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# Inflationary cosmology via quantum corrections in M-theory

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We investigate inflationary cosmology by solving the effective action of M-theory, which consists of 11-dimensional supergravity and quartic terms of the Weyl tensor. The metric is simply expressed by two scale factors, one for the spatial directions and the other for the internal directions. Since the effective action of M-theory is constructed perturbatively, we also solve the equations of motion perturbatively. We will show that the classical solution of 11-dimensional supergravity does not represent inflationary expansion, but if we include the quantum corrections, the behavior of the solution around the very early time is modified and the inflationary scenario is realized. The inflation naturally ends when the scalar curvature becomes small and quartic terms of the Weyl tensor are negligible.

Subject Index B29

## 1. Introduction

The inflationary scenario in the very early Universe provides a resolution to the problems of big bang cosmology, such as the flatness problem and the horizon problem [1–4]. Numerous models have been proposed to describe the inflation, and many of them have introduced inflaton fields which slowly roll down potentials to realize de-Sitter-like vacua [5–9] (see also Refs. [10–14] and references therein). The origin of the inflaton field is not clear, but there are models in which the inflaton potentials are inspired by D-branes in superstring theory [15–20] (see also Ref. [21] and references therein). Also, there are a lot of models where the inflation is realized by modifying the gravity theory [1,22–25].

Recently it has become possible to restrict models of inflation by comparing cosmological parameters with observations of the cosmic microwave background. In particular, observational bounds for the spectral index $n_{\mathrm{s}}$ and tensor to scalar ratio $r$ are measured as $n_{\mathrm{s}} = 0.968 \pm 0.006$ [26] and $r < 0.07$ [27,28]. Among other models of inflation, the predictions of the Starobinsky model [1], which contains a curvature squared term in the action, are in good agreement with the observations. In fact, it predicts $n_{\mathrm{s}} = 0.967$ and $r = 0.003$ when the number of e-folds is 60.

Since the curvature squared term in the Starobinsky model is considered as the quantum effect of gravity, it is natural to question the origin of the quantum effect in more fundamental theories, such as superstring theory or M-theory. Actually, heterotic superstring theory contains the Gauss–Bonnet term, and type II superstring theories or M-theory contain quartic terms of the Riemann tensor [29–34]. As examples, a study of the inflationary solutions in heterotic superstring theory was performed in Ref. [35], and studies of the inflationary solutions in M-theory in Refs. [36–39]. They assumed that the scale factor for spatial directions and that for internal ones behave like $e^{Ht}$ and $e^{Gt}$,


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respectively, and solved equations of motion to obtain constant values of $H$ and $G$. There are some solutions which seem to be consistent with the inflationary scenario, but in general it is difficult to obtain solutions after the inflation which are not expressed by the ansatz of $e^{Ht}$ and $e^{Gt}$.

In this paper, we investigate the effective action of M-theory which consists of 11-dimensional supergravity and quartic terms of the Weyl tensor. The metric is simply expressed by the scale factor $a(t)$ for the spatial directions and the scale factor $b(t)$ for the internal directions. Since the effective action of M-theory is constructed perturbatively, we also solve the equations of motion perturbatively. We will show that the classical solution of 11-dimensional supergravity does not represent inflationary expansion, but if we include the quantum corrections, the behavior of the solution around the very early time is modified and the inflationary scenario is realized. The inflation naturally ends when the scalar curvature becomes small and quartic terms of the Weyl tensor are negligible.

The paper is organized as follows. In Sect. 2 we review the effective action of M-theory, and derive the equations of motion for two scale factors, $a(t)$ and $b(t)$. The 10-dimensional space directions are divided into $d$ spatial directions and $(10-d)$ internal directions. Then we solve equations of motion perturbatively and obtain classical and quantum solutions. In Sect. 3 we examine the case of $d=3$ in detail, and show that inflation occurs in the early stage. In Sect. 4 we study $d=1,\ldots,9$ in general, and confirm that the inflationary scenario is universal except for the case of $d=1$. Section 5 is devoted to conclusions and discussions. Details of the calculations in Sect. 2 are given explicitly in Appendix A.

## 2. Inflationary solution

### 2.1. *Equations of motion with quantum corrections*

M-theory is a theory of membranes in 11-dimensional spacetime and its low-energy limit is approximated by 11-dimensional supergravity. There is a parameter of Planck length $\ell_{\rm p}$ in M-theory, and its effective action is expanded with respect to $\ell_{\rm p}$. From the duality between M-theory and type IIA superstring theory, it is known that the leading correction to supergravity starts from $R^4$ terms, which are products of four Riemann tensors. Actually, the bosonic part of the M-theory effective action which is relevant to the geometry is given by [33,34]

$$\begin{aligned} S_{11} &= \frac{1}{2\kappa_{11}^2}\int d^{11}x\, e\Big\{R + \Gamma\Big(t_8 t_8 W^4 - \frac{1}{4!}\epsilon_{11}\epsilon_{11}W^4\Big)\Big\} \\ &= \frac{1}{2\kappa_{11}^2}\int d^{11}x\, e\Big\{R + 24\Gamma\big(W_{abcd}W^{abcd}W_{efgh}W^{efgh} - 64W_{abcd}W^{aefg}W^{bcdh}W_{efgh} \\ &\qquad + 2W_{abcd}W^{abef}W^{cdgh}W_{efgh} + 16W_{acbd}W^{aebf}W^{cgdh}W_{egfh} \\ &\qquad - 16W_{abcd}W^{aefg}W^{b}{}_{ef}{}^{h}W^{cd}{}_{gh} - 16W_{abcd}W^{aefg}W^{b}{}_{fe}{}^{h}W^{cd}{}_{gh}\big)\Big\}, \end{aligned} \tag{1}$$

where $a,b,c,\ldots = 0,1,\ldots,10$ are local Lorentz indices. Indices are lowered or raised by the flat metric $\eta_{ab}$ or $\eta^{ab}$. $t_8$ are the products of four Kronecker deltas with eight indices, and $\epsilon_{11}$ is an antisymmetric tensor with 11 indices. Of course, there is a field redefinition ambiguity, so the coefficients of terms which include Ricci or scalar curvatures cannot be fixed. In this paper, we simply deal with the effective action by employing the Weyl tensor prescription. It is confirmed that the $W^4$ terms are certainly invariant under the local supersymmetry if we add appropriate fermionic terms [40–46].

In the above effective action there are two parameters, the gravitational constant $\kappa_{11}$ and the expansion parameter $\Gamma$, which can be expressed as

$$2\kappa_{11}^2 = (2\pi)^8 \ell_p^9, \qquad \Gamma = \frac{\pi^2 \ell_p^6}{2^{11}3^2}. \tag{2}$$

The Planck length is written by $\ell_\mathrm{p} = g_\mathrm{s}^{1/3}\ell_\mathrm{s}$ in terms of the string length $\ell_\mathrm{s}$ and string coupling constant $g_\mathrm{s}$. The numerical factor of $\Gamma$ is determined by employing the result of the one-loop four-graviton amplitude in type IIA superstring theory. Since the M-theory effective action contains an infinite series of higher-order terms with respect to $\ell_\mathrm{p}$, the leading correction in Eq. (1) is reliable when $\Gamma$ is small, or in other words, the typical length scale of a system is large compared to $\ell_\mathrm{p}$.

Below we consider the equations of motion for the effective action, Eq. (1), up to the linear order of $\Gamma$. By varying the effective action in Eq. (1), equations of motion are obtained as [47]

$$E_{ab} \equiv R_{ab} - \frac{1}{2}\eta_{ab}R + \Gamma\Big\{ -\frac{1}{2}\eta_{ab}Z + R_{cdea}Y^{cde}{}_b - 2D_{(c}D_{d)}Y^c{}_{ab}{}^d \Big\} = 0. \tag{3}$$

Here, $D_a$ is a covariant derivative for local Lorentz indices, and $X_{abcd}$, $Y_{abcd}$, and $Z$ are defined as follows:

$$\begin{aligned} X_{abcd} = 96\big(&W_{abcd}W_{efgh}W^{efgh} - 4W_{abce}W_{dfgh}W^{efgh} + 4W_{abde}W_{cfgh}W^{efgh} \\ &-4W_{cdae}W_{bfgh}W^{efgh} + 4W_{cdbe}W_{afgh}W^{efgh} + 2W_{abef}W_{cdgh}W^{efgh} + 4W_{ab}{}^{ef}W_{ce}{}^{gh}W_{dfgh} \\ &+4W_{cd}{}^{ef}W_{ae}{}^{gh}W_{bfgh} + 8W_{aecg}W_{bfdh}W^{efgh} - 8W_{becg}W_{afdh}W^{efgh} - 8W_{abeg}W_{cf}{}^{e}{}_{h}W_{d}{}^{fgh} \\ &-8W_{cdeg}W_{af}{}^{e}{}_{h}W_{b}{}^{fgh} - 4W^{ef}{}_{ag}W_{efch}W^{g}{}_{b}{}^{h}{}_{d} + 4W^{ef}{}_{ag}W_{efdh}W^{g}{}_{b}{}^{h}{}_{c} + 4W^{ef}{}_{bg}W_{efch}W^{g}{}_{a}{}^{h}{}_{d} \\ &-4W^{ef}{}_{bg}W_{efdh}W^{g}{}_{a}{}^{h}{}_{c}\big), \end{aligned} \tag{4}$$

