



Flavon stabilization in models with discrete flavor symmetry

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ARTICLE INFO

Article history:

Received 28 October 2018

Received in revised form 27 November 2018

Accepted 27 November 2018

Available online 29 November 2018

Editor: J. Hisano

ABSTRACT

We propose a simple mechanism for stabilizing flavon fields with aligned vacuum structure in models with discrete flavor symmetry. The basic idea is that flavons are stabilized by the balance between the negative soft mass and non-renormalizable terms in the potential. We explicitly discuss how our mechanism works in A_4 flavor model, and show that the field content is significantly simplified. It also works as a natural solution to the cosmological domain wall problem.

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1. Introduction

There are many proposals that try to explain the observed pattern of neutrino masses and mixings using discrete flavor symmetry (for reviews, see Refs. [1–4]). In these classes of models, the Lagrangian is assumed to be invariant under some discrete symmetry and charged leptons and neutrinos obtain masses through vacuum expectation values (VEVs) of scalar fields which are charged under the discrete symmetry, called flavons. Thus the discrete symmetry is spontaneously broken by the VEVs of flavons.

Models with the spontaneous breakdown of discrete symmetry necessarily have several discrete degenerate vacua. In general, such a model suffers from the cosmological domain wall problem [5]. Domain wall problem in the context of discrete flavor symmetry has been discussed in a few papers [6–9]. Ref. [6] considered some possibilities that may avoid the domain wall problem in the A_4 model based on the scalar potential proposed in Ref. [10]. The inflation scale could be lower than the flavor symmetry breaking scale so that the flavor symmetry breaking occurs before inflation, but there is a flat direction in the scalar potential in the model of Ref. [10], and hence this solution does not work unless the inflation scale is lower than the soft supersymmetry (SUSY) breaking scale. One possibility is that (a part of) the discrete symmetry is explicitly broken so that the VEV of flat direction does not pro-

duce exactly stable domain walls.¹ Ref. [6] also briefly discussed a possibility that the flat direction in the scalar potential obtains a large field value during/after inflation due to the Hubble mass correction, so that domain walls are inflated away. However, we find out that actually it is difficult to stabilize the flat direction in the desired way as far as we work with the scalar sector of Ref. [10], because the flat direction necessarily has unwanted higher dimensional operators that is not suitable for the purpose of solving the domain wall problem.

In this paper, we propose a very simple alternative mechanism to stabilize the flavon potential with desired alignment structure. In our model, flavons are stabilized by the balance between the negative soft SUSY breaking mass and non-renormalizable terms in the potential. The field content is then significantly simplified, and we do not need any additional field (so-called driving field). In addition, the cosmological domain wall problem is naturally solved independent of the inflation scale.

The structure of the paper is as follows. In the next section, we first review A_4 model briefly. In Sec. 3, we explain the basic idea of our novel mechanism of the flavon stabilization and study a concrete model with A_4 symmetry in Sec. 4. We show how the domain wall problem is naturally avoided in our model in Sec. 5. Sec. 6 is devoted to our conclusions and discussion. A_4 representations are summarized in the Appendix.

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¹ Present authors (S.C. and K.N.) have shown that the effect of quantum anomaly under the color SU(3) is not enough to completely solve the degeneracy of the discrete vacua at least in many of the known discrete flavor models [8].

Table 1

Charge assignments under A_4 , Z_{12} and $U(1)_R$ for leptons and various Higgs and flavon fields. Here $\rho \equiv e^{\pi i/6}$ is an element of Z_{12} rotation.

	ℓ	e^c	μ^c	τ^c	H_u	H_d	φ_T	φ_S	ξ
A_4	$\mathbf{3}$	$\mathbf{1}$	$\mathbf{1}''$	$\mathbf{1}'$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{1}$
Z_{12}	ρ^5	ρ^7	ρ^7	ρ^7	1	1	1	ρ^2	ρ^2
$U(1)_R$	$\frac{5}{6}$	$\frac{5}{6}$	$\frac{5}{6}$	$\frac{5}{6}$	0	0	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$

2. Brief description of A_4 model

We assume the superpotential of the form

$$W = W_\ell + W_f, \quad (1)$$

where W_f is related to the stabilization of the flavon fields discussed in the next section, and W_ℓ is given by [10]

$$W_\ell = \frac{y_e}{\Lambda} e^c H_d (\varphi_T \ell) + \frac{y_\mu}{\Lambda} \mu^c H_d (\varphi_T \ell)' + \frac{y_\tau}{\Lambda} \tau^c H_d (\varphi_T \ell)'' + \frac{x_a}{\Lambda^2} H_u H_u \xi (\ell \ell) + \frac{x_b}{\Lambda^2} H_u H_u (\varphi_S \ell \ell) + \dots, \quad (2)$$

where ℓ denotes the A_4 triplet lepton doublets, while A_4 singlets e^c , μ^c , τ^c , H_u , and H_d represent the right-handed electron, muon, tau, the up-type Higgs, and the down-type Higgs, respectively. Also, φ_T , φ_S , and ξ are flavon fields and Λ is the cutoff scale. We assign the charges of the fields under the A_4 symmetry as in Ref. [10] (see Table 1). Here (\dots) , $(\dots)'$, and $(\dots)''$ represent the contraction of various $\mathbf{3}$ representations that transform respectively as $\mathbf{1}$, $\mathbf{1}'$, and $\mathbf{1}''$. In addition, the dots in Eq. (2) stand for the higher dimensional operators. In this paper, we take a basis of the triplet representation that diagonalizes generators of the subgroup $Z_3 \subset A_4$. For details of the convention of the basis and the product of representations, see Appendix A.

