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Left–right symmetry, orbifold S^1/Z_2, and radiative breaking of $U(1)_{\mathrm{R}} \times U(1)_{\mathrm{B-L}}$

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We study the origin of electroweak symmetry under the assumption that $SU(4)_{\mathrm{C}} \times SU(2)_{\mathrm{L}} \times SU(2)_{\mathrm{R}}$ is realized on a five-dimensional space-time. The Pati–Salam-type gauge symmetry is reduced to $SU(3)_{\mathrm{C}} \times SU(2)_{\mathrm{L}} \times U(1)_{\mathrm{R}} \times U(1)_{\mathrm{B-L}}$ by the orbifold breaking mechanism on the orbifold S^1/Z_2. The breakdown of residual gauge symmetries occurs radiatively via the Coleman–Weinberg mechanism, such that the $U(1)_{\mathrm{R}} \times U(1)_{\mathrm{B-L}}$ symmetry is broken down to $U(1)_{\mathrm{Y}}$ by the vacuum expectation value of an $SU(2)_{\mathrm{L}}$ singlet scalar field and the $SU(2)_{\mathrm{L}} \times U(1)_{\mathrm{Y}}$ symmetry is broken down to the electric one $U(1)_{\mathrm{EM}}$ by the vacuum expectation value of an $SU(2)_{\mathrm{L}}$ doublet scalar field regarded as the Higgs doublet. The negative Higgs squared mass term originates from an interaction between the Higgs doublet and an $SU(2)_{\mathrm{L}}$ singlet scalar field as a Higgs portal. The vacuum stability is recovered due to the contributions from the Kaluza–Klein modes of gauge bosons.

Subject Index B40, B43

1. Introduction

The discovery of the Higgs boson [1,2], the last of the standard model (SM) particles, kicks off a new stage of physics beyond the SM. Mysteries concerning the Higgs boson have thickened because no evidence from new physics such as supersymmetry or compositeness have yet been discovered.

Among the big mysteries is the origin of the electroweak scale, or how the vacuum expectation value (VEV) of the Higgs boson, $v = 246\,\mathrm{GeV}$, is understood. To solve the riddle, we need to uncover the origin of the Higgs potential, and in particular a mass term therein. Another mystery is why the vacuum is stable enough after the breakdown of electroweak symmetry. With the Higgs quartic coupling constant $\lambda \doteqdot 0.129$ estimated from the observed Higgs mass $m_h \doteqdot 125.1\,\mathrm{GeV}$, we encounter the vacuum stability problem that λ becomes negative at around 10^7 GeV and the vacuum can decay.

In this paper we tackle these problems through extensions of gauge symmetries and space-time. Concepts such as simplicity and variety are also adopted on a case-by-case basis. The SM gauge symmetry can be extended to contain a left–right symmetry. A typical one is the gauge group $G_{\mathrm{PS}} \equiv SU(4)_{\mathrm{C}} \times SU(2)_{\mathrm{L}} \times SU(2)_{\mathrm{R}}$ in the Pati–Salam model [3]. The space-time can be expanded to include extra dimensions. The orbifold S^1/Z_2 is used as an extra space, because it is simple and has several advantages. Different breaking mechanisms are utilized for the breakdown of the gauge symmetry G_{PS} into $SU(3)_{\mathrm{C}} \times U(1)_{\mathrm{EM}}$, presuming that nature respects diversity.

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We give an outline of our model. Particle physics above some high-energy scale M_{PS} is described by a gauge theory with G_{PS} on the five-dimensional (5D) space-time including S^1/Z_2 as an extra dimension. The gauge symmetry G_{PS} is reduced to $G_{3211} \equiv SU(3)_{\mathrm{C}} \times SU(2)_{\mathrm{L}} \times U(1)_{\mathrm{R}} \times U(1)_{\mathrm{B-L}}$ by the orbifold breaking mechanism.[1] The breakdown of residual gauge symmetries occurs radiatively via the Coleman–Weinberg mechanism.[2] Concretely, the $U(1)_{\mathrm{R}} \times U(1)_{\mathrm{B-L}}$ symmetry is broken down to $U(1)_{\mathrm{Y}}$ by the VEV v_{R} of an $SU(2)_{\mathrm{L}}$ singlet scalar field. Then, a gauge boson corresponding to the broken $U(1)$ symmetry acquires a mass $M_{Z_{\mathrm{LR}}}$ of $O(v_{\mathrm{R}})$. The $SU(2)_{\mathrm{L}} \times U(1)_{\mathrm{Y}}$ symmetry is broken down to the electric one $U(1)_{\mathrm{EM}}$ by the VEV of an $SU(2)_{\mathrm{L}}$ doublet scalar field regarded as the Higgs doublet. If the $SU(2)_{\mathrm{L}}$ singlet scalar field is replaced by its VEV, we obtain the Higgs potential, including a negative squared mass term originating from an interaction between the Higgs doublet and the $SU(2)_{\mathrm{L}}$ singlet scalar field as a Higgs portal. The vacuum stability is recovered due to the contributions from the Kaluza–Klein modes of gauge bosons appearing at a compactification scale M_{c} and above there.

This paper is organized as follows. In the next section, we formulate a 5D Pati–Salam model. We examine the Coleman–Weinberg mechanism and the vacuum stability in a four-dimensional (4D) model with G_{3211} in Sect. 3. In the last section, we give conclusions and discussions.

2. Five-dimensional Pati–Salam model

The space-time is assumed to be factorized into a product of 4D Minkowski space-time M^4 and the orbifold S^1/Z_2, whose coordinates are denoted by x^μ (or x), $\mu = 0, 1, 2, 3$, and y, respectively. The 5D notation x^M ($M = 0, 1, 2, 3, 5$) is also used with $x^5 = y$. The S^1/Z_2 is obtained by dividing the circle S^1 (with the identification $y \sim y + 2\pi R$) by the Z_2 transformation $y \to -y$. Then, the point y is identified with $-y$ on S^1/Z_2, and the space is regarded as an interval with length πR (R being the radius of S^1).

In the following, we formulate a Pati–Salam model on $M^4 \times S^1/Z_2$. First we present the particle contents in Table 1. In most cases, we pay attention to bosons under the assumption that matter fields (quarks and leptons) live on the 4D brane at $y = 0$.

The gauge bosons possess several components such that

$$G_M(x, y) = \sum_{a=1}^{15} G_M^a(x, y) T_{\mathrm{C}}^a,$$
$$W_{\mathrm{L}M}(x, y) = \sum_{a=1}^{3} W_{\mathrm{L}M}^a(x, y) T_{\mathrm{L}}^a, \quad W_{\mathrm{R}M}(x, y) = \sum_{a=1}^{3} W_{\mathrm{R}M}^a(x, y) T_{\mathrm{R}}^a, \tag{1}$$

[1] The orbifold breaking mechanism was originally proposed in superstring theory [4,5]. The Z_2 orbifolding was used in superstring theory [6] and heterotic M-theory [7,8]. In field-theoretical models, it was applied to the reduction of global supersymmetry [9,10], which is an orbifold version of the Scherk–Schwarz mechanism [11,12], and then to the reduction of gauge symmetry [13,14]. The left–right symmetric models on 5D space-time were proposed in Refs. [15,16], and phenomenologies of gauge bosons and matter fields were studied intensively based on the gauge group $SU(2)_{\mathrm{L}} \times SU(2)_{\mathrm{R}} \times U(1)_{\mathrm{B-L}}$.

[2] The Coleman–Weinberg mechanism was originally proposed by S. Coleman and E. Weinberg [17], and used in left–right symmetric models [18–21] and a minimal extension of the SM with an SM singlet and an extra $U(1)$ symmetry [22].

Table 1. Gauge quantum numbers of bosons in the 5D Pati–Salam model.