$$Y_{abcd} = X_{abcd} - \frac{1}{9}(\eta_{ac}X_{bd} - \eta_{bc}X_{ad} - \eta_{ad}X_{bc} + \eta_{bd}X_{ac}) + \frac{1}{90}(\eta_{ac}\eta_{bd} - \eta_{ad}\eta_{bc})X, \tag{5}$$

$$\begin{aligned} Z = 24\big(&W_{abcd}W^{abcd}W_{efgh}W^{efgh} - 64W_{abcd}W^{aefg}W^{bcdh}W_{efgh} + 2W_{abcd}W^{abef}W^{cdgh}W_{efgh} \\ &+16W_{acbd}W^{aebf}W^{cgdh}W_{egfh} - 16W_{abcd}W^{aefg}W^{b}{}_{ef}{}^{h}W^{cd}{}_{gh} - 16W_{abcd}W^{aefg}W^{b}{}_{fe}{}^{h}W^{cd}{}_{gh}\big), \end{aligned} \tag{6}$$

where $X_{ab} = X^c{}_{acb}$ and $X = X^a{}_a$. Note that $X_{abcd} = -X_{bacd} = -X_{abdc} = X_{cdab}$ and $X_{ab} = X_{ba}$. It is possible to show that $W_{cdea}X^{cde}{}_b = W_{cdeb}X^{cde}{}_a$ and $R_{cdea}Y^{cde}{}_b = R_{cdeb}Y^{cde}{}_a$ after some calculations, and hence $E_{ab} = E_{ba}$.

Since we are interested in solutions that would inflate some of the spatial directions, we decompose the ten spatial directions into $d$ spatial ones and $(10-d)$ spatial ones. Then the ansatz for the metric is written as

$$ds^2 = -dt^2 + a(t)^2 dx_i^2 + b(t)^2 dy_m^2, \tag{7}$$

where $i = 1, \dots, d$ and $m = d+1, \dots, 10$. Without loss of generality, we choose $d = 1, 2, 3, 4$, or 5 in the rest of this paper. $a(t)$ and $b(t)$ are scale factors for $d$ spatial directions and $(10-d)$ spatial ones, respectively. In this paper we regard shrinking directions as toroidal internal space, although it is interesting to compactify the internal space with nontrivial manifolds. Substituting the ansatz of Eq. (7) into Eq. (3), we obtain differential equations for $H(t) = \dot{a}(t)/a(t)$ and $G(t) = \dot{b}(t)/b(t)$ as follows:

$$
\begin{aligned}
E_{00} &= \frac{d(d-1)}{2}H^2 + \frac{(10-d)(9-d)}{2}G^2 + d(10-d)HG \\
&\quad + \Gamma\Big[\frac{1}{2}Z - 2dH\dot{Y}_1 - 2(10-d)G\dot{Y}_2 + 2d\big\{\dot{H} - (d-2)H^2 - (10-d)HG\big\}Y_1 \\
&\quad + 2(10-d)\big\{\dot{G} - (8-d)G^2 - dHG\big\}Y_2 - 2d(d-1)H^2Y_3 - 4d(10-d)HGY_4 \\
&\quad - 2(10-d)(9-d)G^2Y_5\Big], \\
E_{ii} &= -(d-1)\dot{H} - (10-d)\dot{G} - \frac{d(d-1)}{2}H^2 - \frac{(10-d)(11-d)}{2}G^2 - (d-1)(10-d)HG \\
&\quad + \Gamma\Big[-\frac{1}{2}Z - 2(\dot{H}+H^2)Y_1 + 2(d-1)H^2Y_3 + 2(10-d)HGY_4 \\
&\quad - 2\Big\{\frac{d}{dt} + (d-1)H + (10-d)G\Big\}\{-\dot{Y}_1 - (d-1)H(Y_1+Y_3) - (10-d)G(Y_1+Y_4)\}\Big], \\
E_{mm} &= -d\dot{H} - (9-d)\dot{G} - \frac{d(d+1)}{2}H^2 - \frac{(10-d)(9-d)}{2}G^2 - d(9-d)HG \\
&\quad + \Gamma\Big[-\frac{1}{2}Z - 2(\dot{G}+G^2)Y_2 + 2dHGY_4 + 2(9-d)G^2Y_5 \\
&\quad - 2\Big\{\frac{d}{dt} + dH + (9-d)G\Big\}\{-\dot{Y}_2 - dH(Y_2+Y_4) - (9-d)G(Y_2+Y_5)\}\Big]. \qquad (8)
\end{aligned}
$$

Here we defined $Y_1 = Y_{0i0i}$, $Y_2 = Y_{0m0m}$, $Y_3 = Y_{ijij}$, $Y_4 = Y_{imim}$, and $Y_5 = Y_{mnmn}$ for $i, j = 1, \ldots, d$ and $m, n = d+1, \ldots, 10$. $Z$ is given by Eq. (6), and the dot represents the time derivative. The details of the calculations can be found in Appendix A.

### 2.2. *Classical solution*

In the following, we will construct the inflationary solution of Eq. (8) up to the linear order of $\Gamma$. Since we solve Eq. (8) order by order, first let us consider the classical equations of motion, which are obtained by setting $\Gamma = 0$. Then the equations of motion in Eq. (8) are simplified as

$$
G = \frac{-d(10-d) \pm 3\sqrt{d(10-d)}}{(10-d)(9-d)}H, \qquad \dot{H} + \frac{-d \pm 3\sqrt{d(10-d)}}{9-d}H^2 = 0. \qquad (9)
$$

From these equations, $H$ and $G$ are solved as

$$
\begin{aligned}
H(t) &= \frac{H_{\mathrm{I}}}{\frac{-d\pm3\sqrt{d(10-d)}}{9-d}H_{\mathrm{I}}\,t + 1}, \\
G(t) &= \frac{-d(10-d) \pm 3\sqrt{d(10-d)}}{(10-d)(9-d)} \frac{H_{\mathrm{I}}}{\frac{-d\pm3\sqrt{d(10-d)}}{9-d}H_{\mathrm{I}}\,t + 1}. \qquad (10)
\end{aligned}
$$

$H(0) = H_{\mathrm{I}}$ is an integral constant. Then $a(t)$ and $b(t)$ are solved as [39,48,49]

$$
\begin{aligned}
a(t) &= a_{\mathrm{I}}\Big(\frac{-d \pm 3\sqrt{d(10-d)}}{9-d}H_{\mathrm{I}}\,t + 1\Big)^{\frac{d\pm3\sqrt{d(10-d)}}{10d}}, \\
b(t) &= b_{\mathrm{I}}\Big(\frac{-d \pm 3\sqrt{d(10-d)}}{9-d}H_{\mathrm{I}}\,t + 1\Big)^{\frac{10-d\mp3\sqrt{d(10-d)}}{10(10-d)}}, \qquad (11)
\end{aligned}
$$

where $a_{\mathrm{I}}$ and $b_{\mathrm{I}}$ are integral constants. We assume that the spacetime appears at $t = 0$ with $a(0) = a_{\mathrm{I}}$ and $b(0) = b_{\mathrm{I}}$. Since the value in parentheses should be positive for $0 \leq t$, we choose $0 \leq H_{\mathrm{I}}$ for the upper sign and $H_{\mathrm{I}} \leq 0$ for the lower sign. Thus, $a(t)$ expands and $b(t)$ shrinks for the upper sign. On the other hand, $a(t)$ shrinks and $b(t)$ expands for the lower sign. Note that the solution becomes rather special in the case of the upper sign with $d = 1$, i.e., $a(t) = a_{\mathrm{I}}(H_{\mathrm{I}}\, t + 1)$ and $b(t) = b_{\mathrm{I}}$.

Below we introduce the dimensionless parameter $\tau$ as

$$\tau = \frac{-d \pm 3\sqrt{d(10-d)}}{9-d} H_{\mathrm{I}}\, t + 1. \tag{12}$$

The range of $\tau$ becomes $1 \leq \tau$. By using $\tau$, the classical solution is summarized as

$$\begin{aligned} H(\tau) &= \frac{H_{\mathrm{I}}}{\tau}, & G(\tau) &= \frac{-d(10-d) \pm 3\sqrt{d(10-d)}}{(10-d)(9-d)} \frac{H_{\mathrm{I}}}{\tau}, \\ a(\tau) &= a_{\mathrm{I}}\, \tau^{\frac{d \pm 3\sqrt{d(10-d)}}{10d}}, & b(\tau) &= b_{\mathrm{I}}\, \tau^{\frac{10-d \mp 3\sqrt{d(10-d)}}{10(10-d)}}. \end{aligned} \tag{13}$$

As mentioned above, $a(\tau)$ or $b(\tau)$ expand as $\tau$ increases, but it is different from the inflationary expansion. This is consistent with the no-go theorem, which states that it is impossible to describe accelerated expansion of the Universe in the framework of supergravity theory [50–52].