If the flavon fields obtain VEVs of the following structure,

$$\langle \varphi_T \rangle = (v_T, 0, 0), \quad \langle \varphi_S \rangle = (v_S, v_S, v_S), \quad \langle \xi \rangle = v_\xi, \quad (3)$$

the diagonal mass matrix of the charged lepton is obtained, while the neutrino mixing matrix becomes the so-called tri-bimaximal form [10–12]. Here and below, we use the same notation φ_T , φ_S , and ξ for the scalar component of the corresponding superfield. The recent neutrino oscillation data requires deviation from the tri-bimaximal form. Extensions of the A_4 model to account for the observed data are found, e.g. in Refs. [13,14]. For a while, however, we study this minimal setup to avoid unnecessary complexity and discuss modification related to the deviation from the tri-bimaximal structure in Sec. 4.4.

3. Basic idea for the novel simple flavon stabilization

In the most popular type of the SUSY A_4 model [10], the flavon fields are stabilized at the renormalizable level by introducing several driving fields in addition to the flavon fields in order to obtain the alignment structure of Eq. (3), and the resulting superpotential is complicated. Moreover, there may remain a flat direction in the scalar potential that requires additional stabilization mechanism and it makes domain wall problem serious.

We propose a simple superpotential that successfully realizes the alignment structure of Eq. (3). The basic idea is that the flavon fields are stabilized by the balance between the negative soft SUSY breaking mass and non-renormalizable terms in the potential. Schematically, we just assume

$$W_f = \frac{\varphi^n}{n\Lambda^{n-3}}, \quad (4)$$

where φ collectively denotes the flavon fields. Giving the tachyonic soft SUSY breaking mass term as $V_{SB} = -m_\varphi^2 |\varphi|^2$, we can stabilize flavons at

$$\langle \varphi \rangle \sim \left(\frac{m_\varphi \Lambda^{n-3}}{n-1} \right)^{1/(n-2)}, \quad (5)$$

up to the complex phase, which will be fixed after taking into account the supergravity correction. We do not need any additional field to stabilize the flavon. This type of potential is considered in the context of thermal inflation [15]. Interestingly, this stabilization mechanism can naturally solve the cosmological domain wall problem as explained in Sec. 5. Of course, it is non-trivial whether or not we can correctly obtain a desired alignment structure of Eq. (3) for this type of potential. We will see that it is actually possible with showing a concrete example in the next section.²

Note that in this case, we have

$$\frac{\langle \varphi \rangle}{\Lambda} \sim \left(\frac{1}{n-1} \frac{m_\varphi}{\Lambda} \right)^{1/(n-2)} \simeq \begin{cases} 5.8 \times 10^{-4} \left(\frac{m_\varphi}{10^3 \text{ TeV}} \right)^{1/2} \left(\frac{10^{12} \text{ GeV}}{\Lambda} \right)^{1/2} & \text{for } n=4 \\ 6.7 \times 10^{-3} \left(\frac{m_\varphi}{10 \text{ TeV}} \right)^{1/4} \left(\frac{10^{12} \text{ GeV}}{\Lambda} \right)^{1/4} & \text{for } n=6 \end{cases}, \quad (6)$$

where we show the numerical values for $n=4$ and $n=6$ as examples. In order for the tau Yukawa coupling y_τ to be within the perturbative range, we need to have $\langle \varphi \rangle / \Lambda \gtrsim 10^{-3} / \cos \beta$ with $\tan \beta \equiv v_{H_u} / v_{H_d}$ being the ratio of the VEVs of up- and down-type Higgs. Thus, relatively small $\tan \beta$ ($\sim \mathcal{O}(1)$) may be favored. For large n (say, $n \gtrsim 6$) and/or large soft masses, this inequality is rather easily satisfied. Neutrino masses are of the order of

$$m_\nu \sim \frac{\langle \varphi \rangle v_{H_u}^2}{\Lambda^2} \sim 3 \times 10^{-2} \text{ eV} \left(\frac{10^{12} \text{ GeV}}{\Lambda} \right) \left(\frac{\langle \varphi \rangle / \Lambda}{10^{-3}} \right) \sin^2 \beta, \quad (7)$$

for x_a and x_b in Eq. (2) being of order unity.

