Diffusion of Conserved Charges in Relativistic Heavy Ion Collisions

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(Received 1 December 2017; published 12 June 2018)

We demonstrate that the diffusion currents do not depend only on gradients of their corresponding charge density, but that the different diffusion charge currents are coupled. This happens in such a way that it is possible for density gradients of a given charge to generate dissipative currents of another charge. Within this scheme, the charge diffusion coefficient is best viewed as a matrix, in which the diagonal terms correspond to the usual charge diffusion coefficients, while the off-diagonal terms describe the coupling between the different currents. In this Letter, we calculate for the first time the complete diffusion matrix for hot and dense nuclear matter, including baryon, electric, and strangeness charges. We find that the baryon diffusion current is strongly affected by baryon charge gradients but also by its coupling to gradients in strangeness. The electric charge diffusion current is found to be strongly affected by electric and strangeness gradients, whereas strangeness currents depend mostly on strange and baryon gradients.

DOI: 10.1103/PhysRevLett.120.242301

Introduction.—Ultrarelativistic hadronic collisions, performed in the largest particle accelerators, allow us to study the properties of hot and dense hadronic and quark matter. These experiments have played a crucial role in uncovering novel transport properties of the quark-gluon plasma (QGP), the state of nuclear matter in which quarks and gluons are no longer confined inside hadrons. In particular, several phenomenological studies [1–8] demonstrated that the QGP has one of the smallest shear viscosity to entropy density ratios in nature—a surprising result that is still not well understood from first principles. Additional studies [4,9–14] have also improved our understanding of the bulk viscosity, unravelling novel behavior near the deconfinement transition of nuclear matter. Recently, much attention was paid to the electric conductivity; several studies on the lattice [15–17], in perturbative QCD [18–20] and effective theories [21–23] have been carried out.

On the other hand, at this stage, very little is known about net-charge diffusion in hot and dense nuclear matter. This is due to the fact that in high energy heavy ion collisions the net-charge density of the matter produced is extremely small in almost all space-time points, and it becomes very difficult to observe any dissipative effects due to diffusion [24]. Recently, the Relativistic Heavy-Ion Collider (RHIC) started to perform hadronic collisions at lower energies within the beam energy scan (BES) program in order to investigate the phase diagram and transport properties of nuclear matter at finite net-baryon (and net-electric charge) density [25–27]. At beam energies down to, e.g., $\sqrt{s_{\mathrm{NN}}} = 7.7$ GeV in the RHIC BES, the baryon chemical potential can reach values up to $\mu_B \sim 400$ MeV which is significant compared to the temperatures that are reached [28,29], and strong gradients in the chemical potential of conserved charges are expected. Therefore, one can expect that low energy collisions are particularly useful to explore the properties of net-charge diffusion of nuclear matter that were out of reach in higher energy collisions.

In relativistic Navier-Stokes-Fourier theory, a net-charge (q) diffusion 4-current, j_q^μ, is determined by the following constitutive relation,

$$j_q^\mu = \kappa_q \nabla^\mu \alpha_q, \tag{1}$$

where $\alpha_q \equiv \mu_q/T$ is the thermal potential, with μ_q being the charge chemical potential, T the temperature, and κ_q the corresponding net-charge diffusion coefficient. We further defined the transverse gradient $\nabla^\mu \equiv \Delta^{\mu\nu}\partial_\nu$ and the projection operator $\Delta^{\mu\nu} \equiv g^{\mu\nu} - u^\mu u^\nu$, where u^μ is the local fluid velocity and $g^{\mu\nu}$ the space-time metric. We remark that this relativistic constitutive relation also includes the effects of heat flow.