### 2.3. Quantum solution

Now we solve the equations of motion in Eq. (8) up to the linear order of $\Gamma$. In order to execute this task, we expand $H$ and $G$ around the classical solution in Eq. (13) as

$$H(\tau) = \frac{H_{\mathrm{I}}}{\tau} + \Gamma H_{\mathrm{I}}^7 h(\tau), \qquad G(\tau) = \frac{-d(10-d) \pm 3\sqrt{d(10-d)}}{(10-d)(9-d)} \frac{H_{\mathrm{I}}}{\tau} + \Gamma H_{\mathrm{I}}^7 g(\tau). \tag{14}$$

$h(\tau)$ and $g(\tau)$ are dimensionless functions with respect to $\tau$. Then, by substituting Eq. (14) into Eq. (8) and comparing the linear terms with respect to $\Gamma$, we obtain equations for $h(\tau)$ and $g(\tau)$. (We employed Mathematica since the calculations are straightforward but tedious.) $g(\tau)$ is expressed in terms of $h(\tau)$ as

$$\begin{aligned} g(\tau) = &\frac{-d(10-d) \pm 3\sqrt{d(10-d)}}{(10-d)(9-d)} h(\tau) + \frac{5184}{5(10-d)^4(9-d)^7\tau^7} \Big\{ \\ &-12d(10-d)\big(659d^8 - 64449d^7 + 2054460d^6 - 31189820d^5 + 251678205d^4 \\ &-1092597255d^3 + 2448054900d^2 - 2647363500d + 590490000\big) \\ &\pm\sqrt{d(10-d)}\big(30181d^9 - 1625421d^8 + 34944195d^7 - 375884005d^6 + 2011588245d^5 \\ &-3752608095d^4 - 8785306575d^3 + 46520278425d^2 - 71493576750d + 18600435000\big)\Big\}, \end{aligned} \tag{15}$$

and the differential equation for $h(\tau)$ is given by

$$0 = \frac{dh(\tau)}{d\tau} + \frac{2}{\tau} h(\tau) + \frac{5c_h}{\tau^8}, \tag{16}$$

$$\begin{aligned} c_h \equiv &\frac{-2592}{25(10-d)^3(9-d)^6} \big\{ 92731d^9 - 4614210d^8 + 89812929d^7 - 824993140d^6 + 2943251745d^5 \\ &+6230342070d^4 - 81640349145d^3 + 234351046800d^2 - 294211642500d + 72630270000 \end{aligned}$$

$$\pm\sqrt{d(10-d)}\big(-70213d^8+4636560d^7-120791847d^6+1618078810d^5-12012374835d^4 +49358796660d^3-106905156345d^2+111881124450d-23678649000\big)\big\}.$$

By integrating the above equation, $h(\tau)$ is easily solved. And by inserting $h(\tau)$ into Eq. (15), $g(\tau)$ can be obtained. The result becomes

$$h(\tau)=\frac{c}{\tau^2}+\frac{c_h}{\tau^7},\qquad g(\tau)=\frac{-d(10-d)\pm3\sqrt{d(10-d)}}{(10-d)(9-d)}\frac{c}{\tau^2}+\frac{c_g}{\tau^7}, \tag{17}$$

$$c_g\equiv\frac{-2592}{25(10-d)^4(9-d)^6}\big\{d(10-d)\big(92731d^8-3648072d^7+50804481d^6-251912470d^5 -435418515d^4+8147315340d^3-25279230465d^2+33786853650d-8089713000\big) \pm\sqrt{d(10-d)}\big(70213d^9-4683156d^8+122597463d^7-1642616950d^6+12125749755d^5 -49064670000d^4+102693985785d^3-99618834150d^2+8070030000d+3542940000\big)\big\}.$$

Note that the constant $c$ appears in $h(\tau)$ and $g(\tau)$, but this corresponds to the moduli parameter of the classical solution, Eq. (13), up to the linear approximation, and it can be absorbed by shifting $H_{\rm I}$. Therefore it is possible to set $c=0$ without loss of generality. From dimensional analysis, it is expected that there are terms of $\Gamma^k H_{\rm I}^{6k+1}/\tau^{6k+1}$ from higher-derivative corrections in general. Since we have poor knowledge about terms for $2\le k$, we simply consider the region $\Gamma H_{\rm I}^6\ll 1$ and neglect those terms.

Finally, $a(\tau)$ and $b(\tau)$ are given as

$$a(\tau)=a_{\rm I}\,\tau^{\frac{d\pm3\sqrt{d(10-d)}}{10d}}\exp\left[\frac{d\pm3\sqrt{d(10-d)}}{60d}c_h\Gamma H_{\rm I}^6\Big(1-\frac{1}{\tau^6}\Big)\right],$$
$$b(\tau)=b_{\rm I}\,\tau^{\frac{10-d\mp3\sqrt{d(10-d)}}{10(10-d)}}\exp\left[\frac{d\pm3\sqrt{d(10-d)}}{60d}c_g\Gamma H_{\rm I}^6\Big(1-\frac{1}{\tau^6}\Big)\right], \tag{18}$$

where $a_{\rm I}$ and $b_{\rm I}$ are initial values of $a(\tau)$ and $b(\tau)$ at $\tau=1$. The exponential parts come from quantum corrections, and they cause inflation or deflation during $1\le\tau<2$. Of course, the above expressions are reliable up to $\mathcal{O}((\Gamma H_{\rm I}^6)^2)$. If we take into account the $\Gamma^k H_{\rm I}^{6k+1}/\tau^{6k+1}$ terms in Eq. (14), we should add terms like $\Gamma^k H_{\rm I}^{6k}(1-1/k^{6k})$ in the argument of the exponentials in the above. These terms would also contribute to inflation or deflation, but as long as we consider the region $\Gamma H_{\rm I}^6\ll 1$, the leading behaviors of the inflation and deflation are well controlled by the solution in Eq. (18).

## 3. Inflationary solution in four-dimensional spacetime

In this section, we consider four-dimensional spacetime with seven internal directions. Namely, we examine the solutions obtained in the previous section by setting $d=3$. Since we are interested in the inflationary solution in four-dimensional spacetime, we also choose the upper sign in the equations in the previous section.

First, the classical solution, Eq. (13), becomes

$$H(\tau)=\frac{H_{\rm I}}{\tau},\qquad G(\tau)=\frac{-7+\sqrt{21}}{14}\frac{H_{\rm I}}{\tau},$$
$$a(\tau)=a_{\rm I}\,\tau^{\frac{1+\sqrt{21}}{10}},\qquad b(\tau)=b_{\rm I}\,\tau^{-\frac{3\sqrt{21}-7}{70}}. \tag{19}$$

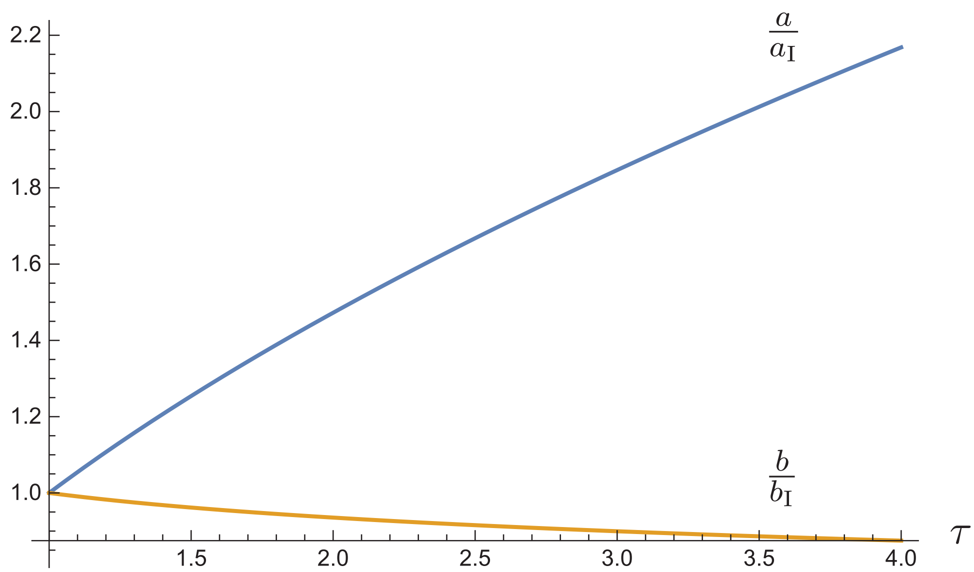


**Fig. 1.** Plots of $a(\tau)/a_I$ and $b(\tau)/b_I$ from Eq. (19). $a(\tau)/a_I$ is not inflating exponentially.

However, this does not represent inflationary expansion in three spatial directions. The scale factor behaves like $a(t) \sim t^{\frac{1+\sqrt{21}}{10}}$ for large $t$, so it rather represents a radiation-dominant era. Plots of $a(\tau)/a_I$ and $b(\tau)/b_I$ are given in Fig. 1.

