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Some comments on the holographic heavy quark potential in a thermal bath

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Abstract

The heavy quark potential of a thermal Yang–Mills theory in strong coupling limit is explored in terms of the holographic principle. With a fairly general AdS/QCD metric the heavy quark potential displays a kink-like screening in the plasma phase. This behavior may conflict the causality of a field theory that is mathematically equivalent to the thermal Yang–Mills.

1. Introduction

The heavy-quark potential (the interaction energy between a quark and its antiparticle in the infinite mass limit) is a very important quantity of QCD. Not only does it provide the information of the quarkonium dissociation which signals the formation of QGP in heavy ion collisions [1], but also is one of the basic probes to explore the phase diagram with nonzero temperature and baryon density. While the potential is Coulomb like for a short separation, $r << \Lambda_{\rm QCD}^{-1}$, the potential rises linearly at large distance, $r >> \Lambda_{\rm QCD}^{-1}$ [2] in the confined phase without light

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quarks as is suggested by the Regge behavior of meson spectra and is expected to be screened within a radius of the order T^{-1} in the deconfined phase.

Mathematically, the heavy-quark potential can be extracted from the expectation value of a Wilson loop, or the correlator between two Polyakov loops [3]. The highly nonperturbative nature of infrared QCD makes it analytically intractable, especially in the confined phase. The lattice simulation of the Wilson loops and/or the Polyakov loops has accumulated sufficient evidence supporting the linear potential below the deconfinement temperature and the screened potential in the plasma phase [4–12]. In particular, the data of [7] for the quenched QCD fits extremely well with a Debye like screening potential in the plasma phase. Furthermore a resummation of perturbative series yields an exponentially screening at high temperature in weak coupling [13,14].

The advent of the holographic principle [15,16], especially the AdS/CFT correspondence [17] opens a new avenue towards analytic treatments of the strong coupling limit of a gauge theory, in particular, the $N = 4$ supersymmetric $SU(N_c)$ Yang–Mills theory at large N_c and large 't Hooft coupling, $\lambda \equiv N_c g_{\rm YM}^2$ with $g_{\rm YM}$ the Yang–Mills coupling. The heavy-quark potential [17–30] at zero temperature is Coulomb like with the strength proportional to $\sqrt{\lambda}$ [17,20,21]. At a nonzero temperature [18,19,22], the potential displays a kink-like screening with a radius of $r_s \simeq 0.754(\pi T)^{-1}$, i.e. the potential is flattened out beyond r_s. This transition is interpreted as string melting in [18] and will be referred to as the kink-like screening in this paper. But the super Yang–Mills is not QCD and the conformal property of the former makes the Coulomb like behavior the only possible outcome at zero temperature following a dimensional argument. A cousin of AdS/CFT with an infrared cutoff, AdS/QCD, has been actively investigated and is able to provide a linear heavy quark-potential at zero temperature. The nonzero temperature behavior were calculated and compared with the lattice data [23–25].

In this paper, we shall explore analytically the property of the heavy-quark potential within the general framework of AdS/QCD. In particular we want to see if a smooth screening potential can emerge in the plasma phase and our conclusion is no. In the next section, the heavy-quark potential of the $N = 4$ super Yang–Mills at a nonzero temperature following AdS/CFT correspondence will be reviewed, where we shall also set up our notations. In the section 3, we shall show that under a fairly general conditions of the metric underlying AdS/QCD, the screening remains kink-like, like that of the super Yang–Mills. In other words, AdS/QCD cannot provide a exponentially screening potential in the plasma phase. Different scenarios of a smooth screening behaviors proposed in the literature will be discussed in the section 4, where we shall also point out a potential gap between the kink-like screening potential and the fundamental principles of quantum field theories.

2. The heavy-quark potential in a $N = 4$ super Yang–Mills plasma in strong coupling

AdS/CFT correspondence relates the type IIB superstring theory in $AdS_5 \times S^5$ background to the $N = 4$ supersymmetric $SU(N_c)$ Yang–Mills theory at the AdS boundary. In particular, the thermal expectation value of a Wilson loop C, $< W[C] >$, at large N_c and large 't Hooft coupling constant $\lambda \equiv N_c g_{\rm YM}^2$ corresponds to the minimum area of a string world sheet in the Schwarzschild-AdS_5 metric,

$$ds^2 = \frac{1}{z^2}\left[\left(1 - \frac{z^4}{z_h^4}\right)d\tau^2 + \left(1 - \frac{z^4}{z_h^4}\right)^{-1} dz^2 + d\vec{x}^2\right] \tag{2.1}$$

bounded by the loop C at the boundary $z = 0$, i.e.