However, we emphasize that Eq. (1) cannot be employed to describe diffusion processes in the presence of more than one conserved charge. This is exactly what happens in matter produced in heavy ion collisions, in which we must always consider at least three conserved charges: baryon number (B), electric charge (Q), and strangeness (S). Since several hadrons (and quarks) carry more than one of these charges, it is not possible for one type of charge to diffuse independently of any other, leading to a mixing between the diffusion currents, with gradients of every single charge density being able to generate a diffusion current of any other charge. The most general expression for the diffusion current then is

$$\begin{pmatrix} j_B^\mu \\ j_Q^\mu \\ j_S^\mu \end{pmatrix} = \begin{pmatrix} \kappa_{BB} & \kappa_{BQ} & \kappa_{BS} \\ \kappa_{QB} & \kappa_{QQ} & \kappa_{QS} \\ \kappa_{SB} & \kappa_{SQ} & \kappa_{SS} \end{pmatrix} \cdot \begin{pmatrix} \nabla^\mu \alpha_B \\ \nabla^\mu \alpha_Q \\ \nabla^\mu \alpha_S \end{pmatrix}, \tag{2}$$

with the diffusion coefficient being a matrix instead of a number $\kappa_{qq'}$. Therefore, in order to describe diffusion processes in heavy ion collisions, it is not sufficient to simply know the usual baryon, electric, and strangeness diffusion coefficients, i.e., the diagonal terms in the matrix $\kappa_{BB}, \kappa_{QQ}, \kappa_{SS}$. One must also calculate the off-diagonal terms or couplings terms $\kappa_{QB}, \kappa_{SB}, \kappa_{SQ}$ (it is sufficient to calculate only these three off-diagonal terms since Onsager's theorem [30,31] guarantees that the diffusion matrix is symmetric, see also Ref. [32]).

The dynamics of the thermal potentials α_B, α_S, and α_Q and their respective currents in heavy ion collisions is currently not well known. It is expected that the influence of diffusion currents on the hydrodynamical evolution of the net-charge currents can be very pronounced at lower collision energies, leading to significant effects on certain observables [24]. However, this cannot be properly investigated without first understanding the order of magnitude of the cross-diffusion effects discussed above. In this Letter, we derive from kinetic theory the generalized expression for the diffusion current shown in Eq. (2) and calculate for the first time the complete charge diffusion matrix for the three charges listed above. We perform this task for a hadron resonance gas (HRG) and for a kinetic theory toy model of the QGP. In contrast to previous work [22], we disregard any external field. We find that the coupling terms, currently neglected in all heavy ion collision simulations, can be as large as the diagonal terms, and consequently, these simulations may be missing crucial ingredients. Furthermore, it may not be a good approximation to perform simulations including only the dynamics of one charge since its gradients will necessarily give rise to diffusion currents of the remaining charges. We use natural units $\hbar = c = k_B = 1$ and Minkowski metric $g^{\mu\nu} = (1, -1, -1, -1)$.

First order Chapman-Enskog expansion.—We consider a dilute gas consisting of $N_{\rm sp}$ particle species (either hadrons or quarks and gluons), with the ith particle species having degeneracy g_i, electric charge Q_i, strangeness charge S_i, baryonic charge B_i, and 4-momentum k_i^μ. The state of the system is characterized by the single-particle momentum distribution function of each particle species $f_i(x,k) \equiv f_{\mathbf{k}}^i$, with the time evolution of $f_{\mathbf{k}}^i$ being given by the relativistic Boltzmann equation.

The single-particle distribution of each particle species is expanded in a Chapman-Enskog series, i.e., in a gradient expansion [33,34]. In this case, the Boltzmann equation is written as

$$\epsilon k_i^\mu \partial_\mu f_{\mathbf{k}}^i = -\sum_{j=1}^{N_{\rm sp}} C_{ij}(x^\mu, k^\mu), \tag{3}$$

with $C_{ij}(x^\mu, k^\mu)$ being the collision term and ϵ a book-keeping parameter that will be set to 1 at the end of the calculation. The Chapman-Enskog expansion is just an expansion in powers of ϵ, $f_{\mathbf{k}}^i \sim f_{0\mathbf{k}}^i + \epsilon f_{1\mathbf{k}}^i + \epsilon^2 f_{2\mathbf{k}}^i + \cdots$, where $f_{j\mathbf{k}}^i$ is the jth order solution of the expansion. The zeroth order solution of this series is the local equilibrium distribution function, leading to the equations of ideal fluid dynamics, while the first order solution contains terms that are of first order in gradients of velocity, temperature, and chemical potential, leading to the equations of relativistic Navier-Stokes theory and the diffusion equation [33,34]. For the purposes of this Letter, it is sufficient to calculate the first order contribution, which is the order that determines the diffusion coefficients. Without loss of generality, we only retain the terms of the expansion that contribute directly to the diffusion terms, omitting all others that contribute to shear and bulk viscosity.