Now we consider the quantum solutions, Eqs. (17) and (18). By setting $d = 3$ and choosing the upper sign, $H$ and $G$ are written as

$$H(\tau) = \frac{H_I}{\tau} + \frac{c_h \Gamma H_I^7}{\tau^7}, \qquad c_h = \frac{13824(477087 - 97732\sqrt{21})}{8575} \sim 47111,$$
$$G(\tau) = \frac{-7+\sqrt{21}}{14}\frac{H_I}{\tau} + \frac{c_g \Gamma H_I^7}{\tau^7}, \qquad c_g = -\frac{41472(532196 - 110451\sqrt{21})}{60025} \sim -17996. \tag{20}$$

And the scale factors $a(\tau)$ and $b(\tau)$ are obtained as

$$a(\tau) = a_I\, \tau^{\frac{1+\sqrt{21}}{10}} \exp\left[\frac{1+\sqrt{21}}{60} c_h \Gamma H_I^6 \left(1 - \frac{1}{\tau^6}\right)\right],$$
$$b(\tau) = b_I\, \tau^{-\frac{3\sqrt{21}-7}{70}} \exp\left[\frac{1+\sqrt{21}}{60} c_g \Gamma H_I^6 \left(1 - \frac{1}{\tau^6}\right)\right], \tag{21}$$

where $a_I$ and $b_I$ are initial values of $a(\tau)$ and $b(\tau)$ at $\tau = 1$. These values should be chosen so that the late-time behaviors are consistent with observations, and the typical scale of the internal directions should be Planck scale at early time. The exponential parts come from quantum corrections, and they cause inflation for three spatial directions and deflation for seven internal directions during $1 \le \tau < 2$. The above expressions are reliable up to $\mathcal{O}((\Gamma H_I^6)^2)$, but as long as we consider the region $\Gamma H_I^6 \ll 1$, the leading behaviors of the inflation or deflation are well controlled by the solution in Eq. (21).

From Eq. (21), the e-folding number $N_e$ is defined by

$$N_e = \frac{1+\sqrt{21}}{60} c_h \Gamma H_I^6 \sim 4383\, \Gamma H_I^6, \tag{22}$$

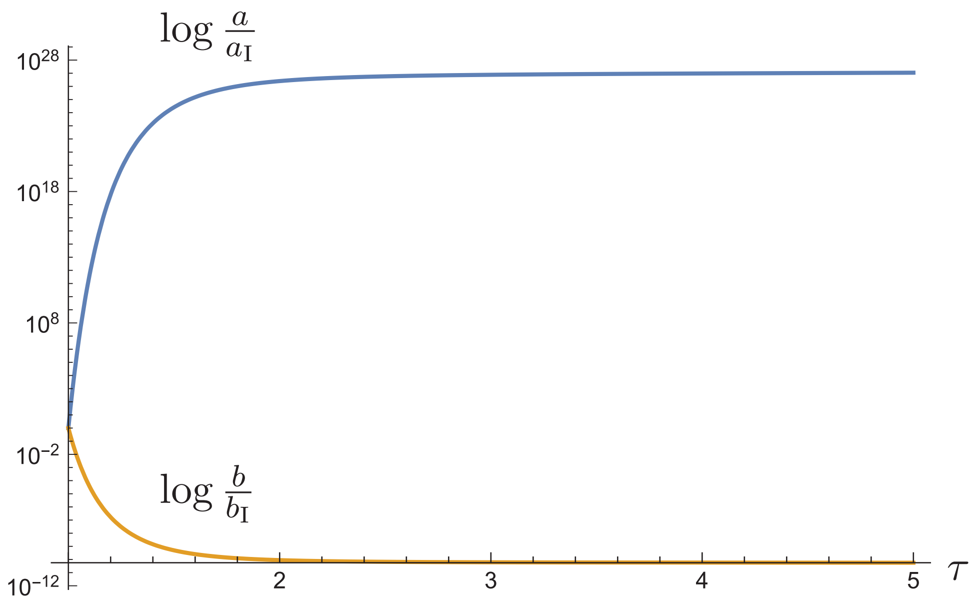


**Fig. 2.** Plots of $\log\big(a(\tau)/a_I\big)$ and $\log\big(b(\tau)/b_I\big)$ in Eq. (21) with $\Gamma H_I^6 \sim 0.014$. The ratio of the scale factor $a(\tau)/a_I$ in three spatial directions increases rapidly during $1 \leq \tau < 2$.

and if we set $N_e = 60$ we obtain $\Gamma H_I^6 \sim 0.014$. Then, $\frac{1+\sqrt{21}}{60} c_g \Gamma H_I^6 \sim -23$ for the deceleration. Plots of $\log\big(a(\tau)/a_I\big)$ and $\log\big(b(\tau)/b_I\big)$ in Eq. (21) with $\Gamma H_I^6 \sim 0.014$ are shown in Fig. 2. The ratio of the scale factor $a(t)/a_I$ in three spatial directions increases rapidly during $1 \leq \tau < 2$. Particularly when $\tau$ is close to 1, the scale factor expands exponentially with respect to $\tau$. After the end of the inflation, the expansion rate in three spatial directions becomes smaller and behaves like the classical solution, $a(\tau)/a_I \sim \tau^{\frac{1+\sqrt{21}}{10}}$. In other words, the inflation ends naturally when the curvature radius becomes large and quantum corrections are negligible. On the other hand, the ratio of the scale factor $b(\tau)/b_I$ in seven internal directions decreases rapidly during $1 \leq \tau < 2$. The deflation ends naturally when quantum corrections are negligible, and the ratio of the scale factor behaves like $b(\tau)/b_I \sim \tau^{-\frac{3\sqrt{21}-7}{70}}$.

Finally, let us examine the slow roll parameter $\epsilon = -\dot{H}/H^2$, which is one of the important parameters for inflationary cosmology. The inflation lasts during $\epsilon < 1$. The explicit form of the function is given by

$$\epsilon = \frac{-1+\sqrt{21}}{2}\Big(1 + \frac{7c_h \Gamma H_I^6}{\tau^6}\Big)\Big(1 + \frac{c_h \Gamma H_I^6}{\tau^6}\Big)^{-2} \tag{23}$$

if we neglect higher-order terms of $\mathcal{O}((\Gamma H_I^6)^2)$ in $H(\tau)$. A plot of the slow roll parameter in Eq. (23) is shown in Fig. 3. From this we see that the inflation ends around $\tau = 2$ because the slow roll parameter $\epsilon$ becomes almost 1 there.

## 4. Analyses of $(d+1)$-dimensional spacetime

In the previous section, we obtained an inflationary solution in four-dimensional spacetime. Then it is natural to ask whether it is true or not in other dimensions.

Below we focus on the case where $d$-dimensional spatial directions are expanding and $(10-d)$ internal directions are shrinking. The classical solutions are given by Eq. (13). For $d = 1, 2, 3, 4, 5$, the scale factor for the expansion is given by $a(\tau)/a_I$ with the upper sign. And for $d = 6, 7, 8, 9$, the

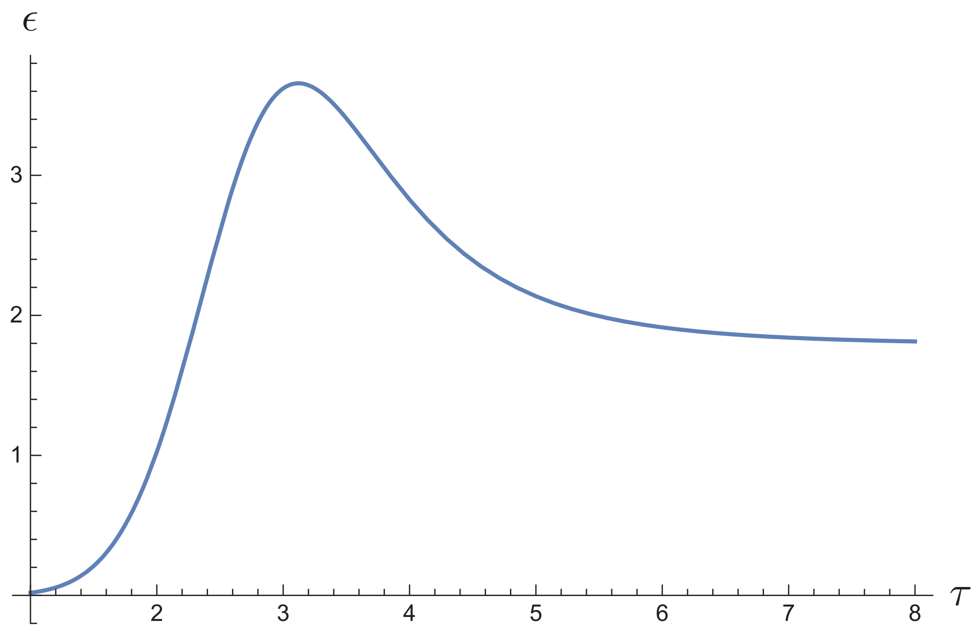


**Fig. 3.** Slow roll parameter $\epsilon$ for $\Gamma H_{\mathrm{I}}^{6} = 0.014$. Inflation ends around $\tau = 2$ since $\epsilon$ becomes almost 1 there.

scale factor for the expansion is expressed by $b(\tau)/b_{\mathrm{I}}$ with the lower sign. The explicit forms of the expanding scale factors are written as

$$\frac{a(\tau)}{a_{\mathrm{I}}} = \begin{cases} \tau & (d=1) \\ \tau^{\frac{7}{10}} & (d=2) \\ \tau^{\frac{1+\sqrt{21}}{10}} & (d=3) \\ \tau^{\frac{2+3\sqrt{6}}{20}} & (d=4) \\ \tau^{\frac{2}{5}} & (d=5) \end{cases}, \qquad \frac{b(\tau)}{b_{\mathrm{I}}} = \begin{cases} \tau^{\frac{1+\sqrt{6}}{10}} & (d=6) \\ \tau^{\frac{7+3\sqrt{21}}{70}} & (d=7) \\ \tau^{\frac{1}{4}} & (d=8) \\ \tau^{\frac{1}{5}} & (d=9) \end{cases}. \tag{24}$$

Plots of the above are shown in Fig. 4. The scale factor is expanding for each case, but it does not represent inflationary expansion.

