

Research Article

Generalized Dirac Oscillator in Cosmic String Space-Time

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In this work, the generalized Dirac oscillator in cosmic string space-time is studied by replacing the momentum p_μ with its alternative $p_\mu + m\omega\beta f_\mu(x_\mu)$. In particular, the quantum dynamics is considered for the function $f_\mu(x_\mu)$ to be taken as Cornell potential, exponential-type potential, and singular potential. For Cornell potential and exponential-type potential, the corresponding radial equations can be mapped into the confluent hypergeometric equation and hypergeometric equation separately. The corresponding eigenfunctions can be represented as confluent hypergeometric function and hypergeometric function. The equations satisfied by the exact energy spectrum have been found. For singular potential, the wave function and energy eigenvalue are given exactly by power series method.

1. Introduction

In quantum mechanics, there has been an increasing interest in finding the analytical solutions that play an important role for getting complete information about quantum mechanical systems [1–3]. The Dirac oscillator proposed in [4] is one of the important issues in this relativistic quantum mechanics recently. In this quantum model, the coupling proposed is introduced in such a way that the Dirac equation remains linear in both spatial coordinates and momenta and recovers the Schrödinger equation for a harmonic oscillator in the nonrelativistic limit of the Dirac equation [4–11]. As a solvable model of relativistic quantum mechanical system, the Dirac oscillator has many applications and has been studied extensively in different field such as high-energy physics [12–15], condensed matter physics [16–18], quantum Optics [19–25], and mathematical physics [26–33]. On the other hand, the analysis of gravitational interactions with a quantum mechanical system has recently attracted a great deal attention and has been an active field of research [5, 6, 34–43]. The study of quantum mechanical problems in curved space-time can be considered as a new kind of interaction between quantum matter and gravitation in the microparticle world. In recent years, the Dirac oscillator

embedded in a cosmic string background has inspired a great deal of research such as the dynamics of Dirac oscillator in the space-time of cosmic string [44–47], Aharonov-Casher effect on the Dirac oscillator [5, 48], and noninertial effects on the Dirac oscillator in the cosmic string space-time [49–51]. It is worth mentioning that based on the coupling corresponding to the Dirac oscillator a new coupling into Dirac equation first has been proposed by Bakke *et al.* [52] and used in different fields [53–57]. This model is called the generalized Dirac oscillator which in special case is reduced to ordinary Dirac oscillator. Inspired by the above work, the main aim of this paper is to analyze the generalized Dirac oscillator model with the interaction functions $f_\mu(x_\mu)$ taken as Cornell potential, singular potential and exponential-type potential in cosmic string space-time and to find the corresponding energy spectrum and wave functions. This work is organized as follows. In Section 2, the new coupling is introduced in such a way that the Dirac equation remains linear in momenta, but not in spatial coordinates in a curved space-time. In Sections 3, 4, and 5, we concentrate our efforts in analytically solving the quantum systems with different function $f_\mu(x_\mu)$ and find the corresponding energy spectrum and spinors, respectively. In Section 6, we make a short conclusion.

2. Generalized Dirac Oscillator with a Topological Defect

In cosmic string space-time, the general form of the cosmic string metric in cylindrical coordinates read [41, 42, 44, 58, 59]

$$ds^2 = -dt^2 + d\rho^2 + \alpha^2 \rho^2 d\varphi^2 + dz^2, \quad (1)$$

with $-\infty < (t, z) < +\infty$, $0 < \rho < +\infty$, and $0 < \varphi < 2\pi$. The parameter α is related to the linear mass density of string η by $\alpha = 1 - 4\eta$ and runs in the interval $(0, 1]$. In the limit as $\alpha \rightarrow 1$ we get the line element of cylindrical coordinates. The Dirac equation in the curved space-time ($\hbar = c = 1$) reads

$$[i\gamma^\mu(x) \partial_\mu - i\gamma_\mu(x) \Gamma_\mu(x) - m] \psi(t, x) = 0, \quad (2)$$

where the γ^μ matrices are the generalized Dirac matrices defining the covariant Clifford algebra $\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu}$, m is mass of the particle, and Γ_μ is the spinor affine connection. We choose the basis tetrad e_a^μ as

$$e_a^\mu = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos \varphi & \sin \varphi & 0 \\ 0 & \frac{\sin \varphi}{\alpha \rho} & \frac{\cos \varphi}{\alpha \rho} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \quad (3)$$

then in this representation the matrices γ^μ [44] can be found to be

$$\begin{aligned} \gamma^0 &= \gamma^t, \\ \gamma^1 &= \gamma^\rho = \gamma^1 \cos \varphi + \gamma^2 \sin \varphi, \\ \gamma^2 &= \gamma^\varphi = -\gamma^1 \sin \varphi + \gamma^2 \cos \varphi, \\ \gamma^3 &= \gamma^z, \\ \gamma^\mu \Gamma_\mu(x) &= \frac{1 - \alpha}{2\alpha\rho} \gamma^\rho. \end{aligned} \quad (4)$$