Bosons	$SU(4)_{\mathrm{C}}$	$SU(2)_{\mathrm{L}}$	$SU(2)_{\mathrm{R}}$
$G_M(x,y)$	**15**	**1**	**1**
$W_{\mathrm{L}M}(x,y)$	**1**	**3**	**1**
$W_{\mathrm{R}M}(x,y)$	**1**	**1**	**3**
$\Phi_{\mathrm{L}}(x,y)$	**4**	**2**	**1**
$\Phi_{\mathrm{R}}(x,y)$	$\overline{\mathbf{4}}$	**1**	**2**
$\Phi_{\mathrm{B}}(x,y)$	**1**	**2**	**2**

where T_{C}^a, T_{L}^a, and T_{R}^a are generators of $SU(4)_{\mathrm{C}}$, $SU(2)_{\mathrm{L}}$, and $SU(2)_{\mathrm{R}}$, respectively. We need a scalar field $\Phi_{\mathrm{B}}(x,y)$ that obeys the bi-fundamental representation under $SU(2)_{\mathrm{L}} \times SU(2)_{\mathrm{R}}$ to construct Yukawa interactions on the brane. The Lagrangian density for bosons is given by

$$\mathcal{L}_{5\mathrm{D}} = -\frac{1}{4}\sum_{a=1}^{15} G_{MN}^a G^{aMN} - \frac{1}{4}\sum_{a=1}^{3} W_{\mathrm{L}MN}^a W_{\mathrm{L}}^{aMN} - \frac{1}{4}\sum_{a=1}^{3} W_{\mathrm{R}MN}^a W_{\mathrm{R}}^{aMN}$$
$$+ (D_M\Phi_{\mathrm{L}})^\dagger (D^M\Phi_{\mathrm{L}}) + (D_M\Phi_{\mathrm{R}})^\dagger (D^M\Phi_{\mathrm{R}}) + \mathrm{tr}(D_M\Phi_{\mathrm{B}})^\dagger (D^M\Phi_{\mathrm{B}}) - V_{5\mathrm{D}}, \tag{2}$$

where G_{MN}^a, $W_{\mathrm{L}MN}^a$, and $W_{\mathrm{R}MN}^a$ are field strengths of $SU(4)_{\mathrm{C}}$, $SU(2)_{\mathrm{L}}$, and $SU(2)_{\mathrm{R}}$ gauge bosons, respectively. The covariant derivative D_M and the scalar potential $V_{5\mathrm{D}}$ are given by

$$D_M = \partial_M + ig_4\sum_{a=1}^{15} G_M^a T_{\mathrm{C}}^a + ig_{\mathrm{L}}\sum_{a=1}^{3} W_{\mathrm{L}M}^a T_{\mathrm{L}}^a + ig_{\mathrm{R}}\sum_{a=1}^{3} W_{\mathrm{R}M}^a T_{\mathrm{R}}^a, \tag{3}$$

$$V_{5\mathrm{D}} = \lambda_{\mathrm{L}}|\Phi_{\mathrm{L}}|^4 + \lambda_{\mathrm{R}}|\Phi_{\mathrm{R}}|^4 + \lambda_{\mathrm{B}1}\mathrm{tr}\left(|\Phi_{\mathrm{B}}|^2|\Phi_{\mathrm{B}}|^2\right) + \lambda_{\mathrm{B}2}\left(\mathrm{tr}|\Phi_{\mathrm{B}}|^2\right)^2$$
$$+ \lambda_{\mathrm{LR}}|\Phi_{\mathrm{L}}|^2|\Phi_{\mathrm{R}}|^2 + \lambda_{\mathrm{LB}}|\Phi_{\mathrm{L}}|^2\mathrm{tr}|\Phi_{\mathrm{B}}|^2 + \lambda_{\mathrm{RB}}|\Phi_{\mathrm{R}}|^2\mathrm{tr}|\Phi_{\mathrm{B}}|^2, \tag{4}$$

respectively. If we require the left–right symmetry that the theory should be invariant under the exchange $(W_{\mathrm{L}M}^a, \Phi_{\mathrm{L}})$ into $(W_{\mathrm{R}M}^a, \Phi_{\mathrm{R}})$, we obtain the following conditions among couplings:

$$g_{\mathrm{L}} = g_{\mathrm{R}},\ \ \lambda_{\mathrm{L}} = \lambda_{\mathrm{R}},\ \ \lambda_{\mathrm{LB}} = \lambda_{\mathrm{RB}}. \tag{5}$$

We suppose that all scalar fields have no bulk masses.

From the requirement that the Lagrangian density should be invariant under the translation T : $y \to y + 2\pi R$ and the Z_2 transformation $P_0 : y \to -y$, or it should be a single-valued function on the 5D space-time, non-trivial boundary conditions (BCs) of fields are allowed on S^1/Z_2.

We impose the following BCs on G_M:

$$G_\mu(x,-y) = G_\mu(x,y),\ \ G_5(x,-y) = -G_5(x,y), \tag{6}$$

$$G_\mu(x,2\pi R - y) = U_{\mathrm{C}} G_\mu(x,y) U_{\mathrm{C}}^{-1},\ \ G_5(x,2\pi R - y) = -U_{\mathrm{C}} G_5(x,y) U_{\mathrm{C}}^{-1}, \tag{7}$$

where $U_C = \mathrm{diag}(1,1,1,-1)$. We use the Z_2 transformation $P_1 : y \to 2\pi R - y$ in place of $T : y \to y + 2\pi R$. Then, the G_M are given by the Fourier expansions:

$$G^a_\mu(x,y) = \frac{1}{\sqrt{2\pi R}} G^{(0)a}_\mu(x) + \frac{1}{\sqrt{\pi R}} \sum_{n=1}^{\infty} G^{(n)a}_\mu(x) \cos\frac{ny}{R} \quad (a = 1,\dots,8,15), \tag{8}$$

$$G^a_\mu(x,y) = \frac{1}{\sqrt{\pi R}} \sum_{n=1}^{\infty} G^{(n)a}_\mu(x) \cos\frac{\left(n-\frac{1}{2}\right)y}{R} \quad (a = 9,\dots,14), \tag{9}$$

$$G^a_5(x,y) = \frac{1}{\sqrt{\pi R}} \sum_{n=1}^{\infty} G^{(n)a}_5(x) \sin\frac{ny}{R} \quad (a = 1,\dots,8,15), \tag{10}$$

$$G^a_5(x,y) = \frac{1}{\sqrt{\pi R}} \sum_{n=1}^{\infty} G^{(n)a}_5(x) \sin\frac{\left(n-\frac{1}{2}\right)y}{R} \quad (a = 9,\dots,14). \tag{11}$$

Only G^a_μ $(a = 1,\dots,8,15)$ have y-independent modes with $n = 0$ called zero modes, and $G^{(0)a}_\mu(x)$ $(a = 1,\dots,8)$ and $G^{(0)15}_\mu(x)$ are identified as the 4D gluons and the 4D $U(1)_{\mathrm{B-L}}$ gauge boson, respectively. We denote them as $G^a_\mu(x)$ and $N_\mu(x)$, respectively.

We impose the following BCs on $W_{\mathrm{L}M}$:

$$W_{\mathrm{L}\mu}(x,-y) = W_{\mathrm{L}\mu}(x,y), \quad W_{\mathrm{L}5}(x,-y) = -W_{\mathrm{L}5}(x,y), \tag{12}$$

$$W_{\mathrm{L}\mu}(x,2\pi R - y) = W_{\mathrm{L}\mu}(x,y), \quad W_{\mathrm{L}5}(x,2\pi R - y) = -W_{\mathrm{L}5}(x,y), \tag{13}$$

and then we obtain the zero modes $W^{(0)a}_{\mathrm{L}\mu}(x)$ $(a = 1,2,3)$ identified as the 4D $SU(2)_\mathrm{L}$ weak bosons and denote them as $W^a_\mu(x)$.