Now we consider the quantum solutions in Eqs. (17) and (18). For $d = 1, 2, 3, 4, 5$, the scale factor for the expansion is given by $a(\tau)/a_{\mathrm{I}}$ with the upper sign. And for $d = 6, 7, 8, 9$, the scale factor for the expansion is expressed by $b(\tau)/b_{\mathrm{I}}$ with the lower sign. The explicit expressions for the scale factors are given by

$$\frac{a(\tau)}{a_{\mathrm{I}}} = \begin{cases} \tau & (d=1) \\ \tau^{\frac{7}{10}} \exp\left\{\frac{88134777}{420175}\Gamma H_{\mathrm{I}}^{6}\left(1-\frac{1}{\tau^{6}}\right)\right\} & (d=2) \\ \tau^{\frac{1+\sqrt{21}}{10}} \exp\left\{\frac{1152(-315057+75871\sqrt{21})}{8575}\Gamma H_{\mathrm{I}}^{6}\left(1-\frac{1}{\tau^{6}}\right)\right\} & (d=3) \\ \tau^{\frac{2+3\sqrt{6}}{20}} \exp\left\{\frac{7344(-430936+188879\sqrt{6})}{15625}\Gamma H_{\mathrm{I}}^{6}\left(1-\frac{1}{\tau^{6}}\right)\right\} & (d=4) \\ \tau^{\frac{2}{5}} \exp\left\{\frac{862812}{25}\Gamma H_{\mathrm{I}}^{6}\left(1-\frac{1}{\tau^{6}}\right)\right\} & (d=5) \end{cases},$$

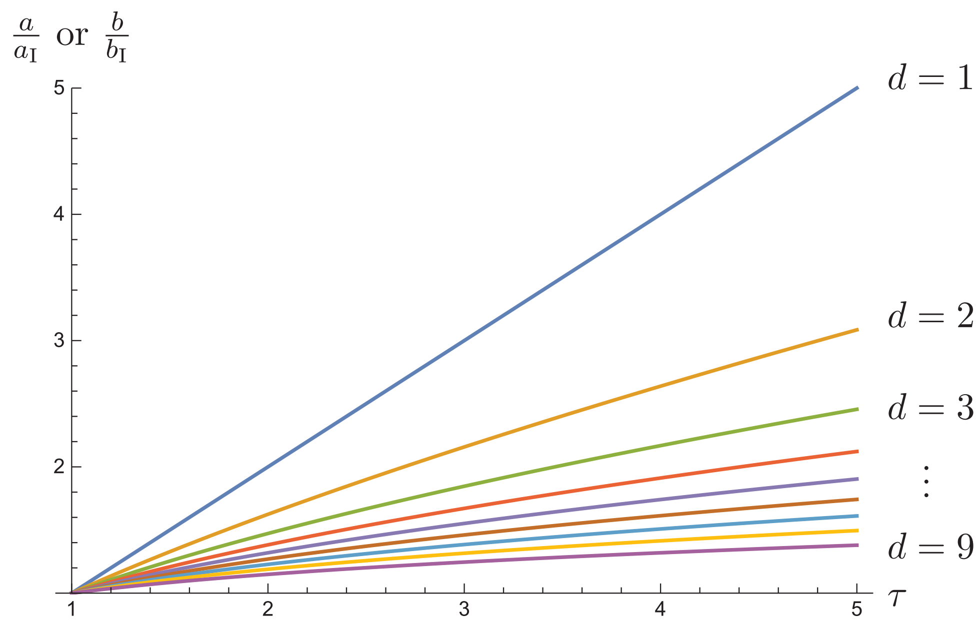


**Fig. 4.** Plots of $a(\tau)/a_{\rm I}$ or $b(\tau)/b_{\rm I}$ in Eq. (24).

$$\frac{b(\tau)}{b_{\rm I}} = \begin{cases} \tau^{\frac{1+\sqrt{6}}{10}} \exp\left\{\frac{288(41176038+17797607\sqrt{6})}{78125}\Gamma H_{\rm I}^6\left(1-\frac{1}{\tau^6}\right)\right\} & (d=6) \\ \tau^{\frac{7+3\sqrt{21}}{70}} \exp\left\{\frac{3456(357455+84349\sqrt{21})}{60025}\Gamma H_{\rm I}^6\left(1-\frac{1}{\tau^6}\right)\right\} & (d=7) \\ \tau^{\frac{1}{4}} \exp\left\{\frac{1293111}{200}\Gamma H_{\rm I}^6\left(1-\frac{1}{\tau^6}\right)\right\} & (d=8) \\ \tau^{\frac{1}{5}} \exp\left\{\frac{458631}{400}\Gamma H_{\rm I}^6\left(1-\frac{1}{\tau^6}\right)\right\} & (d=9) \end{cases}. \tag{25}$$

Note that the definitions of $\tau$ depend on $d$. Also, the parameters $H_{\rm I}$ can be chosen arbitrarily for each $d$. The e-folding numbers are defined by the coefficients of $(1-\frac{1}{\tau^6})$ in the exponentials, and we will fix $\Gamma H_{\rm I}^6$ for $d=2,\dots,9$ by setting the e-folding numbers to be 60. That is, we set

$$\Gamma H_{\rm I}^6 \sim 0.29,\ 0.014,\ 0.0040,\ 0.0017,\ 0.00019,\ 0.0014,\ 0.0093,\ 0.052 \tag{26}$$

for $d=2,\dots,9$. Since the above values are smaller than 1, we may neglect the higher-order corrections. Plots of $\log\big(a(\tau)/a_{\rm I}\big)$ and $\log\big(b(\tau)/b_{\rm I}\big)$ in Eq. (25) by using the above values of $\Gamma H_{\rm I}^6$ are shown in Fig. 5. Inflation does not occur for $d=1$, and for $d=2,\dots,9$ inflation occurs during $1\le\tau<2$. Thus the inflationary scenario is universal if we introduce quantum corrections to the classical gravity.

## 5. Conclusion and discussion

In this paper we have investigated inflationary cosmology in M-theory. In the low-energy limit, M-theory is approximated by 11-dimensional supergravity. If we take into account quantum corrections, however, the 11-dimensional supergravity is corrected by quartic terms of the Weyl tensor at the leading order. We have divided ten spatial directions into $d$ spatial ones and $(10-d)$ internal ones with different scale factors, and investigated the effects of the quantum corrections to the cosmological evolution.

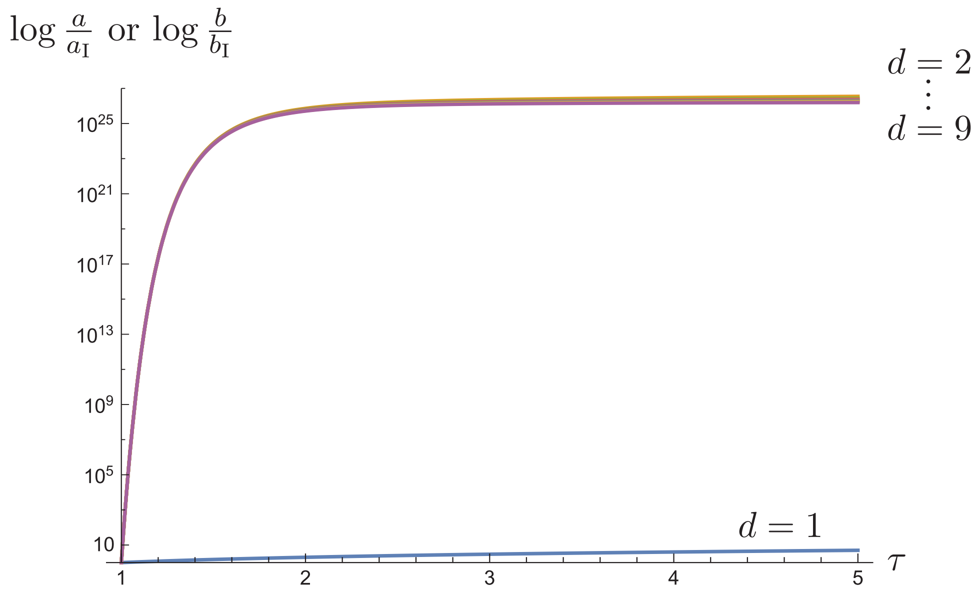


**Fig. 5.** Plots of $\log\left(a(\tau)/a_{\mathrm{I}}\right)$ and $\log\left(b(\tau)/b_{\mathrm{I}}\right)$ in Eq. (25) with $\Gamma H_{\mathrm{I}}^{6}$ as in Eq. (26). Inflation occurs except for $d = 1$.