It is well known that, in both Minkowski space-time and curved space-time, usual Dirac oscillator can be obtained by the carrying out nonminimal substitution $p_\mu \rightarrow p_\mu + m\omega\beta x_\mu$ in Dirac equation where m and ω are the mass and oscillator frequency. In the following, we will construct the generalized oscillator in curved space-time. To do this end, we can replace momenta p_μ in the Dirac equation of curved space-time by

$$p_\mu \rightarrow p_\mu + m\omega\beta f_\mu(x_\mu), \quad (5)$$

where $f_\mu(x_\mu)$ are undetermined functions of x_μ . It is to say that we introduce a new coupling in such a way that the Dirac equation remains linear in momenta but not in coordinates. In particular, in this work, we only consider the radial component the nonminimal substitution

$$f_\mu(x_\mu) = (0, f_\rho(\rho), 0, 0). \quad (6)$$

By introducing this new coupling (6) into (2) and with the help of (4), in cosmic string space-time the eigenvalue equation of generalized Dirac oscillator can be written as

$$\left\{ -i\gamma^t \partial_t + i\gamma^\rho \left(\partial_\rho - \frac{1 - \alpha}{2\alpha\rho} + m\omega\rho f(\rho) \right) + \frac{i\gamma^\varphi \partial_\varphi}{\alpha\rho} + i\gamma^z \partial_z - m \right\} \psi = 0. \quad (7)$$

We choose the following ansatz:

$$\psi = e^{-iEt + i(1+1/2 - \Sigma^3/2)\varphi + ikz} \begin{pmatrix} \chi_1(\rho) \\ \chi_2(\rho) \end{pmatrix}, \quad (8)$$

then we have

$$\begin{aligned} & \left[\alpha_1 \left(\frac{d}{d\rho} + \frac{1}{2\rho} - m\omega\rho f(\rho) \right) - \frac{\lambda}{\rho} \alpha_2 - k\alpha_3 \right] \\ & \cdot \left[\alpha_1 \left(\frac{d}{d\rho} + \frac{1}{2\rho} + m\omega f(\rho) \right) - \frac{\lambda}{\rho} \alpha_2 - k\alpha_3 \right] \chi_1 \end{aligned} \quad (9)$$

$$= (E^2 - m^2) \chi_1,$$

$$\begin{aligned} & \left[\alpha_1 \left(\frac{d}{d\rho} + \frac{1}{2\rho} + m\omega f(\rho) \right) - \frac{\lambda}{\rho} \alpha_2 - k\alpha_3 \right] \\ & \cdot \left[\alpha_1 \left(\frac{d}{d\rho} + \frac{1}{2\rho} - m\omega f(\rho) \right) - \frac{\lambda}{\rho} \alpha_2 - k\alpha_3 \right] \chi_2 \end{aligned} \quad (10)$$

$$= (E^2 - m^2) \chi_2,$$

where

$$\begin{aligned} \alpha_1 &= i(\sigma_1 \cos \varphi + \sigma_2 \sin \varphi), \\ \alpha_2 &= -\sigma_1 \sin \varphi + \sigma_2 \cos \varphi, \\ \alpha_3 &= \sigma_3. \end{aligned} \quad (11)$$

It is straightforward to prove the following relations satisfied above matrices α_i :

$$\begin{aligned} \alpha_1^2 &= -\alpha_2^2 = \alpha_3^2 = -1, \\ \alpha_1 \alpha_2 &= -\alpha_2 \alpha_1 = i\sigma_1 \sigma_2, \\ \alpha_1 \alpha_3 &= -\alpha_3 \alpha_1 = i(\sigma_1 \sigma_3 \cos \varphi + \sigma_2 \sigma_3 \sin \varphi), \\ \alpha_3 &= -\alpha_3 \alpha_2 = -\sigma_1 \sigma_3 \sin \varphi + \sigma_2 \sigma_3 \cos \varphi. \end{aligned} \quad (12)$$

With help of (12) and simple algebraic calculus, (9) becomes

$$\begin{aligned} & \left\{ \partial_\rho^2 + \frac{1}{\rho} \partial_\rho - \frac{1}{\rho^2} \left[\frac{1}{4} + i\lambda \sigma_1 \sigma_2 + \lambda^2 \right] \right\} \chi_1 + \left\{ -2m\omega \right. \\ & \cdot \frac{f(\rho)}{\rho} [ik\rho(\sigma_1 \sigma_3 \cos \varphi + \sigma_2 \sigma_3 \sin \varphi) + i\sigma_1 \sigma_2 \lambda] \left. \right\} \\ & \cdot \chi_1 + \left\{ m^2 + k^2 - E^2 + m\omega \frac{f(\rho)}{\rho} - m^2 \omega^2 f^2(\rho) \right\} \\ & \cdot \chi_1 = 0. \end{aligned} \quad (13)$$

