Realistic tribimaximal neutrino mixing

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We propose a generalized version of the tribimaximal (TBM) ansatz for lepton mixing, leading to a nonzero reactor angle θ_{13} and CP violation. The latter is characterized by two CP phases. The Dirac phase, affecting neutrino oscillations, is nearly maximal ($\delta_{CP} \sim \pm\pi/2$), while the Majorana phase implies narrow allowed ranges for the neutrinoless double beta decay amplitude. The solar angle θ_{12} lies nearly at its TBM value, while the atmospheric angle θ_{23} has the TBM value for a maximal δ_{CP}. Neutrino oscillation predictions can be tested in present and upcoming experiments.

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I. INTRODUCTION

Ever since the discovery of neutrino oscillations, the structure of the leptonic mixing matrix has been an active topic of research. Over the last twenty years or so, there has been a flood of both theoretical and experimental activity aimed at determining and understanding the structure of the leptonic mixing matrix. Solar and atmospheric data, confirmed by accelerator and reactor data, made it clear that the structure of lepton mixing is quite at odds with that of quarks, given the large values of θ_{12} and θ_{23}. These observations were soon encoded in the tribimaximal mixing (TBM) ansatz proposed by Harrison, Perkins, and Scott [1], described by

$$U_0 = \begin{bmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{bmatrix}. \quad (1)$$

Since it was first proposed, the TBM ansatz has been a popular benchmark for describing the pattern of lepton mixing, inspiring a flood of theory papers. It gives $\theta_{12} = \sin^{-1}(\frac{1}{\sqrt{3}})$, $\theta_{23} = \pi/4$, whose status is rather good in view of the latest neutrino oscillation global fit [2,3]. Unfortunately, it predicts $\theta_{13} = 0$, and hence, CP-conservation in neutrino oscillation. Indeed, data from reactors have indicated that such a “bona fide” TBM ansatz cannot be the correct description of nature, since the leptonic mixing angle θ_{13} has been established to be nonzero to a very high significance [4–6]. Moreover, there has been mounting evidence for CP violation in neutrino oscillations, providing further indication that amendment is needed.

Motivated by the need for departing from the simplest “first-order” form for the TBM ansatz, Eq. (1), here, we propose a generalized version of the TBM ansatz (GTBM), which correctly accounts for the nonzero value of θ_{13} and introduces the CP violation as follows:

$$U = \begin{bmatrix} \sqrt{\frac{2}{3}} & \frac{e^{-i\rho}\cos\theta}{\sqrt{3}} & -\frac{ie^{-i\rho}\sin\theta}{\sqrt{3}} \\ -\frac{e^{i\rho}}{\sqrt{6}} & \frac{\cos\theta}{\sqrt{3}} - \frac{ie^{-i\sigma}\sin\theta}{\sqrt{2}} & \frac{e^{-i\sigma}\cos\theta}{\sqrt{2}} - \frac{i\sin\theta}{\sqrt{3}} \\ \frac{e^{i(\rho+\sigma)}}{\sqrt{6}} & -\frac{e^{i\sigma}\cos\theta}{\sqrt{3}} - \frac{i\sin\theta}{\sqrt{2}} & \frac{\cos\theta}{\sqrt{2}} + \frac{ie^{i\sigma}\sin\theta}{\sqrt{3}} \end{bmatrix}. \quad (2)$$

This new ansatz is characterized by just one angle θ and two phases ρ and σ. These are three parameters to be compared with the three angles, plus three (physical) phases characterizing the three-family (unitary) lepton mixing matrix [7]. The latter can be written in the symmetric form as $U = U_{23}(\theta_{23}, \phi_{23}) \cdot U_{13}(\theta_{13}, \phi_{13}) \cdot U_{12}(\theta_{12}, \phi_{12})$, where $U_{ij}(\theta, \phi)$ are matrices corresponding to complex rotations in the ij plane, each characterized by an angle θ_{ij} and an associated phase ϕ_{ij} [7]. In addition to the Dirac CP phase $\delta_{CP} = \phi_{13} - \phi_{12} - \phi_{23}$, one has two

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Majorana phases [8,9] that affect neutrinoless double beta decay. Equation (2) gives all of these six parameters in terms of one angle θ plus two phase parameters ρ, σ. The parameters have ranges

$$0 \le \theta < \pi, \qquad 0 \le \rho < \pi, \qquad 0 \le \sigma < 2\pi. \tag{3}$$

We now turn to the several interesting limiting cases of the above GTBM matrix in Eq. (2).