4. Concrete A_4 model for the flavon scalar potential

Now let us have a closer look at the flavon superpotential. As a concrete example, besides the A_4 symmetry, we impose an additional Z_{12} symmetry and the R -symmetry $U(1)_R$ in order to control the flavon sector. The charge assignments are summarized in Table 1. Actually, this model corresponds to the $n=6$ case described in the previous section, since lower dimensional operators are forbidden by the charge assignments.

We further assume the superpotential for the flavon sector of the following form:

$$W_f = \frac{1}{6\Lambda^3} \left[g_1 (\varphi_T^3)^3 + g_2 (\varphi_T^3)^2 + g_3 (\varphi_S^2)^3 + g_4 (\varphi_S^2)^{''3} + g_5 \xi^6 \right]. \quad (8)$$

Although we can also write down other terms of the sixth order in the flavon fields allowed by the symmetry, their existence does not affect the main results as far as coefficients g_1, \dots, g_5 in Eq. (8) are larger than those with other terms (say, by one order of magnitude). We will come back to this point in Sec. 4.4, and work with

² In the context of non-SUSY flavor models, a similar vacuum alignment mechanism with renormalizable scalar potential and minimal field content was considered in Ref. [16] for A_4 and in Ref. [17] for S_4 .

the superpotential Eq. (8). In addition, flavons are assumed to have soft SUSY breaking masses as

$$V_{\text{SB}} = -m_T^2 \sum_{i=1}^3 |\varphi_{Ti}|^2 - m_S^2 \sum_{i=1}^3 |\varphi_{Si}|^2 - m_\xi^2 |\xi|^2. \quad (9)$$

The flavon scalar potential is then given by

$$V = \sum_{i=1}^3 \left| \frac{\partial W_f}{\partial \varphi_{Ti}} \right|^2 + \sum_{i=1}^3 \left| \frac{\partial W_f}{\partial \varphi_{Si}} \right|^2 + \left| \frac{\partial W_f}{\partial \xi} \right|^2 + V_{\text{SB}} + V_A. \quad (10)$$

Here we also include the so-called A -term potential V_A induced by the supergravity effect:

$$V_A = \frac{A}{2\Lambda^3} \left[g_1(\varphi_T^2)^3 + g_2(\varphi_T^3)^2 + g_3(\varphi_S^2)^3 + g_4(\varphi_S^2)''^3 + g_5\xi^6 \right] + \text{h.c.}, \quad (11)$$

where $|A| = m_{3/2}$ denotes the gravitino mass. For simplicity, we assume that $m_{3/2}$ is smaller than the soft masses m_T , m_S , and m_ξ and also that all coefficients g_1, \dots, g_5 , and A are real.³ It is easy to see the stabilization of ξ , so we discuss the potential of φ_T and φ_S below.

4.1. Potential of φ_T

First, in the φ_T sector, we get a vacuum of the form of Eq. (3) with

$$v_T = \left(\frac{1}{5} \right)^{1/8} \left(\frac{m_T \Lambda^3}{g_1 + 4g_2} \right)^{1/4}. \quad (12)$$

Of course, we can obtain other discrete vacua by transforming $\langle \varphi_T \rangle$ with A_4 group element, but we pick this vacuum up.⁴ Here we ignore the V_A term, which is justified as far as $m_{3/2} \ll m_T$. Expanding the flavon fields around the vacuum as

$$\begin{aligned} \varphi_{T1} &= v_T + \frac{1}{\sqrt{2}}(\varphi_{T1}^R + i\varphi_{T1}^I), & \varphi_{T2} &= \frac{1}{\sqrt{2}}(\varphi_{T2}^R + i\varphi_{T2}^I), \\ \varphi_{T3} &= \frac{1}{\sqrt{2}}(\varphi_{T3}^R + i\varphi_{T3}^I), \end{aligned} \quad (13)$$

we find out that φ_{T1}^R obtains a positive mass-squared of $8m_T^2$. On the other hand, φ_{T2}^R and φ_{T3}^R are mixed with each other. Their mass matrix is

$$\mathcal{M}_R^2 = -\frac{4m_T^2}{5(g_1 + 4g_2)^2} \times \begin{pmatrix} g_1^2 + 12g_1g_2 + 16g_2^2 & -g_1^2 + 16g_2^2 \\ -g_1^2 + 16g_2^2 & g_1^2 + 12g_1g_2 + 16g_2^2 \end{pmatrix}. \quad (14)$$

The condition that both of them have positive mass eigenvalues is

$$g_1(g_1 + 6g_2) < 0 \quad \text{and} \quad g_2(3g_1 + 8g_2) < 0. \quad (15)$$

Parameter regions consistent with this condition are shown in Fig. 1. The same condition also ensures that φ_{T2}^I and φ_{T3}^I have positive mass eigenvalues. All of them have mass-squared of $\mathcal{O}(m_T^2)$. φ_{T1}^I obtains a mass only from the V_A term and its mass-squared