In the low-energy limit, the solution represents that $d$ spatial directions slowly expand and $(10-d)$ internal directions gradually shrink. These qualitative aspects can be understood naively by dimensional analysis. Namely, the Hubble parameter behaves like $W \sim H^2 \sim \tau^{-2}$, so the scale factor behaves like $\log a \sim \log \tau$. This does not match with inflationary cosmology, and it is consistent with the no-go theorem which states that it is difficult to realize accelerated expansion of the Universe in the low-energy limit of string theory [50–52].

If we correct the 11-dimensional supergravity by adding $W^4$ terms, the Hubble parameter is modified like $H^2 \sim \tau^{-2}(1+\Gamma\tau^{-6})^2$, so the scale factor behaves like $\log a \sim \log \tau + \Gamma\tau^{-6}$ up to the linear order of $\Gamma$. Then the quantum corrections give exponential expansion of the scale factor when $\tau$ is close to 1, and the inflation naturally ends when the quantum corrections are negligible compared to the classical supergravity part. This is also true for the internal directions. The scale factor for the internal directions rapidly deflates around $\tau = 1$, and changes to shrink gradually when the quantum corrections are negligible compared to the classical part. The explicit solutions of the scale factors are given by Eq. (18).

For the case of $d = 3$, we showed the plots of $a(\tau)$ and $b(\tau)$ in Fig. 2 by fixing $\Gamma H_{\mathrm{I}}^{6} \sim 0.014$, which is determined by setting the e-folding number $N_{\mathrm{e}} = 60$. The deceleration of the internal directions are also fixed and the e-folding number is estimated as $-23$. We plotted the slow roll parameter $\epsilon$ in Fig. 3, and confirmed that $\epsilon < 1$ when $1 \leq \tau < 2$. For the case of $d = 2, \ldots, 9$, the qualitative features of the scale factors are the same as those for the case of $d = 3$. Thus we conclude that inflation is universal if we take quantum corrections into account in M-theory. Notice that $d = 1$ and $d = 10$ are rather special and do not receive quantum corrections, hence there are no inflationary solutions.

As future work, it is important to examine other cosmological parameters for the case of $d = 3$. We will introduce fluctuations around the background metric, and evaluate the spectral index $n_{\mathrm{s}}$ and the tensor to scalar ratio $r$ to compare them with observations. It is necessary to compare our results around $\tau \sim 1$ with those in Refs. [36–39,53]. If the results are compatible with each other, it is possible to seek inflationary solutions by using a simpler metric ansatz of $e^{Ht}$ and $e^{Gt}$. It will also be interesting to apply the analyses of this paper to heterotic superstring theory with nontrivial internal space, which contains $R^2$ corrections [54] and reveals several problems in string cosmology [55]. Of course, it is also important to connect our approach to the late-time behavior of four-dimensional cosmology, such as the accelerating Universe. For example, it is known that the compactification of 11-dimensional supergravity into four dimensions is related to $f(R)$ gravity with negative powers of curvature [56], which expresses the accelerating Universe in late time as in Ref. [57]. It would be an interesting direction to generalize Ref. [56] by including higher-derivative corrections and checking the consistency with observations.

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## Appendix A. The derivation of Eq. (8)

In this appendix we derive Eq. (8). First, it is straightforward to evaluate the Weyl tensor by using the ansatz in Eq. (7). We define $W_1 = W_{0i0i}$, $W_2 = W_{0m0m}$, $W_3 = W_{ijij}$, $W_4 = W_{imim}$, and $W_5 = W_{mnmn}$ for $i,j = 1,\dots,d$ and $m,n = d+1,\dots,10$, and these are written as

$$\begin{aligned}
W_1 &= \frac{4(d-10)}{45}\dot{H} + \frac{4(10-d)}{45}\dot{G} - \frac{(10-d)(9-d)}{90}H^2 + \frac{(10-d)(d-1)}{90}G^2 + \frac{(5-d)(10-d)}{45}HG,\\
W_2 &= \frac{4d}{45}\dot{H} - \frac{4d}{45}\dot{G} + \frac{d(9-d)}{90}H^2 - \frac{d(d-1)}{90}G^2 - \frac{d(5-d)}{45}HG,\\
W_3 &= \frac{d-10}{45}\dot{H} + \frac{10-d}{45}\dot{G} + \frac{(10-d)(9-d)}{90}H^2 + \frac{(10-d)(11-d)}{90}G^2 - \frac{(10-d)^2}{45}HG,\\
W_4 &= \frac{d-5}{45}\dot{H} + \frac{5-d}{45}\dot{G} - \frac{d(9-d)}{90}H^2 - \frac{(d-1)(10-d)}{90}G^2 - \frac{d^2-10d+5}{45}HG,\\
W_5 &= \frac{d}{45}\dot{H} - \frac{d}{45}\dot{G} + \frac{d(d+1)}{90}H^2 + \frac{d(d-1)}{90}G^2 - \frac{d^2}{45}HG.
\end{aligned} \tag{A1}$$

Of course, $W_1$ and $W_2$ are exchanged with each other if we exchange $H$ with $G$ and $d$ with $(10-d)$. This is also true for $W_3$ and $W_5$.

Second, let us evaluate $Z = t_8 t_8 W^4 - \frac{1}{4!}\epsilon_{11}\epsilon_{11}W^4$. Each term in $Z$ is calculated as follows. Note that $i,j$ are summed from 1 to $d$, $m,n$ are summed from $d+1$ to 10, and $a,b,c,d,e,f,g,h$ are summed from 0 to 10 below.

$$\begin{aligned}
&W_{abcd}W^{abcd}W_{efgh}W^{efgh}\\
&= 4W_{abab}W^{abab}W_{efef}W^{efef}
\end{aligned}$$

$$= 4\left(2W_{0i0i}^2 + 2W_{0m0m}^2 + W_{ijij}^2 + 2W_{imim}^2 + W_{mnmn}^2\right)^2$$
$$= 16d^2W_1^4 + 16(10-d)^2W_2^4 + 4d^2(d-1)^2W_3^4 + 16d^2(10-d)^2W_4^4 + 4(10-d)^2(9-d)^2W_5^4$$
$$+ 32d(10-d)W_1^2W_2^2 + 16d^2(d-1)W_1^2W_3^2 + 32d^2(10-d)W_1^2W_4^2 + 16d(10-d)(9-d)W_1^2W_5^2$$
$$+ 16d(d-1)(10-d)W_2^2W_3^2 + 32d(10-d)^2W_2^2W_4^2 + 16(10-d)^2(9-d)W_2^2W_5^2$$
$$+ 16d^2(d-1)(10-d)W_3^2W_4^2 + 8d(d-1)(10-d)(9-d)W_3^2W_5^2 + 16d(10-d)^2(9-d)W_4^2W_5^2. \quad \text{(A2)}$$

$$W_{abcd}W^{aefg}W^{bcdh}W_{efgh}$$
$$= W_{abab}W^{abab}W_{aeae}W^{aeae}$$
$$= W_{0i0i}^2W_{0e0e}^2 + W_{0i0i}^2W_{ieie}^2 + W_{0m0m}^2W_{0e0e}^2 + W_{0m0m}^2W_{meme}^2$$
$$+ W_{ijij}^2W_{ieie}^2 + W_{imim}^2W_{ieie}^2 + W_{imim}^2W_{meme}^2 + W_{mnmn}^2W_{meme}^2$$
$$= d(d+1)W_1^4 + (10-d)(11-d)W_2^4 + d(d-1)^2W_3^4 + 10d(10-d)W_4^4 + (10-d)(9-d)^2W_5^4$$
$$+ 2d(10-d)W_1^2W_2^2 + 2d(d-1)W_1^2W_3^2 + 2d(10-d)W_1^2W_4^2 + 2d(10-d)W_2^2W_4^2$$
$$+ 2(10-d)(9-d)W_2^2W_5^2 + 2d(d-1)(10-d)W_3^2W_4^2 + 2d(10-d)(9-d)W_4^2W_5^2. \quad \text{(A3)}$$

$$W_{abcd}W^{abef}W^{cdgh}W_{efgh}$$
$$= 8W_{abab}W_{abab}W^{abab}W^{abab}$$
$$= 16W_{0i0i}^4 + 16W_{0m0m}^4 + 8W_{ijij}^4 + 16W_{imim}^4 + 8W_{mnmn}^4$$
$$= 16dW_1^4 + 16(10-d)W_2^4 + 8d(d-1)W_3^4 + 16d(10-d)W_4^4 + 8(10-d)(9-d)W_5^4. \quad \text{(A4)}$$

