarly in Ref. [10]. The denominator N_c will cancel the same N_c factor in the original expressions of w_1 and w_3 (for explicit expressions of w_1 and w_3 , we refer to Ref. [34]) so that they have the proper prefactor $N_c - 1$ arising from the presence of the $N_c - 1$ valence quarks inside a heavy baryon. Theoretically, only the valence part of M_0 requires this scaling factor. However, as far as we fix the values of w_i using the experimental data, it is not possible to fix separately the valence and sea parts. Thus, it is plausible to define \tilde{w}_1 as

$$\tilde{w}_1 = \left[\frac{N_c - 1}{N_c} (w_1 + w_3) - w_3 \right] \sigma, \quad (19)$$

where σ is introduced to compensate possible deviations arising from the relation $M_1^{(-)} = -2M_1^{(+)}$ assumed to be valid in the realistic soliton case, and from the scaling factor $(N_c - 1)/N_c$ introduced to M_0 without separation of the valence and sea parts. We want to mention that σ has been already determined in Ref. [10]: $\sigma \sim 0.85$.

While w_2 and w_3 are kept to be the same as in the case of light baryons, $w_{4,5,6}$ are required to be modified by introducing the same factor $(N_c - 1)/N_c$ as done for M_0 . So, we redefine $\bar{w}_{4,5,6}$ as

$$\bar{w}_i = \frac{(N_c - 1)}{N_c} w_i, \quad i = 4, 5, 6. \quad (20)$$

Employing the numerical values provided in Ref. [29] and Eq. (20), we obtain the following values

$$\begin{aligned} \tilde{w}_1 &= -10.08 \pm 0.24, \\ w_2 &= 4.15 \pm 0.93, \\ w_3 &= 8.54 \pm 0.86, \\ \bar{w}_4 &= -2.53 \pm 0.14, \\ \bar{w}_5 &= -3.29 \pm 0.57, \\ \bar{w}_6 &= -1.34 \pm 0.56. \end{aligned} \quad (21)$$

Using these numerical values, we can now derive the magnetic moments of the baryon sextet. So, we want to emphasize that there is no additional free parameter to fit.

The explicit expressions of the magnetic moments for the baryon sextet with $J' = 1/2$ in Eq. (15) are derived as

$$\mu^{(0)} [6_1^{1/2}, B_c] = -\frac{1}{30} (3Q - 2) \left(\tilde{w}_1 - \frac{1}{2} w_2 - \frac{1}{3} w_3 \right), \quad (22)$$

$$\mu^{(\text{op})} [6_1^{1/2}, B_c] = -\frac{1}{270} \begin{bmatrix} 5Q - 7 \\ 7Q - 2 \\ Q + 3 \end{bmatrix} \bar{w}_4 - \frac{1}{90} \begin{bmatrix} 4Q - 5 \\ 2Q - 1 \\ Q + 3 \end{bmatrix} \bar{w}_5, \quad (23)$$

$$\begin{aligned} \mu^{(\text{wf})} [6_1^{1/2}, B_c] &= -\frac{1}{90\sqrt{2}} \begin{bmatrix} 4(Q - 2) \\ 5Q - 4 \\ Q \end{bmatrix} \\ &\times \left(\tilde{w}_1 + \frac{1}{2} w_2 + w_3 \right) q_{\overline{15}} \\ &+ \frac{1}{90\sqrt{10}} \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} (Q + 1) \\ &\times (\tilde{w}_1 + 2w_2 - 2w_3) q_{\overline{24}}, \end{aligned} \quad (24)$$

in the basis of $[\Sigma_c, \Xi'_c, \Omega_c]$. Q denotes the electric charge of the corresponding baryon. For the baryon sextet with spin $J' = 3/2$, we obtain

$$\mu^{(0)} [6_1^{3/2}, B_c] = -\frac{1}{20} (3Q - 2) \left(\tilde{w}_1 - \frac{1}{2} w_2 - \frac{1}{3} w_3 \right), \quad (25)$$

$$\mu^{(\text{op})} [6_1^{3/2}, B_c] = -\frac{1}{180} \begin{bmatrix} 5Q - 7 \\ 7Q - 2 \\ Q + 3 \end{bmatrix} \bar{w}_4 - \frac{1}{60} \begin{bmatrix} 4Q - 5 \\ 2Q - 1 \\ Q + 3 \end{bmatrix} \bar{w}_5, \quad (26)$$

$$\begin{aligned} \mu^{(\text{wf})} [6_1^{3/2}, B_c] &= -\frac{1}{60\sqrt{2}} \begin{bmatrix} 4(Q - 2) \\ 5Q - 4 \\ Q \end{bmatrix} \\ &\times \left(\tilde{w}_1 + \frac{1}{2} w_2 + w_3 \right) q_{\overline{15}} \\ &+ \frac{1}{60\sqrt{10}} \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} (Q + 1) \\ &\times (\tilde{w}_1 + 2w_2 - 2w_3) q_{\overline{24}}, \end{aligned} \quad (27)$$

in the basis of $[\Sigma_c^*, \Xi_c^*, \Omega_c^*]$ for the charmed sextet.