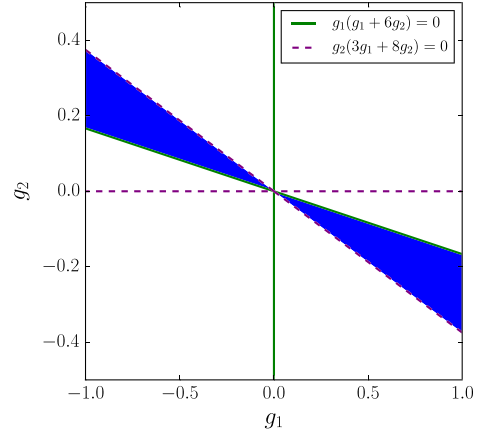


Fig. 1. Parameter region consistent with the condition Eq. (15). Blue region is allowed.

is $\mathcal{O}(m_T m_{3/2})$, lighter than the other scalar components, and has a positive sign for $g_1 + 4g_2 > 0$ ($g_1 + 4g_2 < 0$) if $A > 0$ ($A < 0$). Therefore, we confirm that the φ_T field is indeed stabilized successfully at the vacuum of Eq. (3).

4.2. Potential of φ_S

Next, let us turn to the φ_S sector. It is convenient to work with the transformed basis:

$$\begin{pmatrix} \tilde{\varphi}_{S1} \\ \tilde{\varphi}_{S2} \\ \tilde{\varphi}_{S3} \end{pmatrix} = U \begin{pmatrix} \varphi_{S1} \\ \varphi_{S2} \\ \varphi_{S3} \end{pmatrix}, \quad U \equiv \begin{pmatrix} 1/\sqrt{3} & 1/\sqrt{3} & 1/\sqrt{3} \\ 0 & -1/\sqrt{2} & 1/\sqrt{2} \\ -\sqrt{2}/\sqrt{3} & 1/\sqrt{6} & 1/\sqrt{6} \end{pmatrix}. \quad (16)$$

In this basis, the aligned vacuum in Eq. (3) becomes

$$\langle \tilde{\varphi}_S \rangle = (\tilde{v}_S, 0, 0). \quad (17)$$

We get a vacuum at

$$\tilde{v}_S = \left(\frac{1}{5} \right)^{1/8} \left(\frac{m_S \Lambda^3}{g_3 + g_4} \right)^{1/4}. \quad (18)$$

Again we ignore the V_A term, which is justified as far as $m_{3/2} \ll m_S$. Expanding the flavon fields around the vacuum as

$$\begin{aligned} \tilde{\varphi}_{S1} &= \tilde{v}_S + \frac{1}{\sqrt{2}}(\tilde{\varphi}_{S1}^R + i\tilde{\varphi}_{S1}^I), & \tilde{\varphi}_{S2} &= \frac{1}{\sqrt{2}}(\tilde{\varphi}_{S2}^R + i\tilde{\varphi}_{S2}^I), \\ \tilde{\varphi}_{S3} &= \frac{1}{\sqrt{2}}(\tilde{\varphi}_{S3}^R + i\tilde{\varphi}_{S3}^I), \end{aligned} \quad (19)$$

we find out that $\tilde{\varphi}_{S1}^R$ obtains a mass-squared of $8m_S^2$. On the other hand, φ_{S2}^R and φ_{S3}^R are mixed with each other. Their mass matrix is

$$\mathcal{M}_R^2 = -\frac{m_S^2}{5(g_3 + g_4)^2} \times \begin{pmatrix} 8g_3^2 + 13g_3g_4 + 2g_4^2 & 2\sqrt{3}(g_3 + g_4)g_4 \\ 2\sqrt{3}(g_3 + g_4)g_4 & 3g_4(3g_3 + 2g_4) \end{pmatrix}. \quad (20)$$

The condition that both of them have positive mass eigenvalues is

$$g_3g_4 < 0 \quad \text{and} \quad 8g_3^2 + 17g_3g_4 + 8g_4^2 < 0. \quad (21)$$

Parameter regions consistent with this condition are shown in Fig. 2. The same condition also ensures that φ_{S2}^I and φ_{S3}^I have positive mass-squared eigenvalues. All of them have masses of $\mathcal{O}(m_S^2)$.

³ For example, g_1 , g_3 , g_5 , and A are taken to be real and positive without loss of generality, but g_2 and g_4 are complex in general. The following discussion does not much depend on whether g_2 and g_4 are real or complex.

⁴ As shown later in Sec. 5, the flavons can naturally fall into the desired minimum during inflation.

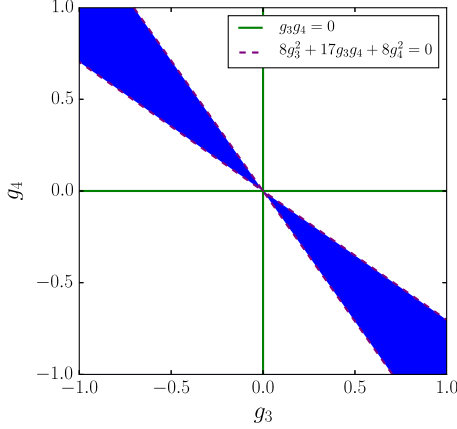


Fig. 2. Parameter region consistent with the condition Eq. (21). Blue region is allowed.