It is easy to prove the following relation [44]:

$$i\sigma_1\sigma_2\lambda + ik\rho(\sigma_1\sigma_3\cos\varphi + \sigma_2\sigma_3\sin\varphi) = -2\vec{s}\cdot\vec{L}, \quad (14)$$

where $\vec{s} = \vec{\sigma}/2$. The eigenvalue of $\vec{s}\cdot\vec{L}$ can be assumed as $(l + 1/2)/2\alpha$ and (13) reads

$$\begin{aligned} & \frac{d^2\chi_1}{d\rho^2} + \frac{1}{\rho}\frac{d}{d\rho}\chi_1 \\ & - \left[\frac{\lambda^2}{\rho^2} + \mu\frac{f(\rho)}{\rho} - m\omega\frac{df(\rho)}{d\rho} + m^2\omega^2f^2(\rho) \right] \chi_1 \quad (*a) \\ & + \nu\chi_1 = 0, \end{aligned}$$

where

$$\begin{aligned} \lambda &= \left(\frac{1+1/2}{\alpha} - \frac{1}{2} \right), \\ \mu &= \frac{-2(1+1/2)m\omega}{\alpha}, \\ \nu &= E^2 - m^2 - k^2. \end{aligned} \quad (15)$$

For the component χ_2 , from (10) an analog equation can be also obtained

$$\begin{aligned} & \frac{d^2\chi_2}{d\rho^2} + \frac{1}{\rho}\frac{d}{d\rho}\chi_2 \\ & - \left[\frac{\lambda^2}{\rho^2} + \mu\frac{f(\rho)}{\rho} + m\omega\frac{df(\rho)}{d\rho} + m^2\omega^2f^2(\rho) \right] \chi_2 \quad (*b) \\ & + \nu\chi_2 = 0, \end{aligned}$$

where

$$\begin{aligned} \lambda &= \left(\frac{1+1/2}{\alpha} + \frac{1}{2} \right), \\ \mu &= \frac{-2(1+1/2)m\omega}{\alpha}, \\ \nu &= E^2 - m^2 - k^2. \end{aligned} \quad (16)$$

In particular, (*a) and (*b) will be reduced to the result obtained in [44] when the function $f(\rho)$ is taken as $f(\rho) = \rho$. As we can see, (*a) and (*b) have the same form. So without loss of generality in remaining parts of this work, our main tasks is only to solve the equation (*a) with different functions $f(\rho)$ and find corresponding eigenvalue and eigenfunction. While with regard to (*b), it is straightforward to obtain the corresponding solution.

3. The Solution with $f(\rho)$ to Be Cornell Potential

The Cornell potential that consists of Coulomb potential and linear potential has gotten a great deal of attention in particle physics and was used with considerable success in

models describing systems of bound heavy quarks [60–62]. In Cornell potential, the short-distance Coulombic interaction arises from the one-gluon exchange between the quark and its antiquark, and the long-distance interaction is included to take into account confining phenomena.

Now we let the function $f(\rho)$ be Cornell potential

$$f(\rho) = a\rho - \frac{b}{\rho}, \quad (17)$$

where a and b are two constants. Substituting (17) into (*a) and (*b) leads to following equation:

$$\frac{d^2\chi}{d\rho^2} + \frac{1}{\rho}\frac{d}{d\rho}\chi + \left[\frac{-\tau_1^2}{\rho^2} - \tau_2\rho^2 + \tau_3 \right] \chi = 0, \quad (18)$$

where

$$\begin{aligned} \tau_1^2 &= \lambda^2 - \mu b + m^2\omega^2b^2 - \omega mb, \\ \tau_2 &= m^2\omega^2a^2, \\ \tau_3 &= \nu + 2abm^2\omega^2 - a\mu + m\omega a. \end{aligned} \quad (19)$$

We make a change in variables $\xi = m\omega a\rho^2$ and then (18) can be rewritten as

$$\xi\frac{d^2\chi}{d\xi^2} + \frac{d}{d\xi}\chi + \left[\frac{-\tau_1^2}{4\xi} - \frac{1}{4}\xi + \frac{\tau_3}{4ma\omega} \right] \chi = 0. \quad (20)$$