II. TBM LIMIT

The first is the limit $\theta, \rho, \sigma \to 0$, in which case our GTBM mixing matrix in Eq. (2) reduces to the simplest celebrated TBM form, U_0 in Eq. (1). This is unrealistic, as it cannot describe reactor neutrino data.

III. COMPLEX TBM LIMIT

In the limit of $\theta \to 0$ and any arbitrary value of ρ, σ, the matrix reduces to a "complex TBM" matrix, which is a TBM matrix with additional CP phases. This matrix is given by

$$U_{CTBM} = \begin{bmatrix} \sqrt{\frac{2}{3}} & \frac{e^{-i\rho}}{\sqrt{3}} & 0 \\ -\frac{e^{i\rho}}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{e^{-i\sigma}}{\sqrt{2}} \\ \frac{e^{i(\rho+\sigma)}}{\sqrt{6}} & -\frac{e^{i\sigma}}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{bmatrix}. \tag{4}$$

The phases ρ and σ are physical parameters only if neutrinos are of the Majorana-type, and can be rotated away otherwise. Indeed, for Dirac neutrinos, there is no difference between TBM and complex TBM. For the Majorana neutrino case, the phases in the symmetric parametrization are given as $\phi_{12} = \rho$ and $\phi_{23} = \sigma$, while the Dirac phase δ_{CP} is unphysical, since $\theta_{13} = 0$.

VI. THE $\mu - \tau$ SYMMETRIC LIMIT

We now discuss the realistic limits of GTBM that lead to $\theta_{13} \neq 0$, as required by current data [4–6]. One of the properties of the TBM matrix was the so-called $\mu - \tau$ symmetry, i.e., $|U_{\mu j}| = |U_{\tau j}|$; $j = 1, 2, 3$ [1,10]. For $\sigma \to 0$ and any arbitrary values of θ, ρ, the GTBM matrix also retains this symmetry, reducing to

$$U = \begin{bmatrix} \sqrt{\frac{2}{3}} & \frac{e^{-i\rho}\cos\theta}{\sqrt{3}} & -\frac{ie^{-i\rho}\sin\theta}{\sqrt{3}} \\ -\frac{e^{i\rho}}{\sqrt{6}} & \frac{\cos\theta}{\sqrt{3}} - \frac{i\sin\theta}{\sqrt{2}} & \frac{\cos\theta}{\sqrt{2}} - \frac{i\sin\theta}{\sqrt{3}} \\ \frac{e^{i\rho}}{\sqrt{6}} & -\frac{\cos\theta}{\sqrt{3}} - \frac{i\sin\theta}{\sqrt{2}} & \frac{\cos\theta}{\sqrt{2}} + \frac{i\sin\theta}{\sqrt{3}} \end{bmatrix}. \tag{5}$$

Indeed, one sees that the matrix in Eq. (5) also has an inherent $\mu - \tau$ symmetry, leading to maximal atmospheric angle $\theta_{23} = \frac{\pi}{4}$ and a maximal CP violating value of the CP phase $\delta_{CP} = \pm\frac{\pi}{2}$. The other two angles are also nonzero and are correlated with each other, as follows:

$$\cos^2\theta_{12}\cos^2\theta_{13} = \frac{2}{3}. \tag{6}$$

Using the 3σ range of the reactor mixing angle $1.96 \times 10^{-2} \le \sin^2\theta_{13} \le 2.41 \times 10^{-2}$ [2,3], we obtain $0.346 \le \sin^2\theta_{12} \le 0.349$ for the solar mixing angle. This is illustrated in Fig. 1, in which the shaded boxes highlight the 1 and 3σ regions indicated by the current neutrino oscillation global fit. This correlation is rather different from the one predicted in [11]. The additional CP phases are physical, both Majorana and Dirac, since $\theta_{13} \neq 0$ also makes ϕ_{13} well-defined. This $\mu - \tau$ symmetric case has implications for m_{ee}, shown in the Fig. 5.