The equation for the first order Chapman-Enskog correction will then be

$$-\sum_{j=1}^{N_{\rm sp}} \hat{C}_{ij}^{(1)} f_{1\mathbf{k}} = \sum_{q \in \{B,Q,S\}} f_{0\mathbf{k}}^i k_i^\mu \nabla_\mu \alpha_q \left(\frac{E_{i,\mathbf{k}} n_q}{\epsilon_0 + P_0} - q_i \right), \tag{4}$$

where $\hat{C}_{ij}^{(1)}$ is the linearized collision operator,

$$\hat{C}_{ij}^{(1)} f_{1\mathbf{k}} = \int_{k'pp'} W_{\mathbf{kk'}-\mathbf{pp'}}^{ij} f_{0\mathbf{k}}^i f_{0\mathbf{k'}}^j \times \left(\frac{f_{1\mathbf{p}}^i}{f_{0\mathbf{p}}^i} + \frac{f_{1\mathbf{p'}}^j}{f_{0\mathbf{p'}}^j} - \frac{f_{1\mathbf{k}}^i}{f_{0\mathbf{k}}^i} - \frac{f_{1\mathbf{k'}}^j}{f_{0\mathbf{k'}}^j} \right), \tag{5}$$

with $W_{\mathbf{kk'}\to\mathbf{pp'}}^{ij} = (1-\delta_{ij}/2)(2\pi)^6 s\sigma^{ij}(s,\theta)\delta^{(4)}(p_i + p'_j - k_i - k'_j)$ being the scattering amplitude, $\sigma^{ij}(s,\theta)$ the differential cross section, and abbreviation $\int_{k_i} \equiv \int d^3k_i/[(2\pi)^3 E_{i,\mathbf{k}}]$. For the sake of simplicity, we only consider elastic $2 \leftrightarrow 2$ collisions between hadrons or quarks and employ classical statistics.

This equation can be solved following the well known procedure outlined in [22,35]. Since the collision operator $\hat{C}_{ij}^{(1)}$ is linear, the solution for $f_{1\mathbf{k}}$ must be of the general form $f_{1\mathbf{k}}^i = \sum_q a_q^i k_i^\mu \nabla_\mu \alpha_q$, where the coefficient a_q^i is a function of the energy in the local rest frame $E_{i,\mathbf{k}} \equiv u_\mu k_i^\mu$. Next, one expands a_q^i in powers of energy $f_{1\mathbf{k}}^i = \sum_q k_i^\mu \nabla_\mu \alpha_q \sum_{m=0}^M a_{q,m}^i \, (E_{i,\mathbf{k}})^m$, where the integer M characterizes the truncation of the Taylor series. Finally, one substitutes this expansion into Eq. (4), multiplies the equation by the nth basis element of the expansion

$\Delta^{\mu}_{\nu} k_i^{\nu} (E_{i,\mathbf{k}})^n$, and integrates it in momentum space. This leads to the following equation for the expansion coefficients $a^j_{q,m}$:

$$\sum_{m=0}^{M} \sum_{j=1}^{N_{\rm sp}} (\mathcal{A}^i_{nm} \delta^{ij} + \mathcal{C}^{ij}_{nm}) a^j_{q,m} = b^i_{q,n}, \tag{6}$$

where we defined

$$\begin{aligned} b^i_{q,n} &= \int_{k_i} f^i_{0\mathbf{k}} \left(\frac{E_{i,\mathbf{k}} n_q}{\epsilon_0 + P_0} - q_i \right) E^{n-1}_{i,\mathbf{k}} \Delta_{\mu\nu} k_i^{\mu} k_i^{\nu}, \\ \mathcal{A}^i_{nm} &= \sum_{j=1}^{N_{\rm sp.}} \int_{k_i k'_j p_i p'_j} W^{ij}_{\mathbf{k}\mathbf{k}' \to \mathbf{p}\mathbf{p}'} f^i_{0\mathbf{k}} f^j_{0\mathbf{k}'} E^{n-1}_{i,\mathbf{k}} \\ &\quad \times \Delta_{\mu\nu} k_i^{\mu} (E^m_{i,\mathbf{p}} p_i^{\nu} - E^m_{i,\mathbf{k}} k_i^{\nu}), \\ \mathcal{C}^{ij}_{nm} &= \int_{k_i k'_j p_i p'_j} W^{ij}_{\mathbf{k}\mathbf{k}' \to \mathbf{p}\mathbf{p}'} f^i_{0\mathbf{k}} f^j_{0\mathbf{k}'} E^{n-1}_{i,\mathbf{k}} \\ &\quad \times \Delta_{\mu\nu} k_i^{\mu} (E^m_{j,\mathbf{p}'} p'^{\nu}_j - E^m_{j,\mathbf{k}'} k'^{\nu}_j). \end{aligned} \tag{7}$$