Taking account of the boundary conditions satisfied by the wave function χ , i.e., $\chi \propto \xi^{\tau_1/2}$ for $\xi \rightarrow 0$ and $\chi \propto e^{-\xi/2}$ for $\xi \rightarrow \infty$, physical solutions χ can be expressed as [44, 60, 63]

$$\chi = \xi^{|\tau_1|/2} e^{-\xi/2} \mathcal{G}(\xi). \quad (21)$$

If we insert this wave function χ into (20), we have the second-order homogeneous linear differential equation in the following form:

$$\begin{aligned} & \xi\frac{d^2\mathcal{G}}{d\xi^2} + (|\tau_1| + 1 - \xi)\frac{d}{d\xi}\mathcal{G} + \left[\frac{\tau_3}{4ma\omega} - \frac{|\tau_1|}{2} - \frac{1}{2} \right] \mathcal{G} \\ & = 0. \end{aligned} \quad (22)$$

It is well known that (22) is the confluent hypergeometric equation and it is immediate to obtain the corresponding eigenvalues and eigenfunctions

$$\mathcal{G}(\xi) = F\left[-\left(\frac{\tau_3}{4ma\omega} - \frac{|\tau_1| + 1}{2}\right), |\tau_1| + 1, \xi\right], \quad (23)$$

$$\begin{aligned} E_n^2 &= \delta_1 - \frac{2(1+1/2)m\omega a}{\alpha} \\ &+ 4ma\omega\left[n + \frac{1}{2} + \frac{|\delta_2|}{2}\right], \end{aligned} \quad (24)$$

with

$$\begin{aligned} \delta_1 &= m^2 + k^2 - 2abm^2\omega^2 + m\omega a, \\ \delta_2 &= \sqrt{\lambda^2 - \mu b + b^2m^2\omega^2 - m\omega b}. \end{aligned} \quad (25)$$

In particular, if we assume that $\alpha = 1$, from (24), the energy levels of generalized Dirac oscillator with $f(\rho)$ to be Cornell potential in the absence of a topological defect can be obtained. In addition if we let $a = 1$, $b = 0$ in (24) the energy levels given here will be reduced to that one obtained in [44].

4. The Solution with $f(\rho)$ to Be Singular Potential

The investigation of singular potentials in quantum mechanics is almost as old as quantum mechanics itself and covers a wide range of physical and mathematical interest because the real world interactions were likely to be highly singular [64]. The singular potentials of $v(r) \propto 1/r^n$ type, with $n \geq 2$, are of great current physical interest and are relevant to many problems such as the three-body problem in nuclear physics [65, 66], point-dipole interactions in molecular physics [67], the tensor force between nucleons [68], and the interaction between a charges and an induced dipole [69], respectively. Recently, in cosmic string background, singular inverse-square potential with a minimal length had been studied [70].

Next let us take $f(\rho)$ to be singular inverse-square-type potential [71]

$$f_\rho(\rho) = a + \frac{b}{\rho} + \frac{c}{\rho^2}. \quad (26)$$

Substituting (26) into (*a), the corresponding radial equation reads

$$\left\{ \frac{d^2}{d\rho^2} + \frac{1}{\rho} \frac{d}{d\rho} - \frac{\delta_1}{\rho} - \frac{\lambda^2 + \delta_2}{\rho^2} - \frac{\delta_3}{\rho^3} - \frac{\delta_4}{\rho^4} + \gamma \right\} \chi = 0, \quad (27)$$

where

$$\begin{aligned} \gamma &= v - m^2 a^2 \omega^2 \\ \delta_1 &= 2abm^2 \omega^2 + a\mu, \\ \delta_2 &= \mu b + m\omega b + b^2 m^2 \omega^2 + 2acm^2 \omega^2, \\ \delta_3 &= \mu c + 2m\omega c + 2bcm^2 \omega^2, \\ \delta_4 &= c^2 m^2 \omega^2. \end{aligned} \quad (28)$$

It is obvious that (27) has the same mathematical structure with the Schrödinger equation of fourth-order inverse-potential in [72]. So (27) can be solved by power series method.

We look for an exact solution of (27) via the following ansatz to the radial wave function [72–74]:

$$\begin{aligned} \chi &= \Theta(\rho) \exp[g(\rho)], \\ g(\rho) &= -\frac{\delta_1}{2\rho} - \frac{\delta_2}{2}\rho - \frac{\delta_3}{2} \log \rho. \end{aligned} \quad (29)$$

Thence, (27) can be transformed into the following form:

$$\begin{aligned} &\left\{ \frac{d^2}{d\rho^2} + \left[-\delta_1 + \frac{1 - \delta_3}{\rho} + \frac{\delta_2}{\rho^2} \right] \frac{d}{d\rho} + \gamma + \frac{\delta_1^2}{4} \right. \\ &\quad + \frac{\delta_1(\delta_3 - 3)}{2\rho} + \frac{\delta_3^2 - 4\delta_2 - 2\delta_1\delta_2 - 4\lambda^2}{4\rho^2} \\ &\quad \left. + \frac{-\delta_2(1 + \delta_3) - \delta_3}{2\rho^3} + \frac{\delta_2^2 - 4\delta_4}{4\rho^4} \right\} \Theta(\rho) = 0. \end{aligned} \quad (30)$$