We impose the following BCs on $W_{\mathrm{R}M}$:

$$W_{\mathrm{R}\mu}(x,-y) = W_{\mathrm{R}\mu}(x,y), \quad W_{\mathrm{R}5}(x,-y) = -W_{\mathrm{R}5}(x,y), \tag{14}$$

$$W_{\mathrm{R}\mu}(x,2\pi R - y) = U_\mathrm{R} W_{\mathrm{R}\mu}(x,y) U_\mathrm{R}^{-1}, \quad W_{\mathrm{R}5}(x,2\pi R - y) = -U_\mathrm{R} W_{\mathrm{R}5}(x,y) U_\mathrm{R}^{-1}, \tag{15}$$

where $U_\mathrm{R} = \mathrm{diag}(1,-1)$. Then, we obtain the zero modes $W^{(0)3}_{\mathrm{R}\mu}(x)$ regarded as a $U(1)$ gauge boson. We denote $W^{(0)3}_{\mathrm{R}\mu}(x)$ and its $U(1)$ gauge group as $R_\mu(x)$ and $U(1)_\mathrm{R}$, respectively.

For scalar fields, the following BCs are imposed:

$$\Phi_\mathrm{L}(x,-y) = -\Phi_\mathrm{L}(x,y), \quad \Phi_\mathrm{L}(x,2\pi R - y) = -U_\mathrm{C}\Phi_\mathrm{L}(x,y), \tag{16}$$

$$\Phi_\mathrm{R}(x,-y) = -U_\mathrm{R}\Phi_\mathrm{R}(x,y), \quad \Phi_\mathrm{R}(x,2\pi R - y) = -U_\mathrm{C}\Phi_\mathrm{R}(x,y), \tag{17}$$

$$\Phi_\mathrm{B}(x,-y) = \Phi_\mathrm{B}(x,y), \quad \Phi_\mathrm{B}(x,2\pi R - y) = U_\mathrm{R}\Phi_\mathrm{B}(x,y). \tag{18}$$

Then, zero modes appear from the lower component of Φ_R and the upper component of Φ_B concerning $SU(2)_\mathrm{R}$, and they are denoted as $\phi_\mathrm{R}(x)$ and $\phi(x)$, respectively. Here, $\phi_\mathrm{R}(x)$ is the $SU(2)_\mathrm{L}$ singlet scalar field and $\phi(x)$ is the $SU(2)_\mathrm{L}$ doublet scalar field. $\phi(x)$ is regarded as the Higgs doublet in the SM.

We list gauge quantum numbers and mass spectra of bosons after compactification in Table 2, where Q_R is the $U(1)_\mathrm{R}$ charge and $Q_{\mathrm{B-L}}$ is the $U(1)_{\mathrm{B-L}}$ charge defined by

$$Q_{\mathrm{B-L}} \equiv \sqrt{\frac{2}{3}} T^{15}_\mathrm{C}, \tag{19}$$

Table 2. Gauge quantum numbers of bosons after compactification in the 5D Pati–Salam model.

Bosons	$SU(3)_{\mathrm{C}}$	$SU(2)_{\mathrm{L}}$	Q_{R}	$Q_{\mathrm{B-L}}$	(P_0, P_1)	Mass
$G_\mu^{(n)a}(x,y)$ $(a=1,\ldots,8)$	$\mathbf{8}$	$\mathbf{1}$	0	0	$(+1,+1)$	$\frac{n}{R}$
$G_\mu^{(n)a}(x,y)$ $(a=9,\ldots,14)$	$\mathbf{3}$	$\mathbf{1}$	0	$\frac{2}{3}$	$(+1,-1)$	$\frac{n-\frac{1}{2}}{R}$
	$\overline{\mathbf{3}}$	$\mathbf{1}$	0	$-\frac{2}{3}$	$(+1,-1)$	$\frac{n-\frac{1}{2}}{R}$
$G_\mu^{(n)15}(x,y)$	$\mathbf{1}$	$\mathbf{1}$	0	0	$(+1,+1)$	$\frac{n}{R}$
$W_{\mathrm{L}\mu}^{(n)a}(x,y)$ $(a=1,2,3)$	$\mathbf{1}$	$\mathbf{3}$	0	0	$(+1,+1)$	$\frac{n}{R}$
$W_{\mathrm{R}\mu}^{(n)3}(x,y)$	$\mathbf{1}$	$\mathbf{1}$	0	0	$(+1,+1)$	$\frac{n}{R}$
$W_{\mathrm{R}\mu}^{(n)+}(x,y)$	$\mathbf{1}$	$\mathbf{1}$	1	0	$(+1,-1)$	$\frac{n-\frac{1}{2}}{R}$
$W_{\mathrm{R}\mu}^{(n)-}(x,y)$	$\mathbf{1}$	$\mathbf{1}$	-1	0	$(+1,-1)$	$\frac{n-\frac{1}{2}}{R}$
$G_5^{(n)a}(x,y)$ $(a=1,\ldots,8)$	$\mathbf{8}$	$\mathbf{1}$	0	0	$(-1,-1)$	$\frac{n}{R}$
$G_5^{(n)a}(x,y)$ $(a=9,\ldots,14)$	$\mathbf{3}$	$\mathbf{1}$	0	$\frac{2}{3}$	$(-1,+1)$	$\frac{n-\frac{1}{2}}{R}$
	$\overline{\mathbf{3}}$	$\mathbf{1}$	0	$-\frac{2}{3}$	$(-1,+1)$	$\frac{n-\frac{1}{2}}{R}$
$G_5^{(n)15}(x,y)$	$\mathbf{1}$	$\mathbf{1}$	0	0	$(-1,-1)$	$\frac{n}{R}$
$W_{\mathrm{L}5}^{(n)a}(x,y)$ $(a=1,2,3)$	$\mathbf{1}$	$\mathbf{3}$	0	0	$(-1,-1)$	$\frac{n}{R}$
$W_{\mathrm{R}5}^{(n)3}(x,y)$	$\mathbf{1}$	$\mathbf{1}$	0	0	$(-1,-1)$	$\frac{n}{R}$
$W_{\mathrm{R}5}^{(n)+}(x,y)$	$\mathbf{1}$	$\mathbf{1}$	1	0	$(-1,+1)$	$\frac{n-\frac{1}{2}}{R}$
$W_{\mathrm{R}5}^{(n)-}(x,y)$	$\mathbf{1}$	$\mathbf{1}$	-1	0	$(-1,+1)$	$\frac{n-\frac{1}{2}}{R}$
$\Phi_{\mathrm{L}}(x,y)$	$\mathbf{3}$	$\mathbf{2}$	0	$\frac{1}{6}$	$(-1,-1)$	$\frac{n}{R}$
	$\mathbf{1}$	$\mathbf{2}$	0	$-\frac{1}{2}$	$(-1,+1)$	$\frac{n-\frac{1}{2}}{R}$
$\Phi_{\mathrm{R}}(x,y)$	$\overline{\mathbf{3}}$	$\mathbf{1}$	$\frac{1}{2}$	$-\frac{1}{6}$	$(-1,-1)$	$\frac{n}{R}$
	$\overline{\mathbf{3}}$	$\mathbf{1}$	$-\frac{1}{2}$	$-\frac{1}{6}$	$(+1,-1)$	$\frac{n-\frac{1}{2}}{R}$
	$\mathbf{1}$	$\mathbf{1}$	$\frac{1}{2}$	$\frac{1}{2}$	$(-1,+1)$	$\frac{n-\frac{1}{2}}{R}$
	$\mathbf{1}$	$\mathbf{1}$	$-\frac{1}{2}$	$\frac{1}{2}$	$(+1,+1)$	$\frac{n}{R}$
$\Phi_{\mathrm{B}}(x,y)$	$\mathbf{1}$	$\mathbf{2}$	$\frac{1}{2}$	0	$(+1,+1)$	$\frac{n}{R}$
	$\mathbf{1}$	$\mathbf{2}$	$-\frac{1}{2}$	0	$(+1,-1)$	$\frac{n-\frac{1}{2}}{R}$

using the 15th components of T_{C}^a. The fifth components of the gauge bosons are would-be Nambu–Goldstone bosons and absorbed by the corresponding 4D gauge bosons.