Before we present the numerical results of the magnetic moments of the heavy baryons, we first discuss general relations we find in Eqs. (26) and (27). In the present mean-field approach, there is no difference between charmed and bottom baryons, since there is no contribution from the heavy quark in the limit of $m_Q \rightarrow \infty$. Thus, even though the electric charges of the charm and bottom baryons are different each other, we have exactly the same numerical values of the magnetic moments for both the charm and bottom baryon belonging to the same representation, which can be written as

$$\mu [\mathcal{R}^J, B_c] = \mu [\mathcal{R}^J, B_b]. \quad (28)$$

Furthermore, we find a general interesting relations between the magnetic moments of the baryon sextet with $J' = 1/2$ and those with $J' = 3/2$. The difference is just fact 3/2 given as

$$\mu [6_1^{3/2}, B_c] = \frac{3}{2} \mu [6_1^{1/2}, B_c]. \quad (29)$$

In fact, this relation was already found in both the bound-state approach of the Skyrme model [17] and the SU(3) quark models [11–13].

Coleman and Glashow found various relations between the magnetic moments of the baryon octet [35], which arise from the isospin invariance. Similar relations have been also obtained in the case of the baryon decuplet [28,36]. We find here the generalized Coleman–Glashow relations for the spin-1/2 baryon sextet as

Table 1Numerical results of the magnetic moments for the charmed baryon sextet with $J' = 1/2$ in units of the nuclear magneton μ_N .

$\mu [6_1^{1/2}, B_c]$	$\mu^{(0)}$	$\mu^{(\text{total})}$	Oh et al. [17]	Scholl and Weigel [18]	Faessler et al. [19]	Lattice QCD [20,22]
Σ_c^{++}	2.00 ± 0.09	2.15 ± 0.1	1.95	2.45	1.76	2.220 ± 0.505
Σ_c^+	0.50 ± 0.02	0.46 ± 0.03	0.41	0.25	0.36	–
Σ_c^0	-1.00 ± 0.05	-1.24 ± 0.05	-1.1	-1.96	-1.04	-1.073 ± 0.269
$\Xi_c^{'+}$	0.50 ± 0.02	0.60 ± 0.02	0.77	–	0.47	0.315 ± 0.141
Ξ_c^0	-1.00 ± 0.05	-1.05 ± 0.04	-1.12	–	-0.95	-0.599 ± 0.071
Ω_c^0	-1.00 ± 0.05	-0.85 ± 0.05	-0.79	–	-0.85	-0.688 ± 0.031

Table 2Numerical results of magnetic moments for charmed baryon sextet with $J' = 3/2$ in units of the nuclear magneton μ_N .

$\mu [6_1^{3/2}, B_c]$	$\mu^{(0)}$	$\mu^{(\text{total})}$	Oh et al. [17]	Lattice QCD [21]
Σ_c^{*++}	3.00 ± 0.14	3.22 ± 0.15	3.23	–
Σ_c^{*+}	0.75 ± 0.04	0.68 ± 0.04	0.93	–
Σ_c^{*0}	-1.50 ± 0.07	-1.86 ± 0.07	-1.36	–
Ξ_c^{*+}	0.75 ± 0.04	0.90 ± 0.04	1.46	–
Ξ_c^{*0}	-1.50 ± 0.07	-1.57 ± 0.06	-1.4	–
Ω_c^{*0}	-1.50 ± 0.07	-1.28 ± 0.08	-0.87	-0.730 ± 0.023

$$\begin{aligned} \mu(\Sigma_c^{++}) - \mu(\Sigma_c^+) &= \mu(\Sigma_c^+) - \mu(\Sigma_c^0), \\ \mu(\Sigma_c^0) - \mu(\Xi_c^0) &= \mu(\Xi_c^0) - \mu(\Omega_c^0), \\ 2[\mu(\Sigma_c^+) - \mu(\Xi_c^0)] &= \mu(\Sigma_c^{++}) - \mu(\Omega_c^0). \end{aligned} \quad (30)$$