Now we take $\Theta(\rho)$ in following series form:

$$\Theta(\rho) = \sum_{n=0}^{\infty} a_n \rho^{n+\lambda+1/2}, \quad a_0 \neq 0, \quad a_1 \neq 0. \quad (31)$$

Substituting (31) into (30) gives rise to following equation:

$$\begin{aligned} &\sum_{n=0}^{\infty} a_n \left\{ - \left[\delta_1 \left(n + \lambda + 2 - \frac{\delta_3}{2} \right) \right] \rho^{n+\lambda-1/2} + \left[\gamma + \frac{\delta_1^2}{4} \right] \right. \\ &\quad \cdot \rho^{n+\lambda+1/2} + \left[\frac{(2n+2\lambda+1)(2n+2\lambda+1-2\delta_3)}{4} \right. \\ &\quad \left. - \frac{\delta_2(\delta_1+2)}{2} - \lambda^2 + \frac{\delta_3^2}{4} \right] \rho^{n+\lambda-3/2} \\ &\quad \left. - \left[\delta_1 \left(n + \lambda + 2 - \frac{\delta_3}{2} \right) \right] \rho^{n+\lambda-1/2} + \left[\gamma + \frac{\delta_1^2}{4} \right] \right. \\ &\quad \left. \cdot \rho^{n+\lambda+1/2} \right\} = 0. \end{aligned} \quad (32)$$

To make (32) be valid for all values of ρ , the coefficients of each term of the polynomial of ρ must be equal to zero separately. We, therefore, obtain

$$2(\delta_1 + 2)(n + \lambda + 2) = 4\lambda^2 - 9 - 4, \quad (33a)$$

$$\delta_2 = -(n + \lambda + 2), \quad (33b)$$

$$\delta_3 = 2(n + \lambda + 2), \quad (33c)$$

$$2\delta_4 = (n + \lambda + 2)^2, \quad (33d)$$

$$\gamma = -\frac{\delta_1^2}{4}. \quad (33e)$$

Using (*a), (33a), and (33e) and after simple algebraic calculation, the corresponding energy can be written as

$$E_n^2 = m^2 + k^2 + m^2 a^2 \omega^2 - \frac{1}{16} \left(\frac{4\lambda^2 - 9}{n + \lambda + 2} \right)^2. \quad (34)$$

The general radial wave functions corresponding to the energy spectra given in (34) are

$$\Theta(\rho) = \sum_{n=0}^{\infty} a_n \rho^{n+\lambda+1/2} \exp \left[-\frac{\delta_1}{2\rho} - \frac{\delta_2}{2}\rho - \frac{\delta_3}{2} \log \rho \right]. \quad (35)$$

With the help of (32) and (33a)–(33e), the expansion coefficients a_n in (35) satisfy the following recursion relation [72]:

$$a_{n+1} = \frac{4\lambda^2 - 9 - 4(n + \lambda + 2)}{2(n + \lambda + 2)^2} a_{n-1}. \quad (36)$$

From the recursion relation (36) we can determine the coefficients a_n ($n \neq 0, 1$) of the power series in terms of a_0 and a_1 . In addition the above recursion relation implies that (35) yields one solution as a power series in even powers of ρ and another in odd powers of ρ .

In addition, (27) can be also mapped to the double-confluent Heun equation by appropriate function transformation [75]. So when $f(\rho)$ is taken as singular inverse-square-type potential, the solutions of (27) can be also given by the solution of the double-confluent Heun equation [75, 76].

5. The Solution with $f(\rho)$ to Be Exponential-Type Potential

The exponential-type potentials are very important in the study of various physical systems, particularly for modeling diatomic molecules. The typical exponential-type potentials include Eckart potentials [77], the Morse potential [78, 79], the Wood–Saxon potential [80], and Hulthén potential [81, 82]. The research work on the Dirac equation with the above potential is mainly concentrated on Minkowski time and space. However, it has been noticed recently that it is also interesting to study this kind quantum systems in a cosmic string background [83]. In this section we will take the $f(\rho)$ as exponential-type function and solve the corresponding Dirac equation in cosmic string space-time.