After the dimensional reduction, we obtain the Lagrangian density:

$$\mathcal{L}_{\mathrm{4D}} = -\frac{1}{4}\sum_{a=1}^{8} G_{\mu\nu}^a G^{a\mu\nu} - \frac{1}{4}\sum_{a=1}^{3} W_{\mu\nu}^a W^{a\mu\nu} - \frac{1}{4}R_{\mu\nu}R^{\mu\nu} - \frac{1}{4}N_{\mu\nu}N^{\mu\nu} + (D_\mu\phi_{\mathrm{R}})^\dagger(D^\mu\phi_{\mathrm{R}}) + (D_\mu\phi)^\dagger(D^\mu\phi) - V_{\mathrm{4D}} + \mathcal{L}_{\mathrm{KK}}, \tag{20}$$

where $G_{\mu\nu}^a$, $W_{\mu\nu}^a$, $R_{\mu\nu}$, and $N_{\mu\nu}$ are the field strengths of the $SU(3)_{\mathrm{C}}$, $SU(2)_{\mathrm{L}}$, $U(1)_{\mathrm{R}}$, and $U(1)_{\mathrm{B-L}}$ gauge bosons, and $\mathcal{L}_{\mathrm{KK}}$ is the Lagrangian density containing the Kaluza–Klein modes. Here, the

Table 3. Gauge quantum numbers of massless fields in the 4D 3211 model.

Particles	$SU(3)_C$	$SU(2)_L$	Q_R	Q_{B-L}	Y	$Y_\perp$
G_μ	**8**	**1**	0	0	0	0
W_μ	**1**	**3**	0	0	0	0
R_μ	**1**	**1**	0	0	0	0
N_μ	**1**	**1**	0	0	0	0
ϕ_R	**1**	**1**	$-\frac{1}{2}$	$\frac{1}{2}$	0	$\frac{5}{2}$
ϕ	**1**	**2**	$\frac{1}{2}$	0	$\frac{1}{2}$	-1
q_{LA}	**3**	**2**	0	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{2}$
u_{RA}	**3**	**1**	$\frac{1}{2}$	$\frac{1}{6}$	$\frac{2}{3}$	$-\frac{1}{2}$
d_{RA}	**3**	**1**	$-\frac{1}{2}$	$\frac{1}{6}$	$-\frac{1}{3}$	$\frac{3}{2}$
l_{LA}	**1**	**2**	0	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{3}{2}$
ν_{RA}	**1**	**1**	$\frac{1}{2}$	$-\frac{1}{2}$	0	$-\frac{5}{2}$
e_{RA}	**1**	**1**	$-\frac{1}{2}$	$-\frac{1}{2}$	-1	$-\frac{1}{2}$

covariant derivative D_μ and the scalar potential V_{4D} are given by

$$D_\mu = \partial_\mu + ig_3 \sum_{a=1}^{8} G_\mu^a T_C^a + ig \sum_{a=1}^{3} W_\mu^a T_L^a + ig_R R_\mu Q_R + ig_{B-L} N_\mu Q_{B-L}, \tag{21}$$

$$V_{4D} = \lambda_r |\phi_R|^4 + \lambda |\phi|^4 + \lambda_m |\phi_R|^2 |\phi|^2, \tag{22}$$

respectively. From the matching conditions between $\mathcal{L}_{5D}$ and $\mathcal{L}_{4D}$ at the compactification scale $M_c(= 1/(2R))$, we obtain the relations:

$$g_3 = \sqrt{\frac{2}{3}} g_{B-L} = g_4 \Big|_{M_c}, \quad g = g_L = g_R|_{M_c}, \tag{23}$$

$$\lambda_r = \lambda_R|_{M_c}, \quad \lambda = \lambda_{B1} + \lambda_{B2}|_{M_c}, \quad \lambda_m = \lambda_{RB}|_{M_c}. \tag{24}$$

Note that fields from zero modes are massless at M_c, and the value of λ_r does not necessarily agree with that of λ there. The system is described by only the zero modes with the above conditions lower than M_c, and matching conditions at M_c can, in general, offer useful information on extra dimensions [23].

3. The $SU(3)_C \times SU(2)_L \times U(1)_R \times U(1)_{B-L}$ model

Let us study the 4D model with the gauge group $SU(3)_C \times SU(2)_L \times U(1)_R \times U(1)_{B-L}$ described by Eq. (20). We refer to it as the 3211 model. The particle contents of the massless fields are listed in Table 3, where the subscript A represents the generation of matter fields on the 4D brane and runs from 1 to 3. For the sake of reference, we denote values of the weak hypercharge defined by $Y \equiv Q_R + Q_{B-L}$ and those of the $U(1)$ charge defined by $Y_\perp \equiv 3Q_{B-L} - 2Q_R = 5Q_{B-L} - 2Y$, which is orthogonal to Y.

3.1. *Running of gauge couplings*

We study the running of gauge couplings. For the sake of completeness, we consider the case that the 4D 3211 model holds beyond M_c. In this case, by solving the renormalization group equations

Table 4. Gauge couplings and their coefficients of β functions.

	$SU(3)_{\mathrm{C}}$	$SU(2)_{\mathrm{L}}$	$U(1)_{\mathrm{R}}$	$U(1)_{\mathrm{B-L}}$	$U(1)_{\mathrm{Y}}$
g_i	g_3	g	g_{R}	$g_{\mathrm{B-L}}$	g_{Y}
α_i	α_3	α_2	α_{R}	$\alpha_{\mathrm{B-L}}$	α_{Y}
b_i	-7	$-\frac{19}{6}$	$\frac{17}{4}$	$\frac{11}{4}$	$\frac{41}{6}$
b'_i	-1	$-\frac{1}{3}$	$\frac{1}{2}$	$\frac{1}{6}$	—
b''_i	$-\frac{1}{3}$	$\frac{1}{3}$	$-\frac{1}{2}$	$-\frac{19}{18}$	—

(RGEs) of gauge couplings g_i at the one-loop level, we obtain the solutions

$$\alpha_i^{-1}(\mu) = \alpha_i^{-1}(\mu_0) - \frac{b_i}{2\pi} \ln \frac{\mu}{\mu_0} - \sum_{n=1}^{\infty} \frac{b'_i}{2\pi} \theta\left(\mu - \frac{n}{R}\right) \ln \frac{\mu}{\frac{n}{R}} - \sum_{n=1}^{\infty} \frac{b''_i}{2\pi} \theta\left(\mu - \frac{n-\frac{1}{2}}{R}\right) \ln \frac{\mu}{\frac{n-\frac{1}{2}}{R}}, \quad (25)$$

where $\alpha_i \equiv g_i^2/(4\pi)$, μ is a renormalization point, b_i are coefficients of the β functions for zero modes, and b'_i and b''_i are coefficients of the β functions for Kaluza–Klein modes with masses n/R and $(n-\frac{1}{2})/R$, respectively. θ is a step function defined by $\theta(x) = 1$ for $x > 0$, $\theta(x) = 0$ for $x < 0$, and $\theta(0) = 1/2$.