The only exception is φ_{S1}^I , which obtains a mass only from the V_A term. Its mass-squared is $\mathcal{O}(m_5 m_{3/2})$, lighter than the other scalar components, and has a positive sign for $g_3 + g_4 > 0$ ($g_3 + g_4 < 0$) if $A > 0$ ($A < 0$). Thus, the φ_S field is successfully stabilized at the vacuum of Eq. (3).

4.3. Fermionic components

So far we obtained the desired aligned vacuum of Eq. (3). We now discuss the mass matrix of the fermionic components of the flavons φ_T and $\tilde{\varphi}_S$, which we call flavinos denoted by ψ_T and $\tilde{\psi}_S$, respectively. Flavinos become massive after flavons obtain VEVs. For the φ_T sector, the flavino mass matrix is given by

$$\mathcal{M}_{\psi_T} = -\frac{m_T}{\sqrt{5}(g_1 + 4g_2)} \times \begin{pmatrix} 5(g_1 + 4g_2) & 0 & 0 \\ 0 & 0 & g_1 - 4g_2 \\ 0 & g_1 - 4g_2 & 0 \end{pmatrix}. \quad (22)$$

For the $\tilde{\varphi}_S$ sector, on the other hand, we have

$$\mathcal{M}_{\tilde{\psi}_S} = -\frac{m_S}{\sqrt{5}(g_3 + g_4)} \times \begin{pmatrix} 5(g_3 + g_4) & 0 & 0 \\ 0 & -g_3 + g_4/2 & -\sqrt{3}g_4/2 \\ 0 & -\sqrt{3}g_4/2 & g_3 - g_4/2 \end{pmatrix}. \quad (23)$$

Thus, all of them have masses of the order of the soft SUSY breaking mass scale.

4.4. Effects of other terms

We neglect many terms that are allowed by our charge assignments in the flavon superpotential Eq. (8). Thanks to the Z_{12} and $U(1)_R$ charges, the φ_T sector, which gives the diagonal mass matrix of the charged leptons, is completely decoupled from the φ_S and ξ sector, which makes the tri-bimaximal form of the neutrino mixing matrix. There are eleven possible contractions of φ_T^6 that transform as $\mathbf{1}$ as a whole, while there are seventeen possible contractions for the φ_S and ξ sector.

The effects of these terms can be estimated using some symmetry argument as follows. First, we focus on the φ_T sector. If we refer to one of the diagonalized elements of A_4 , e.g. T described in Appendix A, we can see that each component of the flavon transforms as $\{\varphi_{T1}, \varphi_{T2}, \varphi_{T3}\} \rightarrow \{\varphi_{T1}, \omega\varphi_{T2}, \omega^2\varphi_{T3}\}$ under T , where

$\omega \equiv e^{2\pi i/3}$. Thus, the φ_T sector of the most general flavon superpotential in our model, which should be invariant under A_4 , takes the form of

$$W_f^{(T)} \sim \frac{1}{\Lambda^3} \left(\varphi_{T1}^6 + \varphi_{T1}^4 \varphi_{T2} \varphi_{T3} + \dots \right), \quad (24)$$

where the dots denote terms with more than two powers of $\{\varphi_{T2}, \varphi_{T3}\}$. Here, we omit all the $\mathcal{O}(1)$ coefficients for simplicity. It is straightforward from Eq. (24) to see that $\varphi_T = (v_T, 0, 0)$ always expresses one of the extrema of the potential with a properly chosen value of v_T thanks to the absence of terms linear in $\{\varphi_{T2}, \varphi_{T3}\}$. Regarding the curvature of the potential around the extremum, it is non-trivial to see whether the point we are considering is a local minimum, a local maximum, or a saddle point. However, we have already shown in Sec. 4.1 that the potential has a minimum at $\varphi_T = (v_T, 0, 0)$ for some parameter spaces spanned by g_1 and g_2 . Therefore, our flavon stabilization mechanism works well at least in the vicinity of such a parameter region.

Next, we consider the φ_S and ξ sector. Again, it is convenient to move to the basis expressed in Eq. (16). In this basis, one of the generators of A_4 , which is denoted as S in Appendix A, becomes diagonal:

$$\tilde{S} \equiv USU^{-1} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}. \quad (25)$$

Therefore, the φ_S and ξ sector of the most general flavon superpotential has the form of

$$W_f^{(S)} \sim \frac{1}{\Lambda^3} \left(\tilde{\varphi}_{S1}^6 + \tilde{\varphi}_{S1}^5 \xi + \tilde{\varphi}_{S1}^4 (\tilde{\varphi}_{S2}^2 + \tilde{\varphi}_{S2} \tilde{\varphi}_{S3} + \tilde{\varphi}_{S3}^2 + \xi^2) + \dots \right), \quad (26)$$

where the dots represent terms with more than two powers of $\{\tilde{\varphi}_{S2}, \tilde{\varphi}_{S3}, \xi\}$. Again, we omit all $\mathcal{O}(1)$ coefficients. It can be seen that the same argument as for the φ_T sector follows, i.e., $\tilde{\varphi}_S = (\tilde{v}_S, 0, 0)$ always remains an extremum of the potential and hence our flavon stabilization mechanism works well in some parameter regions at least in the vicinity of those shown in Sec. 4.2.