As is known to all, the Dirac equation and Schrödinger equation have been studied by resorting different methods. A usual way is transforming the eigenvalue equation of quantum system considered into a solvable equation via suitable variable substitutions and function transformations [84–86]. In order to obtain solution for $f(\rho)$ being exponential-type potential, we firstly consider the following linear second-order differential equation

$$x^2(1-x)^2 \frac{d^2y}{dx^2} + x(1-x)^2 \frac{dy}{dx} - (\mathcal{L}_1 + \mathcal{L}_2x - \mathcal{L}_3x^2)y = 0, \quad (37)$$

where \mathcal{L}_i , ($i = 1, 2, 3$) are constants. It is known that singular points of a differential equation determine the form of solutions. In this equation, there are two singular points, i.e., $x = 0$ and $x = 1$. In order to remove these singularities and get physically acceptable solutions we use the following ansatz:

$$y = x^\Omega (1-x)^\Lambda R(x). \quad (38)$$

where Λ and Ω are two real parameters. Further we make this two parameters to satisfy following relationships:

$$\begin{aligned} \Omega &= \pm \sqrt{\mathcal{L}_1}, \\ \Lambda &= \frac{1}{2} \left[1 \pm \sqrt{1 - 4(\mathcal{L}_3 - \mathcal{L}_2 - \mathcal{L}_1)} \right], \end{aligned} \quad (39)$$

and by substituting (38) into (37), the differential equation for $\chi(x)$ can be written as

$$\begin{aligned} (1-x)x \frac{\partial^2}{\partial x^2} R(x) &+ [2\Omega + 1 - (2\Omega + 2\Lambda + 1)]x \frac{\partial}{\partial x} R(x) \\ &- (\Omega + \Lambda + \Delta)(\Omega + \Lambda - \Delta)R(x) = 0, \end{aligned} \quad (40)$$

with $\Delta = \pm \sqrt{-\mathcal{L}_3}$. In other words, (37) is reduced to the well-known hypergeometric equation, when condition (39) is imposed. Making use of the boundary conditions at $r = 0$ and $r = \infty$ [86, 87], we can find the equation obeyed by the energy eigenvalue:

$$\Omega + \Lambda + \Delta = -n, \quad (41)$$

and the corresponding eigenfunctions is given in terms of the Gauss hypergeometric functions below

$$\begin{aligned} R(x) &= AF(\tau_1, \tau_2; 1 + 2\Omega; x), \\ \tau_1 &= \Omega + \Lambda + \Delta, \\ \tau_2 &= \Omega + \Lambda - \Delta, \end{aligned} \quad (42)$$

where A is normalization constant. Next, we will use the results given here to obtain the solutions of Dirac equation exponential-type interaction in cosmic string space-time. As a direct application of the above method, let us take the function $f(\rho)$ to be as Yukawa potential, Hulthén-Type potential, and generalized Morse potential, respectively.

Case 1 ($f(\rho)$ being Yukawa potential). In Yukawa meson theory, the Yukawa potential firstly was introduced to describe the interactions between nucleons [88]. Afterwards, it has been applied to many different areas of physics such as high-energy physics [89, 90], molecular physics [91], and plasma physics [92]. In recent years, the considerable efforts have also been made to study the bound state solutions by using different methods.

Now let us choose $f(\rho)$ to be Yukawa potential

$$f(\rho) = \frac{a}{\rho} e^{-\beta\rho}, \quad (43)$$

then (*a) takes the form

$$\begin{aligned} \frac{d^2\chi}{d\rho^2} + \frac{1}{\rho} \frac{d\chi}{d\rho} - \left[\frac{\lambda^2}{\rho^2} + \frac{m\omega a}{\rho^2} e^{-\beta\rho} + \frac{\mu a + am\omega\beta}{\rho} e^{-\beta\rho} \right. \\ \left. + \frac{a^2 m^2 \omega^2}{\rho^2} e^{-2\beta\rho} \right] \chi + \nu\chi = 0. \end{aligned} \quad (44)$$

However, the radial equation (44) cannot accept exact solution due to the presence of the centrifugal term [85]. In order to find analytical solution, we have to use some approximation approaches for the centrifugal term potential. Following [86], the approximation for the centrifugal term reads

$$\frac{1}{\rho^2} \approx \frac{\beta^2}{(1 - e^{-\beta\rho})^2}, \quad (45)$$

$$\frac{1}{\rho} \approx \frac{\beta}{1 - e^{-\beta\rho}}.$$

It is worth mentioning that the above approximations are valid for $\beta\rho \ll 1$ [86]. So if we make the control parameter β small enough, then we can guarantee that the above approximations in (45) hold for larger values ρ . In other words, this approximation (45) is valid in our work.