In the $\mu - \tau$ symmetric matrix of Eq. (5), one can further take the $\rho \to 0$ limit, in which case we get an even simpler matrix given by

$$U = \begin{bmatrix} \sqrt{\frac{2}{3}} & \frac{\cos\theta}{\sqrt{3}} & -\frac{i\sin\theta}{\sqrt{3}} \\ -\frac{1}{\sqrt{6}} & \frac{\cos\theta}{\sqrt{3}} - \frac{i\sin\theta}{\sqrt{2}} & \frac{\cos\theta}{\sqrt{2}} - \frac{i\sin\theta}{\sqrt{3}} \\ \frac{1}{\sqrt{6}} & -\frac{\cos\theta}{\sqrt{3}} - \frac{i\sin\theta}{\sqrt{2}} & \frac{\cos\theta}{\sqrt{2}} + \frac{i\sin\theta}{\sqrt{3}} \end{bmatrix}. \tag{7}$$

Notice that this matrix shares many properties of matrix in Eq. (5), e.g., the maximal atmospheric angle, maximal CP violation, and the correlation given in Eq. (6). In addition, the Majorana phase is fixed, since now $\rho = 0$, leading to very sharp predictions for m_{ee} as shown in Fig. 2. For example, for the case of inverse ordering (IO), the neutrinoless double beta decay amplitude is nearly maximal, while for the normal ordering (NO) case, there is a

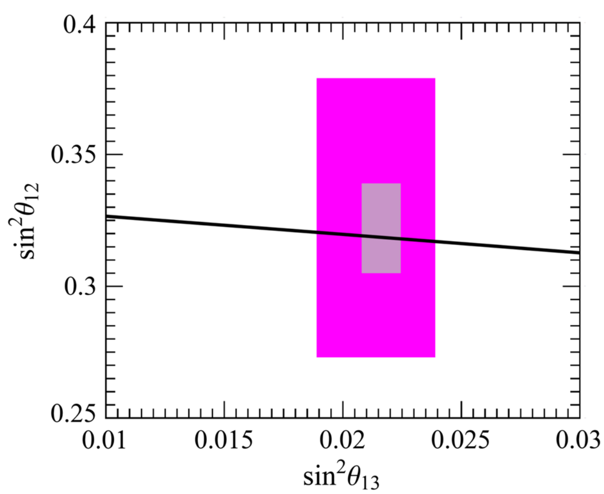

FIG. 1. Correlation between $\sin^2\theta_{13}$ and $\sin^2\theta_{12}$ given in Eq. (6). Notice that in the whole experimentally allowed range [2], the value of $\sin^2\theta_{12}$ remains very close to 1/3.

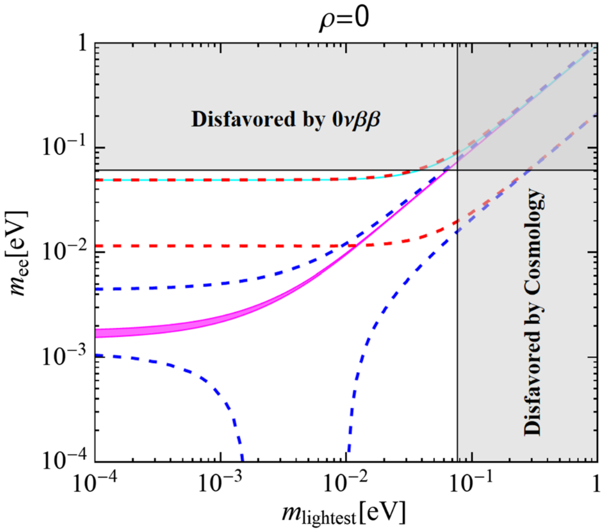

FIG. 2. $|m_{ee}|$ prediction for NO and IO when $\rho = 0$. Here, θ is taken as a free parameter, and we require the three mixing angles to lie in their allowed 3σ regions [2,3]. Note that m_{ee} does not depend on σ.