We would also like to explain the deviation from the tri-bimaximal form of the observed neutrino mixing matrix. For this purpose, we follow the procedure described in Refs. [13,14] and consider a new flavon ξ' that has charges $\{\mathbf{1}', \rho^2, 1/3\}$ under our set up. Again, we can find out an example parameter region that stabilizes the flavon potential: we should just add a term proportional to ξ'^6 to the superpotential Eq. (4) and a term $-m_{\xi'}^2 |\xi'|^2$ to the SUSY breaking potential Eq. (9). In this simple example, ξ' is completely decoupled from other fields in the scalar potential and the minimization condition of the potential remains unchanged except for the further condition for the ξ' VEV. When we consider other possible contractions in the superpotential, the number of allowed terms consist of $\{\varphi_S, \xi, \xi'\}$ is increased to twenty-nine. However, the above discussion still holds and there exists a finite volume region of the parameter space that is consistent with our scenario.

5. Solution to the domain wall problem

Although the flavor structure can be well described by the discrete flavor symmetry with proper VEV alignment of flavon fields, the domain wall problem arises if the spontaneous breaking of the flavor symmetry occurs after inflation, because there are many degenerate minima of the flavon potential. In this section, we show that our mechanism of the flavon stabilization described in Sec. 4

results in a simple and natural description of the cosmological dynamics of the flavon fields that does not lead to the formation of domain walls.

The cosmological flavon dynamics depends on the soft mass m_{soft} , which collectively denote the soft SUSY breaking masses of flavons, and the Hubble parameter H . If $m_{\text{soft}} \gtrsim H$ during inflation, flavons fall into one of the potential minima dynamically and such a region expands that covers the whole observable Universe without domain walls.

If $m_{\text{soft}} \lesssim H$ during inflation, on the other hand, we need to include the negative Hubble-induced mass term during and after inflation as [18,19]

$$V_{\text{SB}} = -c_T H^2 \sum_{i=1}^3 |\varphi_{Ti}|^2 - c_S H^2 \sum_{i=1}^3 |\varphi_{Si}|^2 - c_\xi H^2 |\xi|^2, \quad (27)$$

where c_T , c_S , and c_ξ are positive constants of order unity. These terms may come from the Planck-suppressed coupling among the inflaton and flavon fields in the Kähler potential. Since these negative Hubble-induced mass terms have the same form as the negative soft mass terms Eq. (9), the structure of the minima is the same as those discussed in Sec. 4 with m_{soft} being replaced by H , which is typically much larger than m_{soft} . Then, during the inflation, the flavons will settle down into the minimum of the potential at the VEVs,

$$\langle \varphi_T \rangle = (v_T^H, 0, 0), \quad \langle \varphi_S \rangle = (v_S^H, v_S^H, v_S^H), \quad \langle \xi \rangle = v_\xi^H, \quad (28)$$

where

$$v_T^H, v_S^H, v_\xi^H \sim \left(\frac{H \Lambda^{n-3}}{n-1} \right)^{1/(n-2)}, \quad (29)$$

since the flavons have masses of $\mathcal{O}(H)$ around this temporal minimum.⁵ Domain walls are thus inflated away no matter what happens before inflation.

After inflation, as the Hubble parameter decreases, the VEV of the temporal minimum Eq. (28) becomes smaller, and we expect that the flavons will track the temporal minimum. In order for this to happen, there is a lower bound on n in Eq. (4) depending on the background equation of state of the Universe as [21]

$$n \geq \frac{6p}{3p-1} = \begin{cases} 4 & \text{for } p = 2/3, \\ 6 & \text{for } p = 1/2, \end{cases} \quad (30)$$

where p is defined as the power law exponent of the cosmic scale factor: $a(t) \propto t^p$, hence $p = 2/3$ ($1/2$) for the matter (radiation) dominated Universe. If this condition is satisfied, flavons smoothly relax to the true potential minimum as H becomes smaller than m_{soft} . Otherwise, the flavon oscillations around the temporal minimum may be relatively enhanced as the Universe expands and flavons may overshoot the origin of the potential [21], which makes the whole flavon dynamics much more complicated and the domain wall formation may occur.

Since our model Eq. (8) corresponds to the $n = 6$ case, the flavon fields do track the temporal minima of Eq. (28), and no symmetry restoration occurs after inflation if the inflaton effectively

⁵ As explained in Sec. 4, there are several light (pseudo) scalar fields $\varphi_{T1}^l, \varphi_{S1}^l$, and also ξ^l . Unless there is Hubble-induced A -term [20], their mass-squared are of the order of $m_{3/2}H$ in the early Universe and hence much lighter than other flavons. Although they obtain long wavelength isocurvature fluctuations during inflation, they leave basically no observable signatures unless they are directly related to the dark matter production or baryon asymmetry of the Universe.

behaves as matter or radiation before the completion of reheating after inflation. Therefore, there are no domain walls in the observable Universe. This is a simple way to solve the domain wall problem in models with discrete flavor symmetry.