$$< W[C] > = \text{const}.e^{-\min.(S_{\rm NB}[C])} \tag{2.2}$$

where the Nambu–Goto action

$$S_{\rm NB}[C] = \frac{\mathcal{S}}{2\pi\alpha'} = \frac{1}{2\pi\alpha'}\int d^2\sigma\sqrt{g} \tag{2.3}$$

with $\mathcal{S}$ the world sheet area, and g the determinant of the induced world sheet metric, $ds^2 = g_{ab}d\sigma^a d\sigma^b$ $(a, b = 0, 1)$. The horizon radius z_h corresponds to the temperature $T = (\pi z_h)^{-1}$ and the string tension corresponds to the 't Hooft coupling $\sqrt{\lambda} = \alpha^{-1}$.

The Wilson loop for the free energy of a single heavy quark or antiquark is a Polyakov loop winding up the Euclidean time dimension and will be denoted by $\mathcal{C}_1$. The Wilson loop for the free energy of a pair of heavy quark and antiquark consists of two Polyakov loops running in opposite directions and will be denoted by $\mathcal{C}_2$. The heavy quark potential $V(r)$ corresponds to the interaction part of the free energy of the quark pair and is extracted from the ratio

$$\frac{< W[\mathcal{C}_2] >}{| < W[\mathcal{C}_1] > |^2} = \frac{\text{Tr} < \mathcal{P}(\vec{r})\mathcal{P}^\dagger(0) >}{|\text{Tr} < \mathcal{P}(0) > |^2} \equiv e^{-\frac{V(r)}{T}} \tag{2.4}$$

where the Polyakov loop operator[1]

$$\mathcal{P}(\vec{r}) = \mathcal{T}e^{-i\int_0^{T^{-1}} dt A_0(\vec{r},\tau)} \tag{2.5}$$

with $A_0(\vec{r}, \tau)$ the temporal component of the Lie Algebra valued gauge potential and $\mathcal{T}$ the time ordering operator. The solution of the Euler–Lagrange equation for $\mathcal{C}_1$ is a world sheet with constant 3D spatial coordinates $\vec{x}$ extending from the AdS boundary $z = 0$ to the black hole horizon $z = z_h$ and its NG action will be denoted by $\frac{1}{T}\mathcal{A}_1$. The solution of the Euler-Lagrange equation for $\mathcal{C}_2$ can be either a connected nontrivial world sheet shown in Fig. 1a or two parallel world sheets shown in Fig. 1b with each identical to the world sheet of $\mathcal{C}_1$. The NG action of the former will be denoted by $\frac{1}{T}\mathcal{A}_2$ while the NG action the latter is given by $\frac{2}{T}\mathcal{A}_1$ and the minimum of them contributes to the free energy. It follows from (2.4) that

$$V(r) = \min(\mathcal{A}, 0), \tag{2.6}$$

where $\mathcal{A} \equiv \mathcal{A}_2 - 2\mathcal{A}_1$ and this combination cancels the UV divergences pertaining to $\mathcal{A}_1$ and $\mathcal{A}_2$. We shall name $\mathcal{A}(r)$ as the candidate potential.

For $N = 4$ super Yang–Mills, the Euler–Lagrange equation that minimizes $S_{\rm NB}[C]$ yields the following parametric form of the candidate potential $\mathcal{A}(r)$ [17–19]

$$\begin{cases} r = 2\sqrt{z_h^4 - z_c^4}\int_0^{z_c} dz\frac{z^2}{\sqrt{(z_h^4-z^4)(z_c^4-z^4)}} \\ \mathcal{A} = \frac{\sqrt{\lambda z_c^2}}{\pi z_h^2}\left[\int_0^{z_c}\frac{dz}{z^2}\left(\sqrt{\frac{z_h^4-z^4}{z_c^4-z^4}} - 1\right) - \int_0^{z_h} dz\frac{1}{z^2}\right] \end{cases} \tag{2.7}$$

where the parameter z_c is the maximum extension of the world sheet in the bulk. The candidate potential $\mathcal{A}$ as a function of the distance, shown in Fig. 2, consists of two branches. As z_c starts from the AdS boundary, $\mathcal{A}$ starts with an attractive Coulomb like form

[1] Strictly speaking, the "heavy quark" in the super Yang–Mills refers to the heavy W-boson of the symmetric breaking from $SU(N_c)$ to $SU(N_c - 1)$ and the Polyakov loop operator contains the contribution from the scalar field. See e.g. [17] for details.