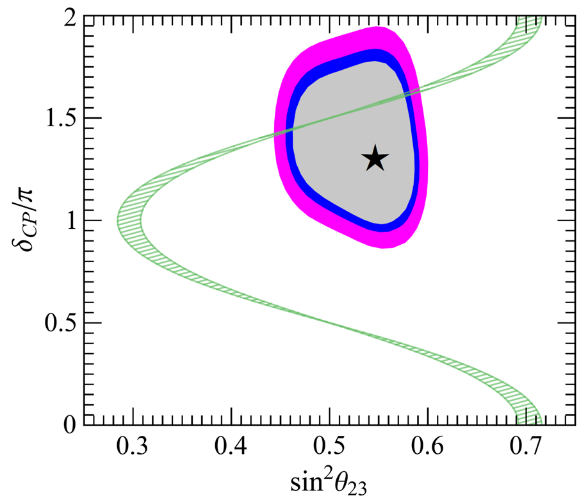

FIG. 3. The correlation between the atmospheric angle θ_{23} and the CP phase δ_{CP} predicted by our generalized TBM matrix in Eq. (2) is given by the hatched band, while the 1, 2, and 3σ regions allowed by the current neutrino oscillation global fit are indicated by the shaded areas [2,3].

lower bound for this amplitude, since destructive interference is prevented.

V. THE $\rho \to 0$ LIMIT

So far, the limits we have discussed all lead to the maximal atmospheric mixing angle, i.e., they all predict $\theta_{23} = \pi/4$. While this is consistent with current data, there is a slight preference for the second octant [2,3]. Our proposed GTBM matrix is flexible enough to allow for deviations from the maximal θ_{23}. The possibility of a nonmaximal θ_{23} can be seen in the limiting case where $\rho \to 0$, where the mixing matrix is given by

$$U = \begin{bmatrix} \sqrt{\frac{2}{3}} & \frac{\cos\theta}{\sqrt{3}} & -\frac{i\sin\theta}{\sqrt{3}} \\ -\frac{1}{\sqrt{6}} & \frac{\cos\theta}{\sqrt{3}} - \frac{ie^{-i\sigma}\sin\theta}{\sqrt{2}} & \frac{e^{-i\sigma}\cos\theta}{\sqrt{2}} - \frac{i\sin\theta}{\sqrt{3}} \\ \frac{e^{i\sigma}}{\sqrt{6}} & -\frac{e^{i\sigma}\cos\theta}{\sqrt{3}} - \frac{i\sin\theta}{\sqrt{2}} & \frac{\cos\theta}{\sqrt{2}} + \frac{ie^{i\sigma}\sin\theta}{\sqrt{3}} \end{bmatrix}. \quad (8)$$

This matrix still shares some of the properties of the $\mu-\tau$ symmetric matrix of Eq. (5). For example, the correlation in Eq. (6) still holds, relating solar and reactor angles as shown in Fig. 1. However, in contrast to the $\mu-\tau$ symmetric limit, we can now have deviations from the maximal atmospheric mixing, as well as deviations from maximal CP violation. In fact, these departures are correlated with each other, as shown in Fig. 3, which also highlights the 1 and 3σ regions indicated by the current neutrino oscillation global fit [2,3].

The mixing matrix of Eq. (8) also leads to the fixed Majorana phase values given by $\phi_{12} = 0, \phi_{13} = \frac{\pi}{2}$, implying sharp predictions for m_{ee}, as shown in Fig. 2.

VI. GENERAL TRIBIMAXIMAL MIXING

Having discussed the various limits of our proposed GTBM matrix, (2), we now briefly discuss its general properties. The full set of mixing angles and phases is given as

$$\sin^2\theta_{12} = \frac{\cos^2\theta}{\cos^2\theta + 2}, \qquad \sin^2\theta_{23} = \frac{1}{2} + \frac{\sqrt{6}\sin 2\theta \sin\sigma}{2\cos^2\theta + 4},$$
$$\sin^2\theta_{13} = \frac{\sin^2\theta}{3}, \qquad \tan\delta_{CP} = \frac{(\cos^2\theta + 2)\cot\sigma}{5\cos^2\theta - 2}, \quad (9)$$

$$\phi_{12} = \rho, \qquad \phi_{13} = \rho + \frac{\pi}{2}, \quad (10)$$

which implies

$$\left|\sin^2\theta_{23} - \frac{1}{2}\right| = \tan\theta_{13}\sqrt{2 - 4\tan^2\theta_{13}}|\sin\sigma|. \quad (11)$$