$$W_{acbd}W^{aebf}W^{cgdh}W_{egfh}$$
$$= W_{acac}W^{aeae}W^{cgcg}W_{egeg} + W_{acac}W_{acac}W^{acac}W^{acac}$$
$$= 2W_{0i0i}W_{0e0e}W_{igig}W_{egeg} + 2W_{0i0i}^4 + 2W_{0m0m}W_{0e0e}W_{mgmg}W_{egeg} + 2W_{0m0m}^4$$
$$+ W_{ijij}W_{ieie}W_{jgjg}W_{egeg} + W_{ijij}^4 + 2W_{imim}W_{ieie}W_{mgmg}W_{egeg} + 2W_{imim}^4$$
$$+ W_{mnmn}W_{meme}W_{ngng}W_{egeg} + W_{mnmn}^4$$
$$= 2d(d+1)W_1^4 + 2(10-d)(11-d)W_2^4 + d(d-1)(d^2-3d+4)W_3^4$$
$$+ 2d(10-d)(1+10d-d^2)W_4^4 + (10-d)(9-d)(d^2-17d+74)W_5^4 + 4d(10-d)W_1^2W_2^2$$
$$+ 4d(d-1)^2W_1^2W_3^2 + 4d^2(10-d)W_1^2W_4^2 + 4d(10-d)^2W_2^2W_4^2 + 4(10-d)(9-d)^2W_2^2W_5^2$$
$$+ 4d(d-1)^2(10-d)W_3^2W_4^2 + 4d(10-d)(9-d)^2W_4^2W_5^2 + 8d(d-1)(10-d)W_1W_2W_3W_4$$
$$+ 8d(10-d)(9-d)W_1W_2W_4W_5 + 4d(d-1)(10-d)(9-d)W_3W_4^2W_5. \quad \text{(A5)}$$

$$W_{abcd}W^{aefg}W^{b}{}_{ef}{}^{h}W^{cd}{}_{gh} + W_{abcd}W^{aefg}W^{b}{}_{fe}{}^{h}W^{cd}{}_{gh}$$
$$= 4W_{abab}W^{aeae}W^{b}{}_{e}{}^{b}{}_{e}W^{ab}{}_{ab} - 2W_{abab}W^{abab}W_{a}{}^{ba}{}_{b}W^{ab}{}_{ab}$$
$$= -8W_{0i0i}^2W_{0e0e}W_{ieie} - 4W_{0i0i}^4 - 8W_{0m0m}^2W_{0e0e}W_{meme} - 4W_{0m0m}^4 + 4W_{ijij}^2W_{ieie}W_{jeje}$$
$$- 2W_{ijij}^4 + 8W_{imim}^2W_{ieie}W_{meme} - 4W_{imim}^4 + 4W_{mnmn}^2W_{meme}W_{nene} - 2W_{mnmn}^4$$
$$= -4dW_1^4 - 4(10-d)W_2^4 + 2d(d-1)(2d-5)W_3^4 - 4d(10-d)W_4^4$$
$$+ 2(10-d)(9-d)(15-2d)W_5^4 - 8d(d-1)W_1^3W_3 - 8(10-d)(9-d)W_2^3W_5$$

$$+ 8d(d-1)(10-d)W_3W_4^3 + 8d(10-d)(9-d)W_4^3W_5 + 4d(d-1)W_1^2W_3^2$$
$$+ 4(10-d)(9-d)W_2^2W_5^2 + 4d(d-1)(10-d)W_3^2W_4^2 + 4d(10-d)(9-d)W_4^2W_5^2$$
$$- 8d(10-d)W_1^2W_2W_4 - 8d(10-d)W_1W_2^2W_4 + 8d(10-d)W_1W_2W_4^2. \tag{A6}$$

Combining the above results, $Z$ as defined by Eq. (6) is obtained as

$$\begin{aligned}
Z = 24\big\{ &- 16d(d-4)W_1^4 - 16(10-d)(6-d)W_2^4 + 4d(d-1)(5d^2-45d+76)W_3^4 \\
&- 16d(10-d)(3d^2-30d+32)W_4^4 + 4(10-d)(9-d)(5d^2-55d+126)W_5^4 \\
&- 32d(10-d)W_1^2W_2^2 + 16d(d-1)(5d-16)W_1^2W_3^2 + 32d(10-d)(3d-4)W_1^2W_4^2 \\
&+ 16d(10-d)(9-d)W_1^2W_5^2 + 16d(d-1)(10-d)W_2^2W_3^2 - 32d(10-d)(3d-26)W_2^2W_4^2 \\
&- 16(10-d)(9-d)(5d-34)W_2^2W_5^2 + 16d(d-1)(10-d)(5d-16)W_3^2W_4^2 \\
&+ 8d(d-1)(10-d)(9-d)W_3^2W_5^2 - 16d(10-d)(9-d)(5d-34)W_4^2W_5^2 \\
&+ 128d(d-1)(10-d)W_1W_2W_3W_4 + 128d(10-d)(9-d)W_1W_2W_4W_5 \\
&+ 64d(d-1)(10-d)(9-d)W_3W_4^2W_5 + 128d(d-1)W_1^3W_3 \\
&+ 128(10-d)(9-d)W_2^3W_5 - 128d(d-1)(10-d)W_3W_4^3 - 128d(10-d)(9-d)W_4^3W_5 \\
&+ 128d(10-d)W_1^2W_2W_4 + 128d(10-d)W_1W_2^2W_4 - 128d(10-d)W_1W_2W_4^2\big\}.
\end{aligned} \tag{A7}$$

Third, let us evaluate $X_{abcd}$ as defined by Eq. (4). Since the Weyl tensor $W_{abcd}$ is nonzero only when $(a,b) = (c,d)$ or $(a,b) = (d,c)$, $X_{abcd}$ is simplified as follows:

$$\begin{aligned}
X_{abcd} = 192\big\{&W_{abcd}W_{efef}W^{efef} - 8W_{abcd}(W_{afaf}W^{afaf} + W_{bfbf}W^{bfbf}) \\
&+ 4W_{abcd}(3W_{abab}W^{abab} + W_{acac}W^{acac} + W_{bcbc}W^{bcbc}) \\
&+ 4(W_{aece}W_{bfdf} - W_{bece}W_{afdf})W^{efef} - 8W_{abcd}W_{af}{}^{af}W_{bf}{}^{bf} \\
&- 4(W^{ag}{}_{ag} + W^{bg}{}_{bg})(W_{agcg}W_{gbgd} - W_{agdg}W_{gbgc})\big\}.
\end{aligned} \tag{A8}$$

In the above, we may also use the relations $W_{abcd}W_{acac} = W_{abcd}W_{bdbd}$ or $W_{abcd}W_{abab} = W_{abcd}W_{cdcd}$. From Eq. (A8), we see that nonzero components of $X_{abcd}$ are $X_{0i0i}$, $X_{0m0m}$, $X_{ijij}$, $X_{imim}$, and $X_{mnmn}$. Then these are calculated as follows:

$$\begin{aligned}
X_{0i0i} &= 192\big\{W_{0i0i}W_{efef}^2 - 8W_{0i0i}(W_{0f0f}^2 + W_{ifif}^2) + 16W_{0i0i}^3 + 4W_{0e0e}W_{ifif}W_{efef} \\
&\quad + 8W_{0i0i}W_{0f0f}W_{ifif} - 4(-W_{0g0g} + W_{igig})W_{0g0g}W_{gigi}\big\} \\
&= 192\big[2(4-d)W_1^3 - 2(10-d)W_1W_2^2 + 12(d-1)W_1^2W_3 + (5d-16)(d-1)W_1W_3^2 \\
&\quad + 2(3d-4)(10-d)W_1W_4^2 + (10-d)(9-d)W_1W_5^2 + 4(10-d)W_2^2W_4 - 4(10-d)W_2W_4^2 \\
&\quad + 8(10-d)W_1W_2W_4 + 4(d-1)(10-d)W_2W_3W_4 + 4(10-d)(9-d)W_2W_4W_5\big].
\end{aligned} \tag{A9}$$

$$\begin{aligned}
X_{0m0m} &= 192\big\{W_{0m0m}W_{efef}^2 - 8W_{0m0m}(W_{0f0f}^2 + W_{mfmf}^2) + 16W_{0m0m}^3 + 4W_{0e0e}W_{mfmf}W_{efef} \\
&\quad + 8W_{0m0m}W_{0f0f}W_{mfmf} - 4(-W_{0g0g} + W_{mgmg})W_{0g0g}W_{gmgm}\big\} \\
&= 192\big[2(d-6)W_2^3 - 2dW_1^2W_2 + 4dW_1^2W_4 - 4dW_1W_4^2 + d(d-1)W_2W_3^2 \\
&\quad + 2d(26-3d)W_2W_4^2 + 12(9-d)W_2^2W_5 + (34-5d)(9-d)W_2W_5^2
\end{aligned}$$

$$+ 8dW_1W_2W_4 + 4d(d-1)W_1W_3W_4 + 4d(9-d)W_1W_4W_5\big]. \tag{A10}$$

$$\begin{aligned} X_{ijij} &= 192\big\{W_{ijij}W_{efef}^2 - 8W_{ijij}(W_{ifif}^2 + W_{jfjf}^2) + 16W_{ijij}^3 + 4W_{ieie}W_{jfjf}W_{efef} \\ &\quad - 8W_{ijij}W_{ifif}W_{jfjf} - 4(W^{ig}{}_{ig} + W^{jg}{}_{jg})W_{igig}W_{gjgj}\big\} \\ &= 192\big[8W_1^3 + (5d^2-45d+76)W_3^3 - 8(10-d)W_4^3 + 2(5d-16)W_1^2W_3 \\ &\quad + 2(10-d)W_2^2W_3 + 2(5d-16)(10-d)W_3W_4^2 + (10-d)(9-d)W_3W_5^2 \\ &\quad + 4(10-d)(9-d)W_4^2W_5 + 8(10-d)W_1W_2W_4\big]. \end{aligned} \tag{A11}$$