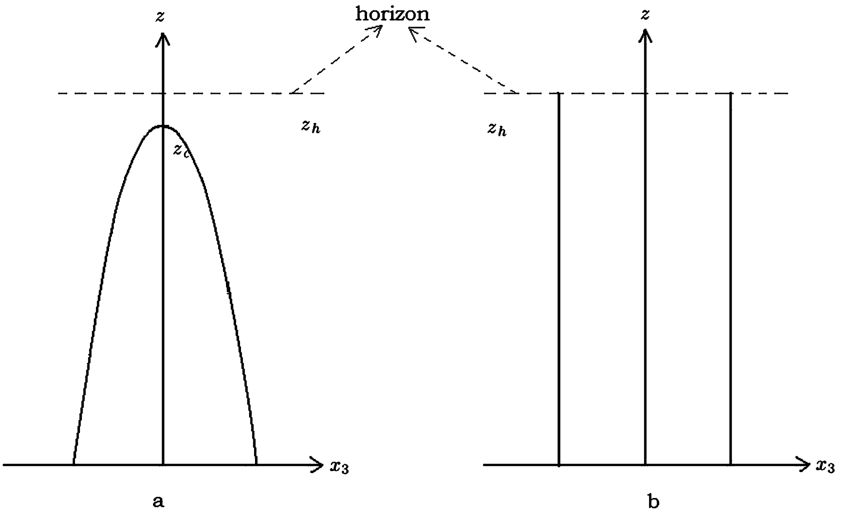

Fig. 1. (a) The connected nontrivial world sheet from the boundary $z = 0$ extending to $z_c < z_h$. (b) Is two parallel world sheets starting from the boundary $z = 0$ and ending at the horizon $z = z_h$. The upper dashed lines represent the black hole horizon.

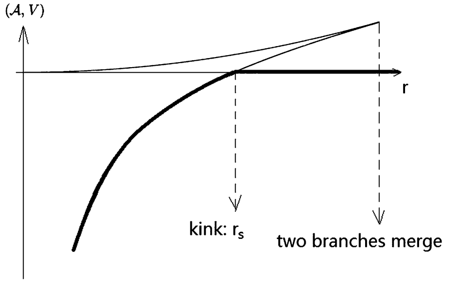

Fig. 2. The candidate potential (thin line) and the heavy quark potential (thick line) of the super Yang–Mills. Both coincide below the r-axis.

$$\mathcal{A} \simeq -\frac{4\pi^2\sqrt{\lambda}}{\Gamma^4(1/4)r} \tag{2.8}$$

for $rT << 1$ along the lower branch. Then $\mathcal{A}$ becomes positive at $r = 0.745/(\pi T) \equiv r_s$ and reaches the end of the lower branch which corresponds to the maximum of r as a function of z_c. Beyond this value of z_c, the potential follows the upper (repulsive) branch and decreases to zero as $z_c \to z_h$ ($r \to 0$). According to (2.6), the potential $V(r)$ is given by $\mathcal{A}$ for $r < r_s$ and vanishes beyond r_s. Numerically, this screening potential can be well approximated by a truncated Coulomb potential

$$V(r) = \begin{cases} \kappa\left(\frac{1}{r} - \frac{1}{a}\right) & \text{for } r \le a \\ 0 & \text{for } r > a \end{cases} \tag{2.9}$$

with $\kappa = -\frac{4\pi^2\sqrt{\lambda}}{\Gamma^4(1/4)}$ and $a = \frac{4\pi^2}{\Gamma^4(1/4)T} \simeq \frac{0.736}{\pi T}$ [31].

3. The heavy-quark potential with a general AdS/QCD metric at a nonzero temperature

Unlike the $N = 4$ super Yang–Mills, the real QCD is characterized by an intrinsic energy scale, $\Lambda_{\rm QCD}$, and the metric of its gravity dual, if exists, should carry a length scale other than the horizon radius. We shall follow the bottom-up approach in this section. The most general metric of AdS/QCD take the form[2]

$$ds^2 = \frac{w(z)}{z^2}[f(z)d\tau^2 + \frac{1}{f(z)}dz^2 + d\vec{x}^2]. \tag{3.1}$$