The parameter σ measures the deviation of θ_{23} from maximal mixing, as shown in Fig. 4. We can read off that σ can only vary within the region $[0, 0.172\pi] \cup [0.828\pi, 1.172\pi] \cup [1.828\pi, 2\pi)$.

The expression for the parameter m_{ee} describing the neutrinoless double beta decay amplitude also takes a rather simple form given by

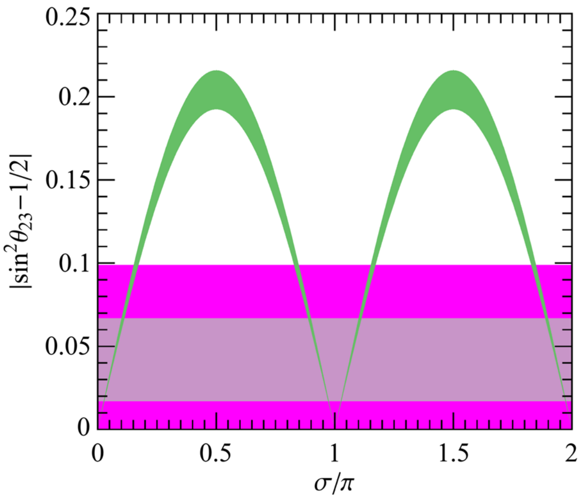

FIG. 4. The predicted dependence of $|\sin^2\theta_{23}-\frac{1}{2}|$ on the parameter σ is indicated by the curved band. Its width comes from varying θ_{13} within its 3σ range, while the horizontal band gives the current determination of θ_{23} [2,3].

$$|m_{ee}| = \frac{1}{3}|2e^{2i\rho}m_1 + m_2\cos^2\theta - m_3\sin^2\theta|. \tag{12}$$

From these mixing angles and phases in Eq. (9), one can further obtain two nontrivial relations given by

$$\cos^2\theta_{12}\cos^2\theta_{13} = \frac{2}{3}, \tag{13}$$

$$\tan 2\theta_{23}\cos\delta_{CP} = \frac{5\sin^2\theta_{13}-1}{4\tan\theta_{12}\sin\theta_{13}}. \tag{14}$$

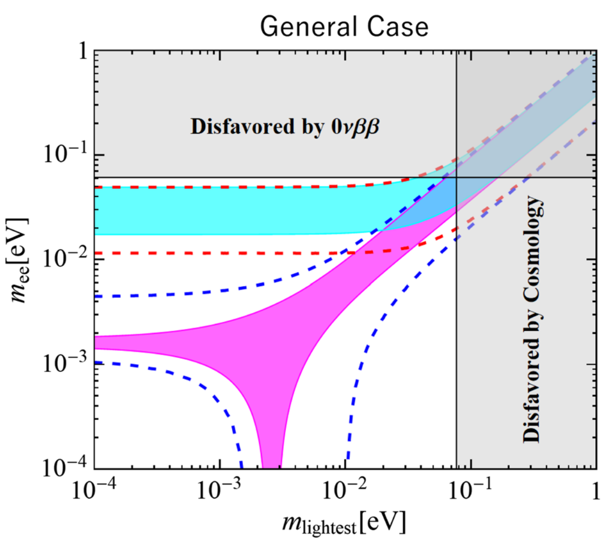

FIG. 5. $|m_{ee}|$ prediction for NO and IO in the most general GTBM ansatz. Here, the parameters ρ and θ are varied within their allowed 3σ ranges [2,3]. Note that m_{ee} does not depend on σ.