Similar relations were also discussed in Ref. [24]. Though the usual Coleman–Glashow relations are satisfied in the chiral limit, the relations in Eq. (30) are the robust ones even when the effects of SU(3) flavor symmetry breaking are taken into account. In the chiral limit, we obtain the relation according to the U -spin symmetry

$$\begin{aligned} \mu(\Sigma_c^0) &= \mu(\Xi_c^0) = \mu(\Omega_c^0) = -2\mu(\Sigma_c^+) = -2\mu(\Xi_c^{'+}) \\ &= -\frac{1}{2}\mu(\Omega_c^0). \end{aligned} \quad (31)$$

Another interesting relation in the chiral limit is the sum rule given as

$$\sum_{B_c \in \text{sextet}} \mu(B_c) = 0. \quad (32)$$

We know that Eq. (32) is very similar to the sum rule for the magnetic moments of the baryon decuplet. In the case of the decuplet, the sum of all the magnetic moments is the same as that of all the electric charges of the corresponding baryons [28]. On the other hand, Eq. (32) is identical to the sum of $2Q - 1$ for all the members of the baryon sextet as shown in Eq. (25), which yields also the null result as given in Eq. (32). We can derive the same relations from Eqs. (30)–(32) for the $J' = 3/2$ sextet baryon and also for the bottom baryons.

Concerning the magnetic moments of the baryon anti-triplet, we already mentioned that those of all members turn out to be zero, since the spin of the soliton with a light-quark pair is zero. Lichtenberg derived a relation based on the quark model, which shows that all the magnetic moments of the baryon anti-triplet are the same [13]

$$\mu(\Lambda_c^+) = \mu(\Xi_c^+) = \mu(\Xi_c^0). \quad (33)$$

This relation is trivially satisfied because all of them vanish in the present work.

4. Numerical results

In Table 1 and 2, we list the results of the magnetic moments for the charmed baryon sextet with $J' = 1/2$ and $J' = 3/2$, respectively. In the second columns, the numerical values of the magnetic moments in the chiral limit are listed whereas in the third columns the total results are given. The effects of flavor SU(3) symmetry breaking contribute to the magnetic moments of the baryon sextet found to be around (5–10)% except for those of Σ_c^0 and Ω_c^0 , for which the effects are found to be around (15–20)%.

In Table 1, we compare the results for the baryon sextet with $J' = 1/2$ with those from the Skyrme model in the bound-state approach [17,18], the relativistic quark model [19], and the lattice QCD [20–22]. While the original Skyrme model in the bound-state approach [17] is constructed, based on the light pseudoscalar meson fields together with the heavy pseudoscalar meson fields, Ref. [18] introduced the light and heavy vector mesons in addition. We find that in general the magnitudes of the present results are slightly larger than those of Ref. [17] except for $\mu(\Xi_c^{'+})$ and $\mu(\Xi_c^0)$. The results of $\mu(\Sigma_c^{++})$ and $\mu(\Sigma_c^0)$ from Ref. [18] are, respectively, around 15% and 60% larger than the present results in magnitude. The results are qualitatively similar to those of Refs. [19], though Ref. [19] used a rather different model, i.e. the relativistic quark model. Comparing the present results with those from the lattice QCD [20–22], we find that the results are systematically larger than those from the lattice QCD except for the Σ_c^{++} magnetic moment.

Since the results of the magnetic moments of the bottom baryons are exactly the same as those of the charmed baryons, it seems redundant to present the corresponding results. The present results for the magnetic moments of the bottom baryon sextet are even in better agreement with those from Ref. [19].

5. Summary and conclusion

In the present work, we have studied the magnetic moments of the lowest-lying singly heavy baryons, based on the chiral quark-soliton model. All the dynamical parameters were fixed in the light baryon sector. The magnetic moments of the baryon anti-triplet

vanish in the present mean-field approach, because the spin of the soliton for the anti-triplet is $J = 0$. Since the first term in the expression of the heavy baryon magnetic moments consists of the leading-order and the $1/N_c$ rotational corrections, we had to decompose them, using the limit of the small soliton size. Having properly considered the scaling factor, we were able to compute the magnetic moments of the baryon sextet with both spins $J' = 1/2$ and $J' = 3/2$. The results were compared with those from other models such as the Skyrme model in bound-state approaches, the relativistic quark model, and the lattice QCD. They are in particular consistent with those from the relativistic quark model. We also compared the present results with those from the lattice QCD. Except for the Σ_c^{++} magnetic moment, we obtained systematically larger values of the magnetic moments than those from the lattice QCD.

The same method can be applied to compute the transition magnetic moments of the singly heavy baryons, which provide essential information on radiative decays of them. The corresponding investigation is under way. The magnetic moments of doubly heavy baryons can be also studied within the present mean-field approach, assuming that $N_c - 2$ valence quarks can produce the modified pion mean fields. The corresponding study is under investigation.

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