Using the approximation in (45) and setting

$$\chi = \frac{1}{\sqrt{\rho}}\Theta(\rho), \quad (46)$$

$$x = e^{-\beta\rho},$$

in the new function χ and new variable x , (44) becomes

$$x^2(1-x)^2 \frac{d^2\Theta}{dx^2} + x(1-x)^2 \frac{d\Theta}{dx} - (\mathcal{L}_1 + \mathcal{L}_2x - \mathcal{L}_3x^2)\Theta = 0, \quad (47)$$

where

$$\mathcal{L}_1 = -\lambda^2\beta^2 + m^2 + k^2 - E^2,$$

$$\mathcal{L}_2 = -m\omega\beta^2 - a\mu\beta + 2m^2 + 2k^2 - 2E^2, \quad (48)$$

$$\mathcal{L}_3 = m^2\omega^2a^2 - a\mu\beta - am\omega\beta^2 - m^2 - k^2 + E^2.$$

Comparing (47) with (37) and using the results given in (41) and (42), it is not difficult to find the equation obeyed by eigenvalues and eigenfunctions and they can be given, respectively,

$$E_n^2 - q_1 - \sqrt{q_2 - E_n^2} + \left[n + \frac{1}{2} \left(1 + \sqrt{1 + q_3 - 16E_n^2} \right) \right] = 0, \quad (49)$$

$$\Theta(\rho) = Ae^{-\beta\Omega\rho} (1 - e^{-\beta\rho})^\Lambda F(\tau_1, \tau_2; 1 + 2\Omega; e^{-\beta\rho}),$$

where

$$q_1 = \lambda^2\beta^2 - m^2 - k^2,$$

$$q_2 = m^2 + k^2 + a\mu\beta + am\omega\beta^2 - m^2\omega^2a^2,$$

$$q_3 = 4(4m^2 + 4k^2 - am^2\omega^2), \quad (50)$$

$$\tau_1 = \Omega + \Lambda + \Delta,$$

$$\tau_2 = \Omega + \Lambda - \Delta.$$

Case 2 ($f(\rho)$ being Hulthén-type potential). In this section, we are interested in considering the Hulthén potential that describes the interaction between two atoms and has been used in different areas of physics and attracted a great of interest for some decades [81, 82, 93]. Next we take the interaction function $f(\rho)$ being Hulthén-Type potential

$$f(\rho) = a + \frac{be^{-\beta\rho}}{1 - e^{-\beta\rho}}, \quad (51)$$

where a , b , and β are real constants. Inserting (45) and (51) into (*a), then (*a) can written as

$$\frac{d^2\chi}{d\rho^2} + \frac{1}{\rho} \frac{d\chi}{d\rho} - \left[\frac{\lambda^2}{\rho^2} + \frac{\mu a}{\rho} + a^2 m^2 \omega^2 + \left(\frac{\mu b}{\rho} + 2abm^2\omega^2 \right) \frac{e^{-\beta\rho}}{1 - e^{-\beta\rho}} \right] \chi + (b^2 m^2 \omega^2 e^{-\beta\rho} + m\omega b\beta) \frac{e^{-\beta\rho}}{(1 - e^{-\beta\rho})^2} \chi + \nu\chi = 0. \quad (52)$$

In the same way as in previous section, taking into consideration approximation (45) for the centrifugal term and using the variable transformation $x = e^{-\beta\rho}$ and function transformation $\chi = (1/\sqrt{\rho})\Theta(\rho)$, (51) changes

$$x^2(1-x)^2 \frac{d^2\Theta}{dx^2} + x(1-x)^2 \frac{d\Theta}{dx} - (\mathcal{L}_1 + \mathcal{L}_2x - \mathcal{L}_3x^2)\Theta = 0, \quad (53)$$

where

$$\mathcal{L}_1 = \lambda^2\beta^2 + m^2\omega^2a^2 + \beta\mu a + m^2 + k^2 - E^2,$$

$$\mathcal{L}_2 = m\omega b\beta + (b - a)(\mu\beta - 2m\omega) + 2m^2 + 2k^2 - 2E^2, \quad (54)$$

$$\mathcal{L}_3 = -m^2\omega^2(a - b)^2 - m^2 - k^2 + E^2.$$

With the help of (38), (41), and (42), the solutions for $f(\rho)$ being Hulthén-Type potential can be easily obtained and the corresponding eigenvalues and eigenfunctions are, respectively,

$$E_n^2 - q_4 - \sqrt{q_5 - E_n^2} + \left[n + \frac{1}{2} \left(1 + \sqrt{1 + q_6 - 16E_n^2} \right) \right] = 0, \quad (55)$$

$$\Theta(\rho) = Ae^{-\beta\Omega\rho} (1 - e^{-\beta\rho})^\Lambda F(\tau_1, \tau_2; 1 + 2\Omega; e^{-\beta\rho}), \quad (56)$$

where

$$\begin{aligned}
q_4 &= \lambda^2 \beta^2 + m^2 \omega^2 a^2 + \beta \mu a + m^2 + k^2, \\
q_5 &= m^2 \omega^2 (a - b)^2 + m^2 + k^2, \\
q_6 &= 4(m^2 + k^2) + 4m^2 \omega^2 (2a^2 - 2ab + b^2) + 4\beta^2 \lambda^2 \\
&\quad + 4\beta \mu a + 4m\omega b \beta + 4(a - b)(\mu \beta - 2m\omega), \\
\tau_1 &= \Omega + \Lambda + \Delta, \\
\tau_2 &= \Omega + \Lambda - \Delta.
\end{aligned} \tag{57}$$