Different proposals for the expressions of the warp factor $w(z)$ and the function $f(z)$ have been explored in the literature. They may be simply specified to warrant an analytic treatment [32–34] or may be dictated by the solution of Einstein equation coupled to a dilaton field [35]. A number of general conditions should be satisfied by $w(z)$ and $f(z)$: 1) The conformal invariance of QCD in UV limit requires that $w(0) = f(0) = 1$; 2) The existence of a horizon with a nonzero Hawking temperature requires that $f(z) > 0$ for $0 < z < z_h$, $f(z) = k(z_h - z)$ with $k > 0$ as $z \to z_h^-$ and there is no curvature singularity for $0 \le z \le z_h$. We further assume that both $w(z)$ and $f(z)$ are infinitely differentiable and nonvanishing outside the horizon, $0 < z < z_h$. Minimizing the Nambu–Goto action of the string world sheet embedded in the background metric (3.1), we find the parametric form of the function $\mathcal{A}(r)$

$$\begin{cases} r = 2\sqrt{F_c}\int_0^{z_c} \frac{dz}{\sqrt{f(F-F_c)}} \\ \mathcal{A} = \frac{\sqrt{\lambda}}{\pi}\left[\int_0^{z_c} dz\sqrt{\frac{F}{f}}\left(\frac{1}{\sqrt{1-\frac{F_c}{F}}} - 1\right) - \int_{z_c}^{z_h} dz\frac{w}{z^2}\right] \end{cases} \tag{3.2}$$

where

$$F \equiv \frac{w^2(z)}{z^4}f(z) \tag{3.3}$$

and $F_c = F(z_c)$. The reality of the potential requires that z_c stays within the domain adjacent to the boundary where $F(z)$ is non-increasing. As $z_c \to 0$, we have $r \to 0$ and end up with the Coulomb like potential (2.8).

As a necessary condition for a smooth screening behavior, there must exists a z_c where $r \to \infty$ while $\mathcal{A}$ stays finite. As we shall see that this is not the case. The integral (3.2) diverges for $z_c = z_h$ but the factor $\sqrt{F_c}$ removes the divergence of the limit $z_c \to z_h$. Consequently, the candidate potential $\mathcal{A}$ will be restricted within a finite r like that of the super Yang–Mills, if the derivative of $F(z)$ is nonvanishing for $0 < z \le z_h$. Alternatively, the integral may also diverge when $z_c < z_h$ but close to the point where the derivative of $F(z)$ vanishes. But in this case, both r of (3.2) and $\mathcal{A}$ of (3.2) share the same divergence and a linear candidate potential at large distance emerges. A special example of the latter case, $f(z) = 1 - \frac{z^4}{z_h^4}$ and $w(z) = e^{\frac{1}{2}cz^2}$ was proposed in [33,34] to generate the Cornell potential at low temperature.[3]

2 Any 5d metric that is translational invariant in time and three spatial coordinates and is rotational invariant in the three spatial coordinates can be casted into this form.

3 At $T = 0$, the combination $\mathcal{A}$ diverges because of the limit $z_h \to \infty$ of the integration of $\mathcal{A}_1$. This reflects the nonexistence of an isolated heavy quark because of the color confinement. At a nonzero T, however, $\mathcal{A}$ becomes finite and there is always a r_s where $\mathcal{A}$ switch sign. In another word, the linearly confining potential is flattened out beyond

To elaborate the above statements, let us examine the behavior near the horizon, where $F(z)$ vanishes according the power law $F(z) \simeq K(z_h - z)^n$ with $K > 0$ and $n > 0$ (monotonically decreasing). It follows from (3.3) that the warp factor $w(z) \sim (z_h - z)^{\frac{n-1}{2}}$. For the super Yang–Mills, $n = 1$ and $K = \frac{4}{z_h^5}$. The exponent can be further constrained by ruling out the curvature singularity at $z = z_h$. It is straightforward to calculate the Riemann tensor of the metric (3.1). Of particular interest is the component

$$R_{ijkl} = -e^{2\phi} f \phi'^2 (\delta_{ik}\delta_{jl} - \delta_{il}\delta_{jk}) \tag{3.4}$$

where $e^{2\phi} \equiv \frac{w(z)}{z^2}$ and the prime denotes the derivative with respect to z. It will contribute a term

$$12 e^{-4\phi} f^2 \phi'^4 \tag{3.5}$$

to the invariant $R^{\mu\nu\rho\lambda} R_{\mu\nu\rho\lambda}$. It is straightforward to verify that this term diverges at $z = z_h$ for all $n > 0$ except that $n = 1$. This divergence will lead $R^{\mu\nu\rho\lambda} R_{\mu\nu\rho\lambda}$ to diverge because the contribution from all components are positive. Therefore we are left with only the case $n = 1$ to consider, which gives qualitatively the same behavior of the potential as the super Yang–Mills in the limit $z_c \to z_h$.