In this work, the expansion in powers of energy is truncated at the lowest level possible by setting $M = 1$. This assumption is mainly employed to simplify the numerical calculations we perform. Nevertheless, we have checked, in simpler examples solved using constant cross sections, that higher truncation values lead to only small corrections to the diffusion coefficients, as was also demonstrated in previous work [22,35] for other transport coefficients.

The qth charge diffusion current is given as

$$j^{\mu}_q = \sum_{i=1}^{N_{\rm sp}} q_i \int \frac{\mathrm{d}^3 k_i}{(2\pi)^3 E_{i,\mathbf{k}}} \Delta^{\mu}_{\nu} k_i^{\nu} f^i_{1\mathbf{k}}. \tag{8}$$

Substituting the expansion for $f^i_{1\mathbf{k}}$ into Eq. (8) and comparing to Eq. (2) leads to the following expression for the diffusion coefficients:

$$\kappa_{qq'} = \frac{1}{3} \sum_{i=1}^{N_{\rm sp}} q_i \sum_{m=0}^{M} a^i_{q',m} \int_{k_i} E^m_{i,\mathbf{k}} \Delta_{\mu\nu} k_i^{\mu} k_i^{\nu} f^i_{0\mathbf{k}}. \tag{9}$$

Therefore, calculating $\kappa_{qq'}$ is reduced to evaluating the integrals in Eq. (7) and then solving the set of linear equations satisfied by $a^i_{q',m}$ in Eq. (6). Both these tasks are performed numerically.

In order to perform these numerical calculations, one has to first specify the differential cross sections describing the particle interactions. Our goal in this Letter is to provide a first estimate for the cross-diffusion coefficients, providing some intuition on how large they can be when compared to the traditional diagonal terms. For this purpose, it is sufficient to employ a simple, but reliable, model in which

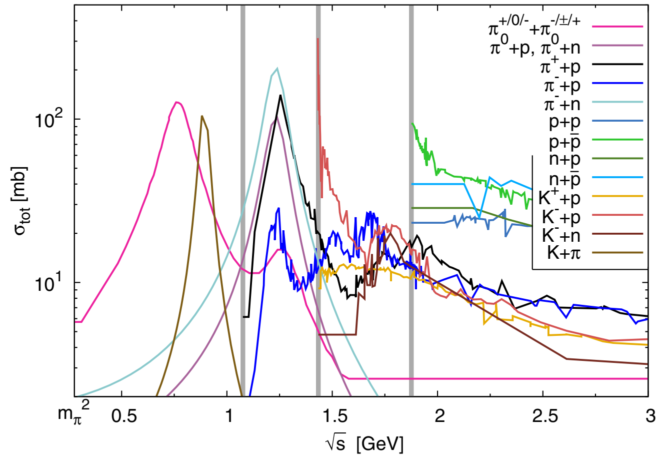

FIG. 1. Tabulated hadronic cross sections over $\sqrt{s}$ from Ref. [36] we used for the Pion-Kaon-Nucleon-Lambda-Sigma gas. The grey bars denote the minimal $\sqrt{s}$ of the particular scattering process. The combinations which are not listed here are assumed to be constant [37–39].

we only consider elastic, isotropic (s-wave) scattering, employing all available $\sqrt{s}$ dependent cross sections from Ref. [36], as shown in Fig. 1. Due to the lack of experimental data, we assume all missing hadronic cross sections to be constant, as done, for example, in hadronic transport models [37–39]. The hyperon cross sections thus take constant values between 3–35 mb.