Case 3 ($f(\rho)$ being generalized Morse potential). The Morse potential [78, 79] as an important molecular potential describes the interaction between two atoms. We choose the interaction function $f(\rho)$ being generalized Morse potential

$$f(\rho) = \frac{a}{\rho^2} (e^{-\beta\rho} - e^{-2\beta\rho}). \tag{58}$$

As before, substitution of form (59) into (*a) and straightforward calculation lead to the following equation:

$$\begin{aligned}
\frac{d^2 \chi}{d\rho^2} + \frac{1}{\rho} \frac{d\chi}{d\rho} - \left[\frac{\lambda^2}{\rho^2} + \frac{\mu a + 2m\omega a \mu}{\rho^3} (e^{-\beta\rho} - e^{-2\beta\rho}) \right. \\
\left. - \frac{a\beta m \omega}{\rho^2} (2e^{-2\beta\rho} - e^{-\beta\rho}) \right] \chi + \frac{a^2 m^2 \omega^2}{\rho^4} (e^{-\beta\rho} \\
- e^{-2\beta\rho})^2 \chi + \nu \chi = 0.
\end{aligned} \tag{59}$$

Letting $x = e^{-\beta\rho}$ and $\chi = (1/\sqrt{\rho})\Theta(\rho)$, the above differential equation (59) changes into the form

$$\begin{aligned}
x^2 (1-x)^2 \frac{d^2 \Theta}{dx^2} + x(1-x)^2 \frac{d\Theta}{dx} \\
- (\mathcal{L}_1 + \mathcal{L}_2 x - \mathcal{L}_3 x^2) \Theta = 0,
\end{aligned} \tag{60}$$

where

$$\begin{aligned}
\mathcal{L}_1 &= \lambda^2 \beta^2 + m^2 + k^2 - E^2, \\
\mathcal{L}_2 &= \mu a (1 + 2m\omega) + 2am\omega\beta - 2m^2 - 2k^2 + 2E^2, \\
\mathcal{L}_3 &= E^2 - m^2 \omega^2 a^2 + 2m\omega a \beta - m^2 - k^2.
\end{aligned} \tag{61}$$

It is easy to see that the differential equation (60) is also similar to (37). So again according to the quantization condition (40) the corresponding expression of eigenvalues can be written as

$$\begin{aligned}
E_n^2 - q_7 - \sqrt{q_8 - E_n^2} + \left[n + \frac{1}{2} \left(1 + \sqrt{1 + q_9 - 16E_n^2} \right) \right] \\
= 0, \\
q_7 &= \lambda^2 \beta^2 + m^2 + k^2, \\
q_8 &= m^2 \omega^2 a^2 - 2m\omega a \beta + m^2 + k^2, \\
q_9 &= 4 \left[\mu a (1 - 2m\omega) - 2m\omega a \beta + a^2 m^2 \omega^2 + \lambda^2 \beta^2 \right].
\end{aligned} \tag{62}$$

The wave function in this case read

$$\Theta(\rho) = A e^{-\beta\Omega\rho} (1 - e^{-\beta\rho})^\Lambda F(\tau_1, \tau_2; 1 + 2\Omega; e^{-\beta\rho}), \tag{63}$$

where

$$\begin{aligned}
\tau_1 &= \Omega + \Lambda + \Delta, \\
\tau_2 &= \Omega + \Lambda - \Delta.
\end{aligned} \tag{64}$$

The above results show that the radial equation of the generalized Dirac oscillator with interaction function $f_\mu(x_\mu)$ to be taken as the exponential-type potential can be mapped into the well-known hypergeometric equation and the analytical solutions can have been found.

6. Conclusion

In this work, the generalized Dirac oscillator has been studied in the presence of the gravitational fields of a cosmic string. The corresponding radial equation of generalized Dirac oscillator is obtained. In our generalized Dirac oscillator model, we take the interaction function $f_\mu(x_\mu)$ to be as Cornell potential, Yukawa potential, generalized Morse potential, Hulthén-Type potential, and singular potential, respectively. By solving the corresponding wave equations the corresponding energy eigenvalues and the wave functions have been obtained and we have showed how the cosmic string leads to modifications in the spectrum and wave function. Based on consideration that Dirac oscillator has been studied extensively in high-energy physics, condensed matter physics, quantum optics, mathematical physics, and even connection with Higgs symmetry it also makes sense to generalize the generalized Dirac oscillator to these fields.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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