To isolate out the leading behavior as $z_c \to z_h$, we introduce δ such that $z_c - z_h << \delta << z_c$ and divide the integration domain $(0, z_c)$ into $(0, z_c - \delta)$ and $(z_c - \delta, z_c)$. In the latter domain, we may make the approximation $f(z) \simeq k(z_h - z)$ and $F(z) = K(z_h - z)$ with both k and K positive constants. Consequently

$$\int\limits_{z_c-\delta}^{z_c} \frac{dz}{\sqrt{f(F - F_c)}} \simeq \frac{1}{\sqrt{kK}} \int\limits_{f_c}^{k\delta} \frac{df}{\sqrt{f(f - f_c)}} \simeq \frac{1}{\sqrt{kK}} \ln \frac{k\delta}{f_c}. \tag{3.6}$$

In the former domain, we may set $F_c = 0$ in the integrand and end up with

$$\int\limits_{0}^{z_c-\delta} \frac{dz}{\sqrt{f(F - F_c)}} \simeq \int\limits_{0}^{z_h-\delta} \frac{dz}{\sqrt{fF}}, \tag{3.7}$$

which is independent of f_c. It follows that

$$r \simeq 2\frac{\sqrt{f_c}}{k} \left(\ln \frac{1}{f_c} + \text{const.} \right) \tag{3.8}$$

as $z_c \to z_h$. Applying the same procedure to the first integral in (3.2), we obtain the leading behavior

$$\mathcal{A} \simeq \frac{\sqrt{\lambda} w_c}{\pi k z_c^2} f_c \left(\ln \frac{1}{f_c} + \text{const.} \right) \tag{3.9}$$

which returns to zero from the positive side. Because of the continuity of r and $\mathcal{A}$ as functions of z_c, there exists a special value of z_c, $z_s \in (0, z_h)$, where the potential changes from attractive for $z_c < z_s$ to repulsive for $z_c > z_s$. If the distance r as a function of z_c consists of m local maxima

r_s. There is no a clear cut distinction between a confined phase and a plasma phase in this regard. The deconfinement transition corresponds to a crossover from $r_s \sim \frac{T^3}{c^2} \exp\left(\frac{c}{2\pi^2 T^2}\right) >> \frac{1}{T}$ at low temperature, $T << \sqrt{c}$, to $r_s \sim \frac{1}{T}$ at high temperature, $T >> \sqrt{c}$.

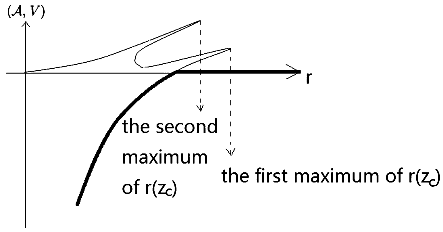

Fig. 3. An example of the candidate potential $\mathcal{A}$ with three branches (thin line) together with the corresponding kink-like heavy quark potential V (thick line) of AdS/QCD.

outside the horizon, the candidate potential function $\mathcal{A}(r)$ will consist of $m+1$ branches and we have $m=1$ for the super Yang–Mills. The potential with three branches is shown in Fig. 3 and an example of the function $r(z_c)$ with two maxima is shown in the Appendix. To reproduce the kink-like screening of Fig. 2, the potential $\mathcal{A}$ as a function of z_c has to be positive at the first local maximum of $r(z_c)$. The heavy quark potential will follow $\mathcal{A}(r)$ for $z<z_s$ and becomes flat afterwards. Next, we explore the case where the derivative of $F(z)$ vanishes somewhere outside the horizon, i.e. $F'(z_0)=0$ for $0<z_0<z_h$. The case when z_0 is minimum of $F(z)$ was considered in [36]. As $z\to z_0$, we may approximate

$$F(z)\simeq a+b(z_0-z)^n \tag{3.10}$$

with positive a and b, and an integer $n\geq 2$. For an even n, z_c has to stay on the side $z_c<z_0$ in order for the square root to be real. As $z_c\to z_0$, both the integral in (3.2) and the first integral for (3.2) diverges. Dividing the integration domain $(0,z_c)$ into $(0,z_c-\delta)$ and $(z_c-\delta,z_c)$ with $\epsilon\equiv|z_0-z_c|<<\delta<<z_0$, we can easily extract the leading behaviors for $z_c\to z_0-0^+$,

$$\begin{cases} r\simeq \frac{2w_0}{\sqrt{b z_0^2}}\left(\ln\frac{z_0}{\epsilon}+\text{const.}\right) \\ \mathcal{A}\simeq\sqrt{\frac{\lambda}{bf_0}}\frac{a}{\pi}\left(\ln\frac{z_0}{\epsilon}+\text{const.}\right)\end{cases} \tag{3.11}$$