6. Conclusions and discussion

We have proposed a simple mechanism to stabilize the flavon potential in the model with discrete flavor symmetry. It is achieved by the balance between the negative soft mass and non-renormalizable terms with appropriate charge assignments of the flavon fields. The flavon sector is significantly simplified and the domain wall problem is naturally solved. Although we have focused on the A_4 model as a concrete setup, a similar mechanism should also work in models with other discrete symmetries in general. We leave this issue as future work.

Finally, we briefly discuss the cosmological aspects of this model. Most flavons have masses of m_{soft} around the vacuum and some flavons may be as light as $\sqrt{m_{3/2}m_{\text{soft}}}$ if $m_{3/2} \lesssim m_{\text{soft}}$ as shown in Sec. 4. Flavons in the φ_T sector eventually decay into a Higgs boson plus 2 leptons (and possibly their superpartners depending on the mass spectrum), and its lifetime is typically much shorter than 1 sec, which does not affect the Big-Bang nucleosynthesis. Flavons in the φ_S and ξ sector decay into 2 Higgs bosons plus 2 leptons (and possibly their superpartners). Since their decay rate is suppressed by Λ^4 , the lifetime can be much longer than 1 sec for $m_S \ll 1000$ TeV and $\Lambda \sim 10^{12}$ GeV and may affect Big-Bang nucleosynthesis. In such a case, we may need relatively low reheating temperature to avoid the overproduction of flavons, which are dominantly in the form of coherent oscillation.⁶ Flavinos, the fermionic superpartners of the flavons, also have masses of $\sim m_{\text{soft}}$. For $m_{3/2} \lesssim m_{\text{soft}}$, flavinos may decay into the gravitino and it can be a dominant source of the gravitino production depending on the reheating temperature. The gravitino decays into lighter SUSY particles unless the gravitino is the lightest, and the final dark matter abundance is determined either by the gravitino decay or thermal production, as usual.

Acknowledgements

This work was supported by the Grant-in-Aid for Scientific Research C (No. 18K03609 [KN]), and Innovative Areas (No. 26104009 [KN], No. 15H05888 [KN], No. 17H06359 [KN]). This work was also supported by JSPS KAKENHI Grant (No. 17J00813 [SC]).

Appendix A. Notes on A_4 representations

It is known that all the elements of the A_4 group can be written as products of two elements S and T , which are the generators of the subgroup Z_2 and Z_3 , respectively. A_4 has a triplet representation $\mathbf{3}$ and three singlet representations $\mathbf{1}$, $\mathbf{1}'$, and $\mathbf{1}''$. We adopt a basis of the triplet representation that diagonalizes T . In this basis, all the elements of A_4 are represented as

$$\mathbf{1} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad T = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega & 0 \\ 0 & 0 & \omega^2 \end{pmatrix}, \quad T^2 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega^2 & 0 \\ 0 & 0 & \omega \end{pmatrix}, \quad (31)$$

⁶ Even for the cosmological scenario described in Sec. 5, there is a finite amount of coherent oscillation around the potential minimum [21], though the oscillation amplitude is suppressed for n satisfying inequality of Eq. (30).

$$S = \frac{1}{3} \begin{pmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{pmatrix}, \quad ST = \frac{1}{3} \begin{pmatrix} -1 & 2\omega & 2\omega^2 \\ 2 & -\omega & 2\omega^2 \\ 2 & 2\omega & -\omega^2 \end{pmatrix},$$

$$ST^2 = \frac{1}{3} \begin{pmatrix} -1 & 2\omega^2 & 2\omega \\ 2 & -\omega^2 & 2\omega \\ 2 & 2\omega^2 & -\omega \end{pmatrix}, \quad (32)$$

$$TS = \frac{1}{3} \begin{pmatrix} -1 & 2 & 2 \\ 2\omega & -\omega & 2\omega \\ 2\omega^2 & 2\omega^2 & -\omega^2 \end{pmatrix},$$

$$TST = \frac{1}{3} \begin{pmatrix} -1 & 2\omega & 2\omega^2 \\ 2\omega & -\omega^2 & 2 \\ 2\omega^2 & 2 & -\omega \end{pmatrix},$$

$$TST^2 = \frac{1}{3} \begin{pmatrix} -1 & 2\omega^2 & 2\omega \\ 2\omega & -1 & 2\omega^2 \\ 2\omega^2 & 2\omega & -1 \end{pmatrix}, \quad (33)$$

$$T^2S = \frac{1}{3} \begin{pmatrix} -1 & 2 & 2 \\ 2\omega^2 & -\omega^2 & 2\omega^2 \\ 2\omega & 2\omega & -\omega \end{pmatrix},$$