We also make a first estimate of the diffusion coefficients of the QGP. For this purpose, we assume three flavors of massless quarks and gluons and choose a unique total cross section $\sigma_{\rm tot}$ in such a way that the shear viscosity to entropy density ratio is fixed to be $\eta/s = 1/(4\pi)$, leading to $\sigma_{\rm tot} \approx 0.72/T^2$ [40,41]. Further details on the choice of the cross sections will be presented in a forthcoming publication [42].

Results.—We first remark that we checked that Onsager's theorem [30,31], which imposes that $\kappa_{qq'} = \kappa_{q'q}$, is fulfilled in all our calculations. We display our results for the diffusion coefficient matrix from Eq. (9) in Fig. 2 for $\mu_B = 0, 300, 600$ MeV. We fix μ_Q and μ_S such that we always retain an exact Isospin symmetry and vanishing net strangeness since this is what approximately occurs in heavy ion collisions [43,44]. For illustrative purposes, we show the HRG results below $T = 160$ MeV and the QGP results above this temperature. [We also want to relate the magnitude of the hadronic results to phenomenological models (e.g., hydrodynamics) which successfully operates often with fixed η/s.] We also compare here to the nonconformal holographic results from Ref. [21,23] since these results are the only ones in the literature that contain all three diagonal coefficients. See Ref. [45] for, i.e., a calculation of the strangeness diffusion coefficient. To the best of our knowledge, the off-diagonal coefficients have never been calculated before in any model.

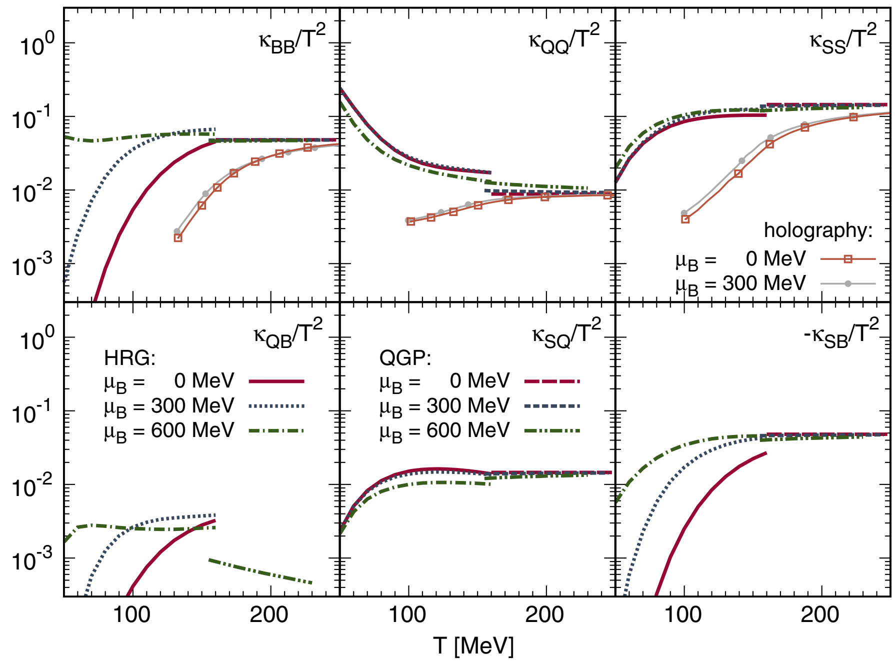

FIG. 2. All diffusion coefficients for baryon, electric, and strangeness diffusion. The hadronic results include resonance cross sections of the lightest 19 hadronic species, whereas the QGP uses massless quarks and gluons with fixed $4\pi\eta/s = 1$. For illustrative purposes, we show the hadron resonance gas results for $T \leq 160$ MeV and above that the QGP calculation. We compare to holographic results from Ref. [21,23].

At this point we remark, that the relativistic kinetic theory which we employ here constitutes an effective model, which is (for temperatures below the phase transition) well in its range of applicability (following the reasoning of very successful transport models such as UrQMD [38], PHSD [46,47], GiBUU [37], B3D [48], or SMASH [49]). Above the phase transition, kinetic theory is perhaps less reliable, but we nevertheless show our results since first principles calculations are not available for most of the coefficients we calculate.