The values of b_i, b'_i, and b''_i are listed in Table 4, where we list $b_{\mathrm{Y}} = 41/6$ in the SM, and "—" represents "not applicable." By taking $\mu = n_\Lambda/(2R) = n_\Lambda M_{\mathrm{c}}$, solutions are written as

$$\begin{aligned}
\alpha_i^{-1}(\mu) &= \alpha_i^{-1}(\mu_0) - \frac{b_i}{2\pi} \ln \frac{\mu}{\mu_0} - \sum_{n=1}^{n_\Lambda} \frac{b'_i}{2\pi} \ln \frac{\mu}{\frac{n}{R}} - \sum_{n=1}^{n_\Lambda} \frac{b''_i}{2\pi} \ln \frac{\mu}{\frac{n-\frac{1}{2}}{R}} \\
&= \alpha_i^{-1}(\mu_0) - \frac{b_i}{2\pi} \ln \frac{\mu}{\mu_0} - \frac{b'_i}{2\pi} \ln \prod_{n=1}^{n_\Lambda} \left(\frac{\mu}{2nM_{\mathrm{c}}}\right) - \frac{b''_i}{2\pi} \ln \prod_{n=1}^{n_\Lambda} \left(\frac{\mu}{(2n-1)\,M_{\mathrm{c}}}\right) \\
&= \alpha_i^{-1}(\mu_0) - \frac{b_i}{2\pi} \ln \frac{\mu}{\mu_0} \\
&\quad - \frac{b'_i}{2\pi} \left(\frac{\mu}{2M_{\mathrm{c}}} \ln \frac{\mu}{2M_{\mathrm{c}}} - \ln \Gamma\left(\frac{\mu}{2M_{\mathrm{c}}} + 1\right)\right) \\
&\quad - \frac{b''_i}{2\pi} \left(\frac{\mu}{2M_{\mathrm{c}}} \ln \frac{\mu}{2M_{\mathrm{c}}} - \ln \Gamma\left(\frac{\mu}{2M_{\mathrm{c}}} + \frac{1}{2}\right) + \ln \sqrt{\pi}\right),
\end{aligned} \quad (26)$$

where Γ is a gamma function defined by

$$\Gamma(z) = \int_0^\infty t^{z-1} e^{-t} dt \quad (27)$$

and we replace $\Pi_{n=1}^{n_\Lambda} n = n_\Lambda!$ and $\Pi_{n=1}^{n_\Lambda}\left(n-\frac{1}{2}\right) = (2n_\Lambda - 1)!!/2^{n_\Lambda}$ into $\Gamma(n_\Lambda + 1)$ and $\Gamma\left(n_\Lambda + \frac{1}{2}\right)/\sqrt{\pi}$, respectively.

Hereafter, we consider the case that the gauge symmetries are partially unified under the Pati–Salam-type gauge group $G_{\mathrm{PS}} \equiv SU(4)_{\mathrm{C}} \times SU(2)_{\mathrm{L}} \times SU(2)_{\mathrm{R}}$ at M_{c}. In this case, we should use the solutions in Eqs. (25) and (26) without the contributions of the Kaluza–Klein modes below M_{c}. We have the following conditions at the breaking scales at M_c and $M_{Z_{LR}}$:

$$\alpha_3 = \frac{2}{3}\alpha_{B-L}\Big|_{M_c}, \quad \alpha_2 = \alpha_R|_{M_c}, \quad \alpha_Y^{-1} = \alpha_R^{-1} + \alpha_{B-L}^{-1}\Big|_{M_{Z_{LR}}}, \tag{28}$$

where $M_{Z_{LR}}$ is the mass of the gauge boson that becomes massive with the breakdown of $U(1)_R \times U(1)_{B-L}$ into $U(1)_Y$. By combining with the solutions of Eq. (26), we obtain the sum rule:

$$\begin{aligned}&\alpha_Y^{-1}(M_Z) - \alpha_2^{-1}(M_Z) - \frac{2}{3}\alpha_3^{-1}(M_Z)\\ &= \frac{b_Y - b_2 - \frac{2}{3}b_3}{2\pi}\ln\frac{M_c}{M_Z} + \frac{-b_Y + b_R + b_{B-L}}{2\pi}\ln\frac{M_c}{M_{Z_{LR}}},\end{aligned} \tag{29}$$

where M_Z is the Z boson mass given by $M_Z \doteqdot 91.19$ GeV. Using the values of b_i, b'_i, and b''_i, and the experimental values such that [24]

$$\alpha_3^{-1}(M_Z) \doteqdot 8.467, \ \ \alpha_2^{-1}(M_Z) \doteqdot 29.59, \ \ \alpha_Y^{-1}(M_Z) \doteqdot 98.36, \tag{30}$$

we obtain the relation:

$$M_c \doteqdot 3.675 \times 10^{13} \times (1.026)^{\xi}\ \mathrm{GeV}, \tag{31}$$

where $M_{Z_{LR}}$ is parametrized as $M_{Z_{LR}} = 10^{\xi} \times M_Z$. From Eq. (31), we find the interesting feature that *the magnitude of M_c, $O(10^{13})$ GeV, is almost irrelevant to the value of $M_{Z_{LR}}$*. This is due to the accidental fact that the coefficient of the second term on the right-hand side of Eq. (29) is tiny, i.e., $(-b_Y + b_R + b_{B-L})/(2\pi) \doteqdot 0.02654$.

Let us estimate the running of gauge couplings beyond M_c, based on a 4D model whose gauge group is G_{PS}. The model contains Kaluza–Klein modes of gauge bosons (G_μ, $W_{L\mu}$, $W_{R\mu}$) and scalar bosons (Φ_L, Φ_R, Φ_B). Under the assumption that Kaluza–Klein modes appear with a mass n/R ($n = 1, 2, \ldots$) for simplicity,[3] the values of $\alpha_4 = g_4^2/(4\pi)$, $\alpha_{2L} = g_{2L}^2/(4\pi)$, and $\alpha_{2R} = g_{2R}^2/(4\pi)$ at $\mu = n_\Lambda/(2R) = n_\Lambda M_c (> M_c)$ are estimated as

$$\begin{aligned}\alpha_i^{-1}(\mu) &= \alpha_i^{-1}(M_c) - \frac{b_i}{2\pi}\ln\frac{\mu}{M_c} - \sum_{n=1}^{n_\Lambda}\frac{b_i^{KK}}{2\pi}\ln\frac{\mu}{\frac{n}{R}}\\ &= \alpha_i^{-1}(M_c) - \frac{b_i}{2\pi}\ln\frac{\mu}{M_c} - \frac{b_i^{KK}}{2\pi}\left(\frac{\mu}{2M_c}\ln\frac{\mu}{2M_c} - \ln\Gamma\left(\frac{\mu}{2M_c}+1\right)\right),\end{aligned} \tag{32}$$

where i stands for 4, 2L, and 2R, and g_4, g_{2L}, and g_{2R} are the gauge couplings of $SU(4)_C$, $SU(2)_L$, and $SU(2)_R$, respectively. The coefficients of the β functions are given by $b_4 = -10$, $b_4^{KK} = -4/3$, $b_{2L} = -5/3$, $b_{2L}^{KK} = 0$, $b_{2R} = -5/3$, and $b_{2R}^{KK} = 0$. From the second relation of Eq. (28), $b_{2L} = b_{2R}$ and $b_{2L}^{KK} = b_{2R}^{KK}$, we find that the relation $\alpha_{2L}(\mu) = \alpha_{2R}(\mu)$ holds beyond M_c.

[3] A mass difference with $1/(2R)$ can be neglected for large x, because $\ln(\Gamma(x+1/2)/\sqrt{\pi}) - \ln\Gamma(x+1) \doteqdot 1$.