$$T^2ST = \frac{1}{3} \begin{pmatrix} -1 & 2\omega & 2\omega^2 \\ 2\omega^2 & -1 & 2\omega \\ 2\omega & 2\omega^2 & -1 \end{pmatrix},$$

$$T^2ST^2 = \frac{1}{3} \begin{pmatrix} -1 & 2\omega^2 & 2\omega \\ 2\omega^2 & -\omega & 2 \\ 2\omega & 2 & -\omega^2 \end{pmatrix}, \quad (34)$$

where we define $\omega \equiv e^{2\pi i/3}$. The product of two triplets are decomposed as $\mathbf{3} \times \mathbf{3} = \mathbf{1} + \mathbf{1}' + \mathbf{1}'' + \mathbf{3}_S + \mathbf{3}_A$. For each contraction, we use the convention of the coefficient as follows:

$$\mathbf{1} \sim a_1 b_1 + a_2 b_3 + a_3 b_2 \equiv (ab), \quad (35)$$

$$\mathbf{1}' \sim a_3 b_3 + a_1 b_2 + a_2 b_1 \equiv (ab)', \quad (36)$$

$$\mathbf{1}'' \sim a_2 b_2 + a_3 b_1 + a_1 b_3 \equiv (ab)'', \quad (37)$$

$$\mathbf{3}_S \sim \begin{pmatrix} 2a_1 b_1 - a_2 b_3 - a_3 b_2 \\ 2a_3 b_3 - a_1 b_2 - a_2 b_1 \\ 2a_2 b_2 - a_3 b_1 - a_1 b_3 \end{pmatrix} \equiv (ab)_{\mathbf{3}_S},$$

$$\mathbf{3}_A \sim \begin{pmatrix} a_2 b_3 - a_3 b_2 \\ a_1 b_2 - a_2 b_1 \\ a_3 b_1 - a_1 b_3 \end{pmatrix} \equiv (ab)_{\mathbf{3}_A}, \quad (38)$$

where $a = (a_1, a_2, a_3)$ and $b = (b_1, b_2, b_3)$ are triplets.

For the product of the same triplet $\varphi = (\varphi_1, \varphi_2, \varphi_3)$, we have

$$\begin{aligned} (\varphi^2) &= \varphi_1^2 + 2\varphi_2\varphi_3, & (\varphi^2)' &= \varphi_3^2 + 2\varphi_1\varphi_2, \\ (\varphi^2)'' &= \varphi_2^2 + 2\varphi_3\varphi_1, \end{aligned} \quad (39)$$

$$(\varphi^2)_{\mathbf{3}_S} = 2 \begin{pmatrix} \varphi_1^2 - \varphi_2\varphi_3 \\ \varphi_3^2 - \varphi_1\varphi_2 \\ \varphi_2^2 - \varphi_3\varphi_1 \end{pmatrix}, \quad (\varphi^2)_{\mathbf{3}_A} = 0, \quad (40)$$

$$(\varphi^3)_{\mathbf{1}} = 2(\varphi_1^3 + \varphi_2^3 + \varphi_3^3 - 3\varphi_1\varphi_2\varphi_3). \quad (41)$$

In terms of the transformed basis $\tilde{\varphi}$ in Eq. (16), they become

$$(\tilde{\varphi}^2) = \tilde{\varphi}_1^2 - \tilde{\varphi}_2^2 + \tilde{\varphi}_3^2, \quad (42)$$

$$(\tilde{\varphi}^2)' = \tilde{\varphi}_1^2 + \frac{1}{2}(\tilde{\varphi}_2^2 + 2\sqrt{3}\tilde{\varphi}_2\tilde{\varphi}_3 - \tilde{\varphi}_3^2), \quad (43)$$

$$(\tilde{\varphi}^2)'' = \tilde{\varphi}_1^2 + \frac{1}{2}(\tilde{\varphi}_2^2 - 2\sqrt{3}\tilde{\varphi}_2\tilde{\varphi}_3 - \tilde{\varphi}_3^2), \quad (44)$$

$$(\tilde{\varphi}^2)_{\mathbf{3}_S} = \begin{pmatrix} \tilde{\varphi}_2^2 + \tilde{\varphi}_3^2 - 2\sqrt{2}\tilde{\varphi}_1\tilde{\varphi}_3 \\ \tilde{\varphi}_2^2 + \tilde{\varphi}_3^2 + \sqrt{2}\tilde{\varphi}_1(\sqrt{3}\tilde{\varphi}_2 + \tilde{\varphi}_3) \\ \tilde{\varphi}_2^2 + \tilde{\varphi}_3^2 + \sqrt{2}\tilde{\varphi}_1(-\sqrt{3}\tilde{\varphi}_2 + \tilde{\varphi}_3) \end{pmatrix}, \quad (\tilde{\varphi}^2)_{\mathbf{3}_A} = 0, \quad (45)$$

$$(\tilde{\varphi}^3)_{\mathbf{1}} = 3\sqrt{3}\tilde{\varphi}_1(\tilde{\varphi}_2^2 + \tilde{\varphi}_3^2). \quad (46)$$

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