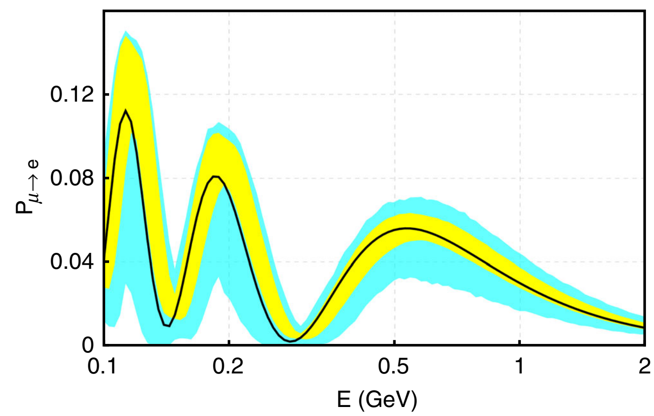

FIG. 6. The allowed range of electron neutrino appearance probability at T2K covers a more restricted region, thanks to the GTBM predictions. Here, the black line corresponds to the best fit, the cyan region is the general three-neutrino result, while the yellow region is the GTBM prediction.

The first is a correlation between θ_{12} and θ_{13}, shown in Fig. 1, while the second is a correlation between θ_{23} and δ_{CP}, depicted in Fig. 3. Owing to the constrained nature of the mixing angles and phases of our ansatz, one also gets predictions for m_{ee}, shown in Fig. 5.

The predictions made by the GTBM ansatz can also be tested in currently running and upcoming neutrino oscillation experiments. The predictions made by GTBM to oscillation experiments is illustrated in Fig. 6. This estimate is for the T2K setup, neglecting matter effects, as an approximation. Clearly, the allowed range of electron neutrino appearance probability at T2K is substantially restricted with respect to the generic expectation.

In conclusion, we have proposed a realistic generalization of the TBM ansatz, which not only accounts for nonzero measured value of θ_{13} but also makes definite and testable predictions for the other parameters of the lepton mixing matrix, including *CP* phases. Our GTBM matrix is characterized in terms of three independent parameters, which determine all six mixing parameters, leading to several testable predictions as we discussed at length. Apart from correcting for θ_{13}, the GTBM matrix retains many of the features of the original TBM matrix from the point of basic underlying symmetries, as we showed by discussing various limits of the GTBM matrix.

VII. *CP* SYMMETRY AS THE ORIGIN OF THE GTBM ANSATZ

Before closing, we comment on the theoretical origin of our GTBM matrix. We note that this ansatz may be derived systematically by the method of generalized *CP* symmetries [12–14]. For example, the mixing matrix in Eq. (7) can be derived from the S_4 flavor symmetry and generalized *CP* [15,16]. In order to derive the GTBM matrix in Eq. (2), one starts from the complex TBM matrix (CTBM) of Eq. (4) and extracts its CP symmetries and flavor symmetries. In the charged lepton diagonal mass basis, the four CP symmetries X_i and the four flavor symmetries G_i are given by

$$X_i = U_{\rm CTBM}\hat{d}_i U^T_{\rm CTBM}, \tag{15}$$

where

$$\hat{d}_1 = {\rm diag}(1,-1,-1), \hat{d}_2 = {\rm diag}(-1,1,-1),$$
$$\hat{d}_3 = {\rm diag}(-1,-1,1), \hat{d}_4 = {\rm diag}(1,1,1). \tag{16}$$

Also, the flavor symmetries G_i; $i = 1, 2, 3, 4$ are given by

$$\begin{aligned} G_1 &= X_2X_3^* = X_3X_2^* = X_4X_1^* = X_1X_4^*,\\ G_2 &= X_1X_3^* = X_3X_1^* = X_4X_2^* = X_2X_4^*,\\ G_3 &= X_1X_2^* = X_2X_1^* = X_4X_3^* = X_3X_4^*,\\ G_4 &= X_1X_1^* = X_2X_2^* = X_3X_3^* = X_4X_4^*. \end{aligned} \tag{17}$$