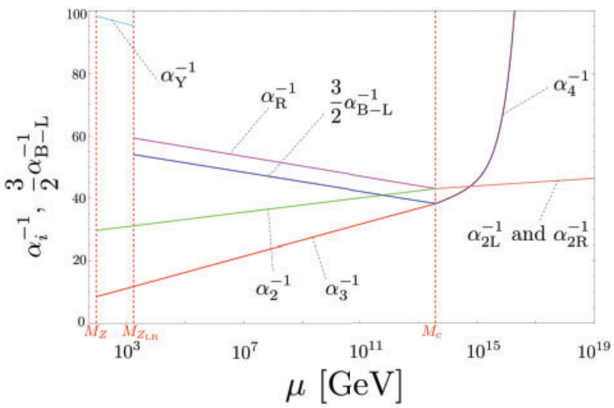

Fig. 1. The running of gauge couplings. The aqua, green, red, purple, and blue lines stand for the evolution of α_Y^{-1}, α_2^{-1}, α_3^{-1}, α_R^{-1}, and $3\alpha_{B-L}^{-1}/2$, respectively. Above M_c, the violet line represents the evolution of α_4^{-1} and the light-brown one represents the evolution of α_{2L}^{-1} and α_{2R}^{-1}.

Typical runnings of α_i^{-1} are depicted in Fig. 1. Here we choose $\xi = 1.275$, i.e., $M_{Z_{LR}} \doteqdot 1718\,\text{GeV}$, as a benchmark.[4] Beyond M_c, the running of the $SU(4)_C$ gauge coupling changes drastically due to the appearance of Kaluza–Klein modes.[5]

3.2. *Scalar potential in the 3211 model*

We study the breakdown of $U(1)_R \times U(1)_{B-L}$ and the electroweak symmetry. The scalar potential at the tree level is given by V_{4D} in Eq. (22). The quartic couplings λ_r, λ_m, and λ and the top Yukawa coupling y_t obey the RGEs at the one-loop level:

$$\frac{d\lambda_r}{d\ln\mu} = \frac{1}{16\pi^2}\left(20\lambda_r^2 + 2\lambda_m^2 - 3g_R^2\lambda_r - 3g_{B-L}^2\lambda_r + \frac{3}{8}g_R^4 + \frac{3}{4}g_R^2 g_{B-L}^2 + \frac{3}{8}g_{B-L}^4\right), \tag{33}$$

$$\frac{d\lambda_m}{d\ln\mu} = \frac{1}{16\pi^2}\left(4\lambda_m^2 + 8\lambda_r\lambda_m + 12\lambda\lambda_m - \frac{9}{2}g^2\lambda_m - 3g_R^2\lambda_m - \frac{3}{2}g_{B-L}^2\lambda_m + 6y_t^2\lambda_m + \frac{3}{8}g_R^4\right), \tag{34}$$

$$\frac{d\lambda}{d\ln\mu} = \frac{1}{16\pi^2}\left(24\lambda^2 + \lambda_m^2 - 3g_R^2\lambda - 9g^2\lambda + \frac{3}{8}g_R^4 + \frac{3}{4}g_R^2 g^2 + \frac{9}{8}g^4 + 12y_t^2\lambda - 6y_t^4\right), \tag{35}$$

$$\frac{dy_t}{d\ln\mu} = \frac{1}{16\pi^2}\left(\frac{9}{2}y_t^3 - \frac{3}{4}g_R^2 y_t - \frac{1}{6}g_{B-L}^2 y_t - \frac{9}{4}g^2 y_t - 8g_3^2 y_t\right), \tag{36}$$

where the contributions from Kaluza–Klein modes are omitted.

[4] The mass bound of an additional neutral gauge boson of $SU(2)_L \times SU(2)_R \times U(1)$ (with $g = g_R$) is 630 GeV from $p\bar{p}$ direct search and 1162 GeV from the electroweak fit [24].

[5] Note that the running of gauge couplings and the unification scale change drastically due to the contributions from Kaluza–Klein modes, including incomplete multiplets [25,26].

For the sake of completeness, we write down the RGEs of the Higgs quartic coupling λ and the top Yukawa coupling y_t in the SM:

$$\frac{d\lambda}{d\ln\mu} = \frac{1}{16\pi^2}\bigg(24\lambda^2 - 3g_{\rm Y}^2\lambda - 9g^2\lambda + \frac{3}{8}g_{\rm Y}^4 + \frac{3}{4}g_{\rm Y}^2 g^2 + \frac{9}{8}g^4 + 12y_t^2\lambda - 6y_t^4\bigg), \tag{37}$$

$$\frac{dy_t}{d\ln\mu} = \frac{1}{16\pi^2}\left(\frac{9}{2}y_t^3 - \frac{17}{12}g_{\rm Y}^2 y_t - \frac{9}{4}g^2 y_t - 8g_3^2 y_t\right). \tag{38}$$

λ and y_t run under the condition that the SM ones match those of 3211 model at $M_{Z_{\rm LR}}$.

We obtain an effective potential improved by the RGEs at the one-loop level:

$$V_{\rm eff}(\mu) = \frac{\lambda_{\rm r}}{4}\varphi_{\rm R}^4 + \frac{B_{\rm r}}{8}\varphi_{\rm R}^4\left(\ln\frac{\varphi_{\rm R}^2}{\mu^2} - \frac{25}{6}\right) + \frac{\lambda_{\rm m}}{4}\varphi^2\varphi_{\rm R}^2 + \frac{B_{\rm m}}{4}\varphi^2\varphi_{\rm R}^2\left(\ln\frac{\varphi\varphi_{\rm R}}{\mu^2} - 3\right) + \frac{\lambda}{4}\varphi^4 + \frac{B}{8}\varphi^4\left(\ln\frac{\varphi^2}{\mu^2} - \frac{25}{6}\right), \tag{39}$$

where $\varphi_{\rm R}^2 = 2\{({\rm Re}\phi_{\rm R})^2 + ({\rm Im}\phi_{\rm R})^2\}$, $\varphi^2 = 2\{({\rm Re}\phi^+)^2 + ({\rm Im}\phi^+)^2 + ({\rm Re}\phi^0)^2 + ({\rm Im}\phi^0)^2\}$, $\varphi_{\rm R}^4 = (\varphi_{\rm R}^2)^2$, $\varphi^4 = (\varphi^2)^2$, and $B_{\rm r}$, $B_{\rm m}$, and B are given by

$$B_{\rm r} = \frac{1}{16\pi^2}\left(20\lambda_{\rm r}^2 + 2\lambda_{\rm m}^2 + \frac{3}{8}g_{\rm R}^4 + \frac{3}{4}g_{\rm R}^2 g_{\rm B-L}^2 + \frac{3}{8}g_{\rm B-L}^4\right), \tag{40}$$

$$B_{\rm m} = \frac{1}{16\pi^2}\left(4\lambda_{\rm m}^2 + 8\lambda_{\rm r}\lambda_{\rm m} + 12\lambda\lambda_{\rm m} + \frac{3}{8}g_{\rm R}^4\right), \tag{41}$$

$$B = \frac{1}{16\pi^2}\left(24\lambda^2 + \lambda_{\rm m}^2 + \frac{3}{8}g_{\rm R}^4 + \frac{3}{4}g_{\rm R}^2 g^2 + \frac{9}{8}g^4 - 6y_t^4\right). \tag{42}$$