First we note that the HRG results are much richer in their T and μ_B dependence because of the multitude of scales involved here (masses and resonances). In contrast, the simple choice of a constant η/s in the QGP leads to the expected flat behavior for all coefficients [20] (it is known that a running strong coupling lets the coefficients increase for higher T in the QGP, see, e.g., Refs. [15,18]). We note that only κ_{QB} vanishes at $\mu_B = 0$ due to the common cross section and the symmetry of the B and Q charges (quarks $[B = 1/3]$ of different flavors carry positive or negative electric charge). At higher μ_B, we see that these coefficients are found to be generally smaller in the QGP phase than they are in the hadronic phase (κ_{SQ} being the exception). This surprising behavior will be investigated in more detail in a forthcoming paper [42].

For the baryon diffusion current j_B^μ, we expect a strong dependence on both μ_B and T, and indeed, this can be seen from the functional behavior of the coefficient κ_{BB} in Fig. 2. For $\mu_B \lesssim 300$ MeV, this coefficient rises rapidly with increasing temperature as the system is less meson dominated at higher temperatures and mesons act purely as a resistance for the diffusion of baryons. This effect is also visible in the off-diagonal coefficients $-\kappa_{SB}$ and κ_{QB}. Comparing κ_{QB} to κ_{BB}, in Fig. 2, we infer that the electric charge gradients contribute to the baryon diffusion current about an order of magnitude less than the baryonic gradients. We conclude that in practice the baryon-electric charge coupling may well be neglected. In contrast, gradients in strangeness can be as important as gradients in the baryon charge, as can be seen in the bottom right panel from the magnitude of the coefficient $-\kappa_{SB}$, which is similar in magnitude to κ_{BB}. We remark that this is due to the hyperons, which carry both B and S charge. The negative sign of κ_{SB} indicates that gradients in strangeness act to reduce the baryon current.

We now discuss the coefficients $\kappa_{QQ}, \kappa_{SQ}, \kappa_{QB}$, which characterize the diffusion of electric charges (at $\mu_B = 0$, κ_{QQ}/T^2 is equal to the electric conductivity σ_{el}/T). We see that κ_{QQ}/T^2 decreases with temperature and for increasing values of μ_B. This happens because the particle density grows, but the ratio of charged to uncharged species stays the same. The small ratio κ_{QB}/κ_{QQ} indicates the little importance of baryon chemical potential gradients to the electric diffusion current, whereas κ_{SQ} is (for $T \gtrsim 100$ MeV) of the same order of magnitude as κ_{QQ}, indicating that strangeness gradients contribute significantly to the electric diffusion current.

Looking at the diffusion coefficients related to strangeness diffusion, we find that κ_{SS} is larger than both $-\kappa_{SB}$ and

κ_{SQ}, being even larger in magnitude than the baryon diffusion coefficient (except for very small values of temperature). However, we find that baryonic gradients act to significantly reduce strangeness currents in both the QGP and HRG since κ_{SB} is negative and its magnitude is only about a factor 2 smaller than κ_{SS}. Therefore, it is possible that cancellation effects due to coupling between the currents can lead to small strangeness diffusion currents. On the other hand, κ_{SQ} is about an order of magnitude smaller than κ_{SS}, indicating that electric gradients are less important for strangeness transport. We remark that the μ_B dependence of κ_{SS}, κ_{QQ}, and κ_{SQ} is very weak; however, their dependence on μ_Q and μ_S can behave differently. This dependence will be addressed in a future publication.

The holographic results from Refs. [21,23] match ours at high T (conformal limit). Their μ_B dependence for the diagonal coefficients is as weak as for our QGP results. It is interesting how a simple kinetic calculation, that simply fixes $\eta/s = 1/4\pi$, is already capable of reproducing the basic trends of such holographic calculations. It would be interesting to see whether this holds for the off-diagonal coefficients.

Conclusion.—We propose that diffusion processes in heavy ion collisions must be described not only by a diffusion coefficient of each conserved charge but also by a diffusion coefficient matrix, which describes how diffusion of different conserved charges couple to each other. We have calculated for the first time the complete diffusion coefficient matrix for the conserved baryon, electric, and strange charges for a hot hadron resonance gas (19 massive species) and QGP (fixed η/s) using kinetic theory. These six transport coefficients include the three off-diagonal transport diffusion coefficients κ_{QB}, κ_{SB}, and κ_{SQ}, which describe the mixing between the different charge currents.