The CP and flavor symmetries corresponding to the real TBM matrix of (1) can be obtained from Eqs. (15) and (17), respectively, by simply taking the limit $\rho, \sigma \to 0$. It is instructive to display explicitly the matrix form of the CP symmetries associated to the real TBM ansatz, which are given by

$$X_1 = \frac{1}{3}\begin{pmatrix} 1 & -2 & 2 \\ -2 & -2 & -1 \\ 2 & -1 & -2 \end{pmatrix}, \quad X_2 = \frac{1}{3}\begin{pmatrix} -1 & 2 & -2 \\ 2 & -1 & -2 \\ -2 & -2 & -1 \end{pmatrix},$$
$$X_3 = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \quad X_4 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}. \tag{18}$$

Notice that the CP symmetry X_3 of (18) is nothing but the famous $\mu-\tau$ reflection symmetry, which the real TBM matrix is known to possess [1,10]. Moreover, the CP symmetry X_4 of (18) is simply the diagonal CP symmetry of phases.

Imposing any three of the CP symmetries in (15) on the neutrino mass matrix,[1] one recovers the complex TBM matrix of Eq. (4). This is clearly ruled out by current neutrino oscillation data. In order to obtain realistic mass matrices, we assume that at the leading order, the neutrino mass matrix $M_\nu^{(0)}$ preserves all four CP symmetries given in Eq. (15). The leading order neutrino mass matrix $M_\nu^{(0)}$ satisfies

$$X_i^T M_\nu^{(0)} X_i = M_\nu^{(0)*}, \tag{19}$$

where X_i; $i = 1, 2, 3, 4$ are the four CP symmetries of (15). This in turn implies that $U^T_{\rm CTBM} M_\nu^{(0)} U_{\rm CTBM}$ is a real diagonal matrix [12], which can be written as

$$U^T_{\rm CTBM} M_\nu^{(0)} U_{\rm CTBM} = {\rm diag}(m_1, m_2, m_3), \tag{20}$$

and leads to

$$M_\nu^{(0)} = U^*_{\rm CTBM}{\rm diag}(m_1, m_2, m_3)U^\dagger_{\rm CTBM}. \tag{21}$$

Thus, as mentioned before, if all four CP symmetries (in fact any subset of three independent ones is sufficient) are imposed simultaneously, we recover back a neutrino mass matrix diagonalized by the complex TBM matrix.

In order to generate realistic mass and mixing patterns, we add perturbation terms, preserving only the X_2, X_3 CP symmetries of (15). This implies that the leptonic mixing matrix is no longer the complex TBM matrix, but a closely related variant of it. After adding the perturbation, the full mass term $M_\nu = M_\nu^{(0)} + \delta M_\nu$ satisfies

$$U^T_{\rm CTBM}(M_\nu^{(0)} + \delta M_\nu)U_{\rm CTBM} = \begin{pmatrix} m_1 & 0 & 0 \\ 0 & m_2 & i\delta m \\ 0 & i\delta m & m_3 \end{pmatrix}. \tag{22}$$

The above matrix can be easily diagonalized by the matrix ${\rm diag}(1, -i, 1)O_{23}$, where

$$O_{23} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & \sin\theta \\ 0 & -\sin\theta & \cos\theta \end{pmatrix}, \tag{23}$$

with

$$\tan 2\theta = \frac{2\delta m}{m_3 + m_2}. \tag{24}$$

Thus, the mixing matrix diagonalizing the full mass matrix M_ν is given by

$$U_{\rm GTBM} = U_{\rm CTBM}{\rm diag}(1, -i, 1)O_{23}Q_\nu. \tag{25}$$

where Q_ν is a diagonal matrix with entries ±1 and $\pm i$, which encode the CP parity of the neutrino states, and in our case, we take it to be

$$Q_\nu = {\rm diag}(1, i, 1). \tag{26}$$

The mixing matrix obtained in Eq. (25) is nothing but the matrix describing our GTBM ansatz in Eq. (2). Having

[1]For simplicity, we work in the basis of diagonal charged lepton mass matrix. In this basis, the whole leptonic mixing is solely due to the neutrino sector.

been obtained from the TBM matrix, the GTBM matrix naturally shares many of the properties, symmetries, and predictions associated with the TBM ansatz. A more detailed discussion of the generalized *CP* methodology and its power to produce other potentially realistic ansatz forms for the lepton mixing matrix will be presented elsewhere.

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