for $n=2$ and

$$\begin{cases} r\simeq \frac{2}{n}B\left(\frac{1}{2}-\frac{1}{n},\frac{1}{2}\right)\sqrt{\frac{a}{bf_0}}\epsilon^{1-\frac{n}{2}} \\ \mathcal{A}\simeq\frac{1}{n\pi}B\left(\frac{1}{2}-\frac{1}{n},\frac{1}{2}\right)\sqrt{\frac{\lambda a^2}{bf_0}}\epsilon^{1-\frac{n}{2}}\end{cases} \tag{3.12}$$

for $n>2$. Similar leading behavior as $z_c\to z_0+0^+$ read

$$\begin{cases} r\simeq \frac{2}{n}\left[B\left(\frac{1}{n},\frac{1}{2}\right)+B\left(\frac{1}{n},\frac{1}{2}-\frac{1}{n}\right)\right]\sqrt{a}bf_0\epsilon^{1-\frac{n}{2}} \\ \mathcal{A}\simeq\frac{1}{n\pi}\left[B\left(\frac{1}{n},\frac{1}{2}\right)+B\left(\frac{1}{n},\frac{1}{2}-\frac{1}{n}\right)\right]\sqrt{\lambda a^2}bf_0\epsilon^{1-\frac{n}{2}}\end{cases} \tag{3.13}$$

for an odd $n\geq 3$. In all cases above, the function $\mathcal{A}(r)$ contains a branch of Cornell potential. The examples of $F(z)$ and the corresponding $\mathcal{A}(r)$ and $V(r)$ for an even n and for an odd n are depicted in Figs. 4–5.

4. Discussions

In previous sections, we have explored all possible forms of the holographic heavy-quark potential at a nonzero temperature with a general metric that is asymptotically AdS and carries

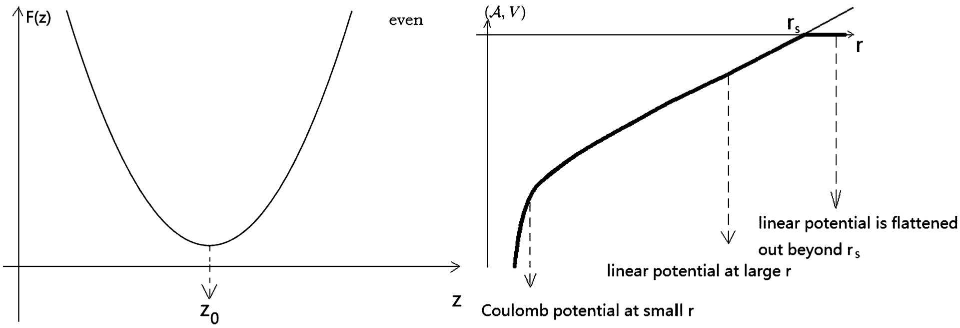

Fig. 4. The left panel shows the example of F(z) whose expansion around z_0 is $F(z) \simeq a + b(z_0 - z)^n$ with an even n. The right panel is the corresponding candidate potential $\mathcal{A}$ as a function of r which is a Coulomb potential for small r combining with a linear potential at large r limit. The thick line represents the heavy quark potential V.

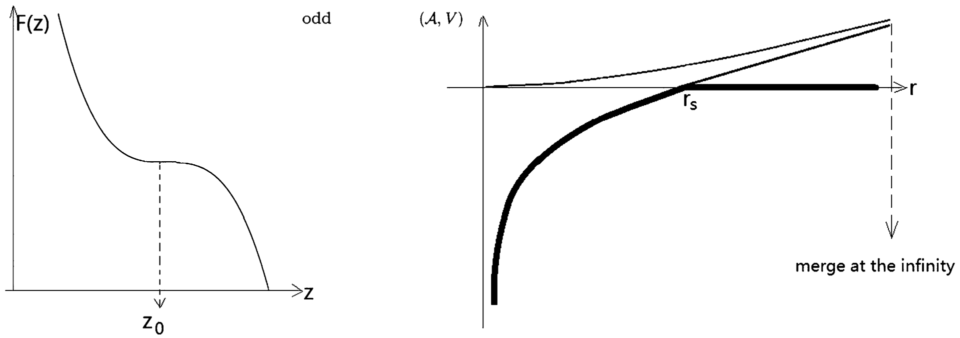

Fig. 5. The left panel shows the example of F(z) whose expansion around z_0 is $F(z) \simeq a + b(z_0 - z)^n$ with an odd n. The right panel shows the two branches of the candidate potential $\mathcal{A}$ as a function of r. Both branches rise linearly with r and will merge at $r \to \infty$. The thick line represents the heavy quark potential V.

a black hole. The candidate potential function $\mathcal{A}(r)$ is either supported within a finite range of the distance r (Figs. 2–3) or takes the linear form for large r when r is allowed to go to infinity (Figs. 4–5). The heavy quark potential $V(r)$ coincides with $\mathcal{A}$ when the free energy of the interacting quark–antiquark pair (represented by the world sheet of Fig. 1a) is lower than that of a non-interacting pair (represented by the world sheet of Fig. 1b) and is flattened out otherwise. A kink-like screening behavior with a discontinuity in the derivative dV/dr is developed then.