The diffusion coefficients can be readily used in, for example, hydrodynamic simulations or other model descriptions of high density heavy ion collisions. Those models are and will be increasingly important for low energy and high density experiments like RHIC BES, NICA, or FAIR, where research has just begun.

Our results emphasize that the mixing between different diffusion currents is important and should not be neglected when simulating low energy heavy ion collisions. For example, the contribution to the baryon diffusion current from gradients of baryon number density can be almost completely canceled by gradients in strangeness of comparable magnitude, whereas we found electric gradients to be almost negligible for baryon transport. Electric diffusion is mainly driven by electric and strangeness gradients. Strangeness diffusion is mostly affected by strangeness and baryon number gradients. The relevance of these effects for experimental observables remains to be investigated.

It would be desirable to compare our results to, for example, lattice QCD results (which at present are only available for the electric conductivity). All coefficients should also be accessible from hadronic transport models or other dynamical approaches.

The authors M. G., J. A. F., and C. G. acknowledge support by the Deutsche Forschungsgemeinschaft (DFG) through Grant No. CRC-TR 211 “Strong-interaction matter under extreme conditions”. M. G. acknowledges the support from the “Helmholtz Graduate School for Heavy Ion research.” J. A. F. acknowledges support from the “Stiftung Polytechnische Gesellschaft”, Frankfurt am Main. The authors thank Harri Niemi for fruitful discussions. This work was supported by the Helmholtz International Center for FAIR within the framework of the LOEWE program launched by the State of Hesse. G. S. D. thanks Conselho Nacional de Desenvolvimento Científico e Tecnológico (CNPq) for financial support.

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[1] P. Romatschke and U. Romatschke, Phys. Rev. Lett. **99**, 172301 (2007).
[2] Z. Xu, C. Greiner, and H. Stocker, Phys. Rev. Lett. **101**, 082302 (2008).
[3] M. Luzum and P. Romatschke, Phys. Rev. C **78**, 034915 (2008); **79**, 039903(E) (2009).
[4] P. Bozek, Phys. Rev. C **81**, 034909 (2010).
[5] H. Song, S. A. Bass, U. Heinz, T. Hirano, and C. Shen, Phys. Rev. Lett. **106**, 192301 (2011); **109**, 139904(E) (2012).
[6] H. Niemi, G. S. Denicol, P. Huovinen, E. Molnar, and D. H. Rischke, Phys. Rev. Lett. **106**, 212302 (2011).
[7] C. Wesp, A. El, F. Reining, Z. Xu, I. Bouras, and C. Greiner, Phys. Rev. **C84**, 054911 (2011).
[8] C. Gale, S. Jeon, B. Schenke, P. Tribedy, and R. Venugopalan, Phys. Rev. Lett. **110**, 012302 (2013).
[9] P. B. Arnold, C. Dogan, and G. D. Moore, Phys. Rev. D **74**, 085021 (2006).
[10] D. Kharzeev and K. Tuchin, J. High Energy Phys. 09 (2008) 093.
[11] J. Noronha-Hostler, G. S. Denicol, J. Noronha, R. P. G. Andrade, and F. Grassi, Phys. Rev. C **88**, 044916 (2013).
[12] M. Nopoush, R. Ryblewski, and M. Strickland, Phys. Rev. C **90**, 014908 (2014).
[13] G. S. Denicol, S. Jeon, and C. Gale, Phys. Rev. C **90**, 024912 (2014).
[14] S. Ryu, J. F. Paquet, C. Shen, G. S. Denicol, B. Schenke, S. Jeon, and C. Gale, Phys. Rev. Lett. **115**, 132301 (2015).
[15] G. Aarts, C. Allton, A. Amato, P. Giudice, S. Hands, and J.-I. Skullerud, J. High Energy Phys. 02 (2015) 186.
[16] B. B. Brandt, A. Francis, B. Jäger, and H. B. Meyer, Phys. Rev. D **93**, 054510 (2016).
[17] H.-T. Ding, O. Kaczmarek, and F. Meyer, Phys. Rev. D **94**, 034504 (2016).
[18] M. Greif, I. Bouras, C. Greiner, and Z. Xu, Phys. Rev. D **90**, 094014 (2014).
[19] A. Puglisi, S. Plumari, and V. Greco, Phys. Rev. D **90**, 114009 (2014).