The effective potential $V_{\rm eff}(\mu)$ satisfies the renormalization conditions such that

$$\left.\frac{\partial^4 V_{\rm eff}}{\partial\varphi_{\rm R}^4}\right|_{\varphi_{\rm R},\varphi=\mu} = \lambda_{\rm r}(\mu), \quad \left.\frac{\partial^4 V_{\rm eff}}{\partial\varphi_{\rm R}^2\partial\varphi^2}\right|_{\varphi_{\rm R},\varphi=\mu} = \lambda_{\rm m}(\mu), \quad \left.\frac{\partial^4 V_{\rm eff}}{\partial\varphi^4}\right|_{\varphi_{\rm R},\varphi=\mu} = \lambda(\mu) \tag{43}$$

and does not depend on μ, that is,

$$\frac{dV_{\rm eff}(\mu)}{d\ln\mu} = \left(\frac{\partial}{\partial\ln\mu} + \frac{d\lambda_{\rm r}}{d\ln\mu}\frac{\partial}{\partial\lambda_{\rm r}} + \frac{d\lambda_{\rm m}}{d\ln\mu}\frac{\partial}{\partial\lambda_{\rm m}} + \frac{d\lambda}{d\ln\mu}\frac{\partial}{\partial\lambda} + \frac{d\varphi_{\rm R}}{d\ln\mu}\frac{\partial}{\partial\varphi_{\rm R}} + \frac{d\varphi}{d\ln\mu}\frac{\partial}{\partial\varphi}\right)V_{\rm eff}(\mu) = 0. \tag{44}$$

The first derivatives of $V_{\rm eff}$ by fields are given by

$$\frac{\partial V_{\rm eff}}{\partial\varphi_{\rm R}} = \left\{\left(\lambda_{\rm r} + B_{\rm r}\ln\frac{\varphi_{\rm R}}{\mu} - \frac{11}{6}B_{\rm r}\right)\varphi_{\rm R}^2 + \frac{1}{2}\left(\lambda_{\rm m} + B_{\rm m}\ln\frac{\varphi\varphi_{\rm R}}{\mu^2} - \frac{5}{2}B_{\rm m}\right)\varphi^2\right\}\varphi_{\rm R}, \tag{45}$$

$$\frac{\partial V_{\rm eff}}{\partial\varphi} = \left\{\left(\lambda + B\ln\frac{\varphi}{\mu} - \frac{11}{6}B\right)\varphi^2 + \frac{1}{2}\left(\lambda_{\rm m} + B_{\rm m}\ln\frac{\varphi\varphi_{\rm R}}{\mu^2} - \frac{5}{2}B_{\rm m}\right)\varphi_{\rm R}^2\right\}\varphi. \tag{46}$$

From the stationary conditions

$$\left\langle \frac{\partial V_{\rm eff}}{\partial \varphi_{\rm R}} \right\rangle = 0, \quad \left\langle \frac{\partial V_{\rm eff}}{\partial \varphi} \right\rangle = 0 \tag{47}$$

we obtain the relations:

$$\tilde{\lambda}_{\rm r} \langle \varphi_{\rm R} \rangle^2 = \frac{1}{2} \tilde{\lambda}_{\rm m} \langle \varphi \rangle^2 \Big|_{\langle \varphi_{\rm R} \rangle}, \quad \tilde{\lambda} \langle \varphi \rangle^2 = \frac{1}{2} \tilde{\lambda}_{\rm m} \langle \varphi_{\rm R} \rangle^2 \Big|_{\langle \varphi_{\rm R} \rangle}, \tag{48}$$

and, by combining them, the relation:

$$\tilde{\lambda}_{\rm r} = \frac{1}{4} \frac{\tilde{\lambda}_{\rm m}^2}{\tilde{\lambda}} \Bigg|_{\langle \varphi_{\rm R} \rangle}, \tag{49}$$

where $\tilde{\lambda}_{\rm r}$, $\tilde{\lambda}_{\rm m}$, and $\tilde{\lambda}$ are defined by

$$\tilde{\lambda}_{\rm r}(\mu) \equiv \lambda_{\rm r} + B_{\rm r} \ln \frac{\langle \varphi_{\rm R} \rangle}{\mu} - \frac{11}{6} B_{\rm r}, \tag{50}$$

$$\tilde{\lambda}_{\rm m}(\mu) \equiv \lambda_{\rm m} + B_{\rm m} \ln \frac{\langle \varphi \rangle \langle \varphi_{\rm R} \rangle}{\mu^2} - \frac{5}{2} B_{\rm m}, \tag{51}$$

$$\tilde{\lambda}(\mu) \equiv \lambda + B \ln \frac{\langle \varphi \rangle}{\mu} - \frac{11}{6} B \tag{52}$$

and $|_{\langle \varphi_{\rm R} \rangle}$ means the value at $\mu = \langle \varphi_{\rm R} \rangle$. We find that *the breakdown of residual gauge symmetries occurs radiatively via the Coleman–Weinberg mechanism, such that the $U(1)_{\rm R} \times U(1)_{\rm B-L}$ symmetry is broken down to $U(1)_{\rm Y}$ at the scale $v_{\rm R} \equiv \langle \varphi_{\rm R} \rangle$ that satisfies Eq. (49) and the $SU(2)_{\rm L} \times U(1)_{\rm Y}$ symmetry is broken down to $U(1)_{\rm EM}$ by $\langle \varphi \rangle$*. The hierarchy between $\langle \varphi_{\rm R} \rangle$ and $\langle \varphi \rangle$ comes from the difference in magnitude among the couplings $\tilde{\lambda}_{\rm r}$, $\tilde{\lambda}_{\rm m}$, and $\tilde{\lambda}$, as seen from Eq. (48).

After the breakdown of $U(1)_{\rm R} \times U(1)_{\rm B-L}$, a gauge boson $Z_{{\rm LR}\mu}(x)$ acquires the mass

$$M_{Z_{\rm LR}} = \frac{1}{2} \sqrt{g_{\rm R}^2 + g_{\rm B-L}^2}\, v_{\rm R}. \tag{53}$$

The $Z_{{\rm LR}\mu}(x)$ and $B_\mu(x)$ (a gauge boson relating to $U(1)_{\rm Y}$) are given as linear combinations such that

$$Z_{{\rm LR}\mu}(x) = R_\mu(x) \cos\theta_{\rm R} - N_\mu(x) \sin\theta_{\rm R}, \tag{54}$$

$$B_\mu(x) = R_\mu(x) \sin\theta_{\rm R} + N_\mu(x) \cos\theta_{\rm R}, \tag{55}$$

where the mixing angle $\theta_{\rm R}$ is defined by $\tan\theta_{\rm R} \equiv g_{\rm B-L}/g_{\rm R}$.

Using the stationary conditions, we obtain the following formulae for mass matrix elements:

$$\left\langle \frac{\partial^2 V_{\rm eff}}{\partial \varphi_{\rm R}^2} \right\rangle\Bigg|_{v_{\rm R}} = \left(2\tilde{\lambda}_{\rm r} + B_{\rm r} - \frac{\tilde{\lambda}_{\rm m}}{4\tilde{\lambda}} B_{\rm m} \right)\Bigg|_{v_{\rm R}} v_{\rm R}^2, \tag{56}$$

$$\left\langle \frac{\partial^2 V_{\rm eff}}{\partial \varphi_{\rm R} \partial \varphi} \right\rangle\Bigg|_{v_{\rm R}} = \left\langle \frac{\partial^2 V_{\rm eff}}{\partial \varphi \partial \varphi_{\rm R}} \right\rangle\Bigg|_{v_{\rm R}} = \left(\tilde{\lambda}_{\rm m} + \frac{B_{\rm m}}{2} \right) \sqrt{-\frac{\tilde{\lambda}_m}{2\tilde{\lambda}}}\Bigg|_{v_{\rm R}} v_{\rm R}^2, \tag{57}$$

$$\begin{aligned} \left\langle \frac{\partial^2 V_{\rm eff}}{\partial \varphi^2} \right\rangle\Bigg|_{v_{\rm R}} &= \left(2\tilde{\lambda} + B - \frac{\tilde{\lambda}}{\tilde{\lambda}_{\rm m}} B_{\rm m} \right) \langle \varphi \rangle^2 \Bigg|_{v_{\rm R}} \\ &= \left(-\tilde{\lambda}_{\rm m} - \frac{\tilde{\lambda}_{\rm m}}{2\tilde{\lambda}} B + \frac{B_{\rm m}}{2} \right)\Bigg|_{v_{\rm R}} v_{\rm R}^2, \end{aligned} \tag{58}$$

where $|_{v_{\rm R}}$ means the values at $\langle \varphi_{\rm R} \rangle = v_{\rm R}$.