In this section we would like first to comment on some proposals in the literature to smooth out the kink-like the heavy quark potential in a thermal bath within the framework of the super Yang–Mills. The authors of [37] suggested that the parameter z_c becomes complex beyond the value when r is maximized. So the potential develops an imaginary part and decays with power law. This, however, cannot be the case at thermal equilibrium using the definition (2.4) since the thermal expectation value $\mathrm{Tr} < \mathcal{P}(\vec{r})\mathcal{P}^\dagger(0) >$ is strictly real as is evident from the following reasoning:

$$\begin{aligned} &\mathrm{Tr} < \mathcal{P}(\vec{r})\mathcal{P}^\dagger(0) >^* = \mathrm{Tr} < \mathcal{P}(0)\mathcal{P}^\dagger(\vec{r}) > \\ &= \mathrm{Tr} < \mathcal{P}(-\vec{r})\mathcal{P}^\dagger(0) > = \mathrm{Tr} < \mathcal{P}(\vec{r})\mathcal{P}^\dagger(0) > \end{aligned} \tag{4.1}$$

where the second equality follows from the translation invariance and the last equality from the rotation invariance (the thermal expectation value should be a function of $|\vec{r}|$). One can obtain a complex potential using other definition or by analytic continuation [12,29]. The authors of [38] suggested an alternative string world sheet by joining the two parallel world sheets of noninteracting quarks with a thin tube to represent the exchange of the lightest supergravity mode. While physically plausible, the area of such a configuration is always larger than that without the tube because the background metric (2.1) is *independent* of the transverse coordinates $\vec{x}$.

Next, we shall question the legitimacy of a kink-like screening potential from field theoretic point of view. Because of the O(4) symmetry of the Lagrangian density, the path integral of a relativistic field theory in R^3 space at temperature T, is mathematically equivalent to the Euclidean field theory under the same Lagrangian density but formulated in $R^2 \times S^1$ space at zero temperature [38,39], provide one of the non-compact spatial dimension of the former, say x^3, is interpreted as the Euclidean time of the latter and the Matsubara time of the former is regarded as the compactified spatial dimension, S^1. The latter field theory will be referred to as the equivalent field theory and is as well defined as the original thermal field theory. Consequently, a static thermal Green's function of the original thermal field theory corresponds a time-dependent Green's function at zero temperature of the equivalent field theory and the analyticity on the complex energy plane of its Fourier transformation should meet the requirements imposed by the unitarity and causality. In particular, the Fourier transformation should have singularities along the real energy axis, signifying the excitation spectrum, and vanish at infinity in order for the retarded (advanced) Green's function to exists. Let us consider the Green's function of the equivalent field theory that corresponds to the Polyakov loop correlator considered in this work, i.e.

$$\begin{aligned}\mathcal{G}(r) &\equiv \frac{\mathrm{Tr} < \mathcal{P}(\vec{r})\mathcal{P}^\dagger(0) >}{|\mathrm{Tr} < \mathcal{P}(0) > |^2} - \lim_{r\to\infty} \frac{\mathrm{Tr} < \mathcal{P}(\vec{r})\mathcal{P}^\dagger(0) >}{|\mathrm{Tr} < \mathcal{P}(0) > |^2} \\ &= \left[e^{-\frac{V(r)}{T}} - 1\right]\theta(r_s - r)\end{aligned} \tag{4.2}$$

From the equivalent field theory perspectives, the Polyakov loop winds up the compactified spatial dimension with $r = \sqrt{\vec{r}_\perp^2 + \tau^2}$, where $\vec{r}_\perp$ and τ are the spatial coordinate difference transverse to the winding dimension and the Euclidean time difference between the two Polyakov loops. The Fourier transformation of (4.2) reads

$$\begin{aligned}\mathcal{G}(i\omega, \vec{r}_\perp) &= \int\limits_{-\infty}^{\infty} d\tau e^{i\omega\tau}\mathcal{G}(r) \\ &= 2\theta(r_s - r_\perp) \int\limits_0^{\sqrt{r_s^2 - r_\perp^2}} d\tau \left[e^{-\frac{V(r)}{T}} - 1\right]\cos\omega\tau\end{aligned} \tag{4.3}$$