[20] P. B. Arnold, G. D. Moore, and L. G. Yaffe, J. High Energy Phys. 11 (2000) 001.
[21] R. Rougemont, J. Noronha, and J. Noronha-Hostler, Phys. Rev. Lett. **115**, 202301 (2015).
[22] M. Greif, C. Greiner, and G. S. Denicol, Phys. Rev. D **93**, 096012 (2016); **96**, 059902(E) (2017).
[23] R. Rougemont, R. Critelli, J. Noronha-Hostler, J. Noronha, and C. Ratti, Phys. Rev. D **96**, 014032 (2017).
[24] A. Monnai, Phys. Rev. C **86**, 014908 (2012).
[25] M. M. Aggarwal *et al.* (STAR), arXiv:1007.2613.
[26] B. Mohanty (STAR), J. Phys. G **38**, 124023 (2011).
[27] J. T. Mitchell (PHENIX), Nucl. Phys. **A904–905**, 903c (2013).
[28] G. Odyniec, J. Phys. Conf. Ser. **455**, 012037 (2013).
[29] L. Adamczyk *et al.* (STAR), Phys. Rev. C **96**, 044904 (2017).
[30] L. Onsager, Phys. Rev. **37**, 405 (1931).
[31] L. Onsager, Phys. Rev. **38**, 2265 (1931).
[32] A. Monnai and T. Hirano, Nucl. Phys. **A847**, 283 (2010).
[33] S. Chapman and T. G. Cowling, *The Mathematical Theory of Non-Uniform Gases: An Account of the Kinetic Theory of Viscosity, Thermal Conduction and Diffusion in Gases* (Cambridge University Press, Cambridge, England, 1970).
[34] G. S. Denicol, J. Phys. G **41**, 124004 (2014).
[35] G. S. Denicol, J. Noronha, H. Niemi, and D. H. Rischke, Phys. Rev. D **83**, 074019 (2011).
[36] K. A. Olive *et al.* (Particle Data Group), Chin. Phys. C **38**, 090001 (2014).
[37] O. Buss, T. Gaitanos, K. Gallmeister, H. van Hees, M. Kaskulov, O. Lalakulich, A. B. Larionov, T. Leitner, J. Weil, and U. Mosel, Phys. Rep. **512**, 1 (2012).
[38] S. A. Bass *et al.*, Prog. Part. Nucl. Phys. **41**, 255 (1998).
[39] M. Bleicher, E. Zabrodin, C. Spieles, S. A. Bass, C. Ernst, S. Soff, L. Bravina, M. Belkacem, H. Weber, H. Stöcker, and W. Greiner, J. Phys. G **25**, 1859 (1999).
[40] Z. Xu and C. Greiner, Phys. Rev. Lett. **100**, 172301 (2008).
[41] I. Bouras, E. Molnar, H. Niemi, Z. Xu, A. El, O. Fochler, C. Greiner, and D. H. Rischke, Phys. Rev. Lett. **103**, 032301 (2009).
[42] J. A. Fotakis, M. Greif, G. S. Denicol, and C. Greiner (to be published).
[43] C. Greiner, P. Koch, and H. Stöcker, Phys. Rev. Lett. **58**, 1825 (1987).
[44] J. Cleymans and H. Satz, Z. Phys. C **57**, 135 (1993).
[45] J. M. Torres-Rincon, Hadronic Transport Coefficients from Effective Field Theories, Ph.D. thesis, Universidad Complutense de Madrid (2012).
[46] W. Cassing and E. L. Bratkovskaya, Phys. Rev. C **78**, 034919 (2008).
[47] W. Cassing and E. L. Bratkovskaya, Nucl. Phys. **A831**, 215 (2009).
[48] J. Novak, K. Novak, S. Pratt, J. Vredevoogd, C. Coleman-Smith, and R. L. Wolpert, Phys. Rev. C **89**, 034917 (2014).
[49] J. Weil *et al.*, Phys. Rev. C **94**, 054905 (2016).