Here we choose $\xi = 1.275$, i.e., $M_{Z_{\rm LR}} \doteqdot 1718$ GeV, as a benchmark. In this case, $v_{\rm R}$ is estimated as

$$v_{\rm R} = \frac{2M_{Z_{\rm LR}}}{\sqrt{g_{\rm R}^2 + g_{\rm B-L}^2}}\Bigg|_{M_{Z_{\rm LR}}} \doteqdot 4584\,{\rm GeV} \tag{59}$$

and the mass matrix elements of the scalar fields are estimated as

$$\begin{pmatrix} \left\langle \frac{\partial^2 V_{\rm eff}}{\partial \varphi_{\rm R}^2} \right\rangle & \left\langle \frac{\partial^2 V_{\rm eff}}{\partial \varphi_{\rm R} \partial \varphi} \right\rangle \\ \left\langle \frac{\partial^2 V_{\rm eff}}{\partial \varphi \partial \varphi_{\rm R}} \right\rangle & \left\langle \frac{\partial^2 V_{\rm eff}}{\partial \varphi^2} \right\rangle \end{pmatrix}\Bigg|_{v_{\rm R}} \doteqdot \begin{pmatrix} 16165 & -777 \\ -777 & 16776 \end{pmatrix} {\rm GeV}^2. \tag{60}$$

After diagonalizing the mass matrix, the mass of the $\varphi_{\rm R}$-dominated component is evaluated as

$$m_{\rm R} \doteqdot 132\,{\rm GeV}. \tag{61}$$

The third term on the right-hand side of Eqs. (39) or (22) and its radiative corrections—the fourth term on the right-hand side of Eq. (39)—are the Higgs portal. By replacing $\varphi_{\rm R}$ in its VEV, we obtain the following approximate squared mass of the Higgs boson:

$$m^2 \approx \frac{1}{2}(\lambda_{\rm m} - 3B_{\rm m}) v_{\rm R}^2. \tag{62}$$

From a numerical analysis, we obtain the negative squared mass because $\lambda_{\rm m} < 3B_{\rm m}$. This can be interpreted that the Higgs mechanism occurs effectively.

The runnings of $\lambda_{\rm r}$, $\lambda_{\rm m}$, and λ are depicted in Fig. 2. The values of $\lambda_{\rm r}$ and $\lambda_{\rm m}$ at $v_{\rm R}$ are estimated using the stationary conditions of Eq. (47) and $\lambda(v_{\rm R})$ with $\langle \varphi \rangle \approx 246$ GeV. Beyond $M_{\rm c}$, contributions from the Kaluza–Klein modes of gauge bosons are added, but those from the Kaluza–Klein modes of scalar fields are not considered because $\lambda_{\rm r}$, $\lambda_{\rm m}$, and λ take tiny values around $M_{\rm c}$ and their effects are negligible. For simplicity, we assume that $\lambda_{\rm B1} = \lambda$ and $\lambda_{\rm B2} \doteqdot 0$ at $M_{\rm c}$. The running of λ is almost the same as that in the SM until $M_{\rm c}$ because the contributions from the gluon and top quark are dominant. We find that the vacuum stability is recovered by the rapid increase of Higgs self-coupling due to contributions from the Kaluza–Klein modes of gauge bosons.

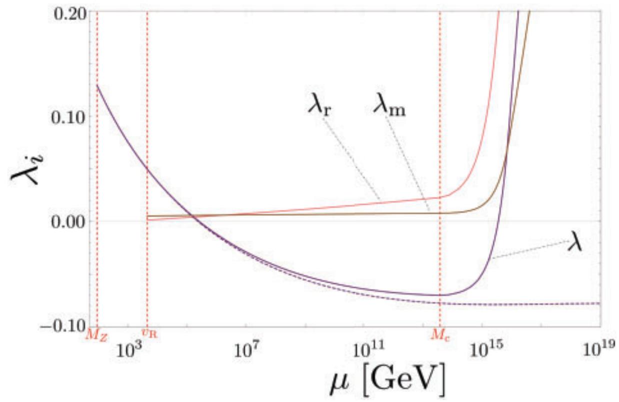

Fig. 2. The running of λ_r, λ_m, and λ. The red, dark brown, and violet lines stand for the evolution of λ_r, λ_m, and λ, respectively. The dotted violet line represents the evolution of λ in the SM.

4. Conclusions and discussions

We have studied the origin of electroweak symmetry under the assumption that $SU(4)_C \times SU(2)_L \times SU(2)_R$ is realized on the 5D space-time $M^4 \times S^1/Z_2$. The Pati–Salam-type gauge symmetry is reduced to $SU(3)_C \times SU(2)_L \times U(1)_R \times U(1)_{B-L}$ at the compactification scale M_c by the orbifold breaking mechanism on S^1/Z_2. The breakdown of residual gauge symmetries occurs radiatively via the Coleman–Weinberg mechanism, such that the $U(1)_R \times U(1)_{B-L}$ symmetry is broken down to $U(1)_Y$ by the VEV of an $SU(2)_L$ singlet scalar field and the $SU(2)_L \times U(1)_Y$ symmetry is broken down to the electric one $U(1)_{EM}$ by the VEV of the Higgs doublet, using the negative squared mass originating from an interaction between the Higgs doublet and an $SU(2)_L$ singlet scalar field as a Higgs portal. The vacuum stability can be recovered by the contributions from Kaluza–Klein modes appearing at M_c and above there.

Our 3211 model has the excellent feature that M_c is almost determined as $M_c = O(10^{13})$ GeV from the gauge coupling unification of $SU(3)_C$ and $U(1)_{B-L}$ into $SU(4)_C$ and the left–right symmetry between $SU(2)_L$ and $SU(2)_R$. On the contrary, the breaking scale v_R of $U(1)_R \times U(1)_{B-L}$ is not fixed from the information of gauge couplings alone. The criterion of naturalness can favor v_R close to the weak scale.

Our 3211 model has almost the same particle contents as those in the minimal $B-L$ extension of the SM proposed in Refs. [27–30]. The main differences between our model and the $B-L$ extended SM are the $U(1)_{B-L}$ charge assignment of $SU(2)_L$ singlet scalar field ϕ_R and the interactions between $U(1)$ gauge bosons and matter fields. In our model, the ν_{RA} and ϕ_R have $U(1)_{B-L}$ charges of $-1/2$ and $1/2$, respectively. Then, the allowed interaction terms between them are not renormalizable ones but non-renormalizable ones, e.g., $(f_{AB}/\Lambda)\phi_R^2 \overline{\nu}_{RA}^c \nu_{RA}$, where Λ is a high-energy scale such as M_c. Hence, small Majorana masses appear after the breakdown of $U(1)_R \times U(1)_{B-L}$ and the seesaw mechanism does not work at the TeV scale. In this paper we have focused on the physics of the gauge symmetry breaking sector. It would be interesting to investigate the flavor physics relating to quarks and leptons in our model. It would also be important to clarify the relationship between our model and the $B-L$ extended SM through the study of gauge kinetic mixing and so on.

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