The finite integration domain makes the analytic continuation to real energy, $i\omega \to \omega$ straightforward and we end up with

$$\mathcal{G}(\omega, \vec{r}_\perp) = 2\theta(r_s - r_\perp) \int\limits_0^{\sqrt{r_s^2 - r_\perp^2}} d\tau \left[e^{-\frac{V(r)}{T}} - 1\right]\cosh\omega\tau$$

(4.4)

which is an entire function of ω and blows up exponentially as $\omega \to \infty$. The Fourier integral for the retarded (advanced) Green's function of the equivalent field theory,

$$G_{R(A)}(t, \vec{r}_\perp) \equiv \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} e^{-i\omega t} \mathcal{G}(\omega \pm i0^+, \vec{r}_\perp) \tag{4.5}$$

with the upper (lower) sign for $R(A)$ no longer exists. Therefore, the kink-like heavy quark potential in the AdS/QCD fails to generate required analyticity of the Green's function in the equivalent field theory. We consider this problem a potential gap between the minimum area framework of the gravity dual and the field theoretic principle. It requires further investigations, perhaps some resummation of the finite N_c or and/or finite λ corrections to the leading form (2.2) to smear the kink of the screening potential in a string theory inspired AdS/QCD. A toy model of such a resummation mechanism is the function

$$\begin{aligned} u(x) &= \frac{1}{2}(x - \sqrt{x^2 + \epsilon^2}) \\ &= \frac{x - |x|}{2} + \sum_{n=1}^{\infty} \frac{(-)^n \Gamma\left(n - \frac{1}{2}\right)}{n!} \left(\frac{\epsilon}{x}\right)^{2n} \end{aligned} \tag{4.6}$$

with x mimicking $r - r_s$ and ϵ mimicking the expansion parameter associate to the large N_c and/or large λ. A kink is developed in to the leading order ($\epsilon \to 0$). Each higher order term diverges at $x = 0$ and the resummation removes the kink. In the case of $N = 4$ super Yang–Mills, the leading order finite λ correction reported in [22] does not show such a divergence and the kink remains but the higher order corrections and/or finite N_c corrections may resolve the issue.

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Appendix A

In this appendix, we will show an example of the functions $F(z)$ and $f(z)$ which can produce 2 maxima value of $r(z_c)$.

Let us consider the following forms of $f(z)$ and $F(z)$ finite temperature case and choose $z_h = 0.05$ for instance, with

$$f(z) = \frac{\frac{n}{\sqrt{\pi}} e^{-(n(z-a))^2} - z + b}{c} \tag{A.1}$$

$$F(z) = \frac{1}{z^4} - \frac{1}{z_h^4} \tag{A.2}$$

with n, a, b and c constants. The black hole horizon is determined by $f(z_h) = 0$ and c is chosen such that $f(0) = 1$. For a sufficiently large n and $0 < a < z_h$, two local maxima of $r(z_c)$ emerge as shown in the Fig. 6 for $n = 2000$, $a = 0.01$ and $b = 0.05$. In this case, we have $z_h \simeq c \simeq b$.

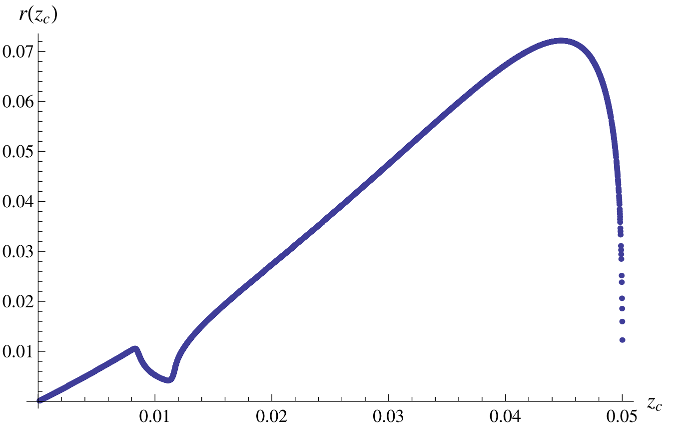

Fig. 6. The figure shows an example of the numerical results of $r(z_c)$ consisting 2 maxima value.

This example meet the general conditions required by $F(z)$ and $f(z)$, and can generate 3 branches of the candidate potential function of $\mathcal{A}(r)$

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