$$\begin{aligned} X_{imim} &= 192\big\{W_{imim}W_{efef}^2 - 8W_{imim}(W_{ifif}^2 + W_{mfmf}^2) + 16W_{imim}^3 + 4W_{ieie}W_{mfmf}W_{efef} \\ &\quad - 8W_{imim}W_{ifif}W_{mfmf} - 4(W^{ig}{}_{ig} + W^{mg}{}_{mg})W_{igig}W_{gmgm}\big\} \\ &= 192\big[2(-3d^2+30d-32)W_4^3 + 4W_1^2W_2 + 4W_1W_2^2 + 2(3d-4)W_1^2W_4 \\ &\quad + 2(26-3d)W_2^2W_4 + (5d-16)(d-1)W_3^2W_4 - 12(d-1)W_3W_4^2 \\ &\quad - 12(9-d)W_4^2W_5 - (5d-34)(9-d)W_4W_5^2 + 4(d-1)W_1W_2W_3 - 8W_1W_2W_4 \\ &\quad + 4(9-d)W_1W_2W_5 + 4(d-1)(9-d)W_3W_4W_5\big]. \end{aligned} \tag{A12}$$

$$\begin{aligned} X_{mnmn} &= 192\big\{W_{mnmn}W_{efef}^2 - 8W_{mnmn}(W_{mfmf}^2 + W_{nfnf}^2) + 16W_{mnmn}^3 + 4W_{meme}W_{nfnf}W_{efef} \\ &\quad - 8W_{mnmn}W_{mfmf}W_{nfnf} - 4(W^{mg}{}_{mg} + W^{ng}{}_{ng})W_{mgmg}W_{gngn}\big\} \\ &= 192\big[8W_2^3 - 8dW_4^3 + (5d^2-55d+126)W_5^3 + 2dW_1^2W_5 + 2(34-5d)W_2^2W_5 \\ &\quad + 4d(d-1)W_3W_4^2 + d(d-1)W_3^2W_5 + 2d(34-5d)W_4^2W_5 + 8dW_1W_2W_4\big]. \end{aligned} \tag{A13}$$

$X_{ac} = X_a{}^d{}_{cd}$ is simplified as

$$X_{ac} = 2304\big(-W_a{}^d{}_{cd}W_{dfdf}W^{dfdf} + 2W_{adcd}W_c{}^d{}_{cd}W^{cdcd} - W_{adcd}W^a{}_{faf}W^{dfdf}\big), \tag{A14}$$

and nonzero components are calculated as follows:

$$\begin{aligned} X_{00} &= 2304\big(-W_{0d0d}W_{dfdf}^2 + 2W_{0d0d}^3 + W_{0d0d}W_{0f0f}W^{dfdf}\big) \\ &= 2304\big[dW_1^3 + (10-d)W_2^3 + d(d-1)W_1^2W_3 - d(d-1)W_1W_3^2 - d(10-d)W_1W_4^2 \\ &\quad - d(10-d)W_2W_4^2 - (10-d)(9-d)W_2W_5^2 + (10-d)(9-d)W_2^2W_5 + 2d(10-d)W_1W_2W_4\big], \end{aligned} \tag{A15}$$

$$\begin{aligned} X_{ii} &= 2304\big(-W_i{}^d{}_{id}W_{dfdf}^2 + 2W_i{}^d{}_{id}W_{idid}^2 - W_{idid}W_{ifif}W_{dfdf}\big) \\ &= 2304\big[(d-2)W_1^3 - (d-1)(2d-5)W_3^3 - (d-2)(10-d)W_4^3 \\ &\quad + (10-d)W_1W_2^2 - 3(d-1)W_1^2W_3 - (10-d)W_2^2W_4 - 3(d-1)(10-d)W_3W_4^2 \\ &\quad - (10-d)(9-d)W_4^2W_5 - (10-d)(9-d)W_4W_5^2 - 2(10-d)W_1W_2W_4\big], \end{aligned} \tag{A16}$$

$$\begin{aligned} X_{mm} &= 2304\big(-W_m{}^d{}_{md}W_{dfdf}^2 + 2W_m{}^d{}_{md}W_{mdmd}^2 - W_{mdmd}W_{mfmf}W_{dfdf}\big) \\ &= 2304\big[(8-d)W_2^3 - d(8-d)W_4^3 - (15-2d)(9-d)W_5^3 \\ &\quad + dW_1^2W_2 - dW_1^2W_4 - 3(9-d)W_2^2W_5 - d(d-1)W_3^2W_4 - d(d-1)W_3W_4^2 \\ &\quad - 3d(9-d)W_4^2W_5 - 2dW_1W_2W_4\big]. \end{aligned} \tag{A17}$$

$X = X^a{}_a$ is evaluated as

$$X = 96\big(12W^{ad}{}_{ef}W_{adgh}W^{efgh} - 24W_{aedg}W^a{}_f{}^d{}_hW^{efgh}\big)$$

$$= 1152\big(4W^{ef}{}_{ef}W_{efef}^2 - 2W_{aeae}W_{afaf}W_{efef}\big)$$
$$= 2304\big[-4dW_1^3 - 4(10-d)W_2^3 + 2d(d-1)W_3^3 + 4d(10-d)W_4^3 + 2(10-d)(9-d)W_5^3$$
$$- 2W_{0i0i}W_{0f0f}W_{ifif} - 2W_{0m0m}W_{0f0f}W_{mfmf} - W_{ijij}W_{ifif}W_{jfjf}$$
$$- 2W_{imim}W_{ifif}W_{mfmf} - W_{mnmn}W_{mfmf}W_{nfnf}\big]$$
$$= 2304\big[-4dW_1^3 - 4(10-d)W_2^3 - d(d-1)(d-4)W_3^3 + 4d(10-d)W_4^3$$
$$- (10-d)(9-d)(6-d)W_5^3 - 3d(d-1)W_1^2W_3 - 3(10-d)(9-d)W_2^2W_5$$
$$- 3d(d-1)(10-d)W_3W_4^2 - 3d(10-d)(9-d)W_4^2W_5 - 6d(10-d)W_1W_2W_4\big]. \tag{A18}$$

Fourth, let us evaluate $Y_{abcd}$ as defined by Eq. (5). There are five kinds of nonzero components, so we define them as $Y_1 = Y_{0i0i}$, $Y_2 = Y_{0m0m}$, $Y_3 = Y_{ijij}$, $Y_4 = Y_{imim}$, and $Y_5 = Y_{mnmn}$. Then these are explicitly written as follows:

$$Y_1 = X_{0i0i} - \frac{1}{9}(-X_{ii} + X_{00}) - \frac{1}{90}X$$
$$= \frac{64}{5}\big[2(40-11d)W_1^3 - 12(10-d)W_2^3 + 2(d-1)(d^2-24d+50)W_3^3$$
$$- 4(7d-10)(10-d)W_4^3 + 2(10-d)(9-d)(6-d)W_5^3 - 10(10-d)W_1W_2^2$$
$$- 2(7d-60)(d-1)W_1^2W_3 + 5(19d-48)(d-1)W_1W_3^2 + 10(11d-12)(10-d)W_1W_4^2$$
$$+ 15(10-d)(9-d)W_1W_5^2 + 40(10-d)W_2^2W_4 + 20(d-3)(10-d)W_2W_4^2$$
$$- 14(10-d)(9-d)W_2^2W_5 + 20(10-d)(9-d)W_2W_5^2 - 6(d-1)(10-d)^2W_3W_4^2$$
$$+ 2(3d-10)(10-d)(9-d)W_4^2W_5 - 20(10-d)(9-d)W_4W_5^2$$
$$- 4(7d-20)(10-d)W_1W_2W_4 + 60(d-1)(10-d)W_2W_3W_4 + 60(10-d)(9-d)W_2W_4W_5\big]. \tag{A19}$$

$$Y_2 = X_{0m0m} - \frac{1}{9}(-X_{mm} + X_{00}) - \frac{1}{90}X$$
$$= \frac{64}{5}\big[-12dW_1^3 + 2(11d-70)W_2^3 + 2d(d-1)(d-4)W_3^3 - 4d(60-7d)W_4^3$$
$$+ 2(d^2+4d-90)(9-d)W_5^3 - 10dW_1^2W_2 - 14d(d-1)W_1^2W_3 + 20d(d-1)W_1W_3^2$$
$$+ 40dW_1^2W_4 + 20d(7-d)W_1W_4^2 + 15d(d-1)W_2W_3^2 + 10d(98-11d)W_2W_4^2$$
$$+ 2(7d-10)(9-d)W_2^2W_5 + 5(142-19d)(9-d)W_2W_5^2 - 20d(d-1)W_3^2W_4$$
$$+ 2d(d-1)(20-3d)W_3W_4^2 - 6d^2(9-d)W_4^2W_5 + 4d(7d-50)W_1W_2W_4$$
$$+ 60d(d-1)W_1W_3W_4 + 60d(9-d)W_1W_4W_5\big]. \tag{A20}$$

$$Y_3 = X_{ijij} - \frac{1}{9}(X_{jj} + X_{ii}) + \frac{1}{90}X$$
$$= \frac{64}{5}\big[-8(6d-25)W_1^3 - 8(10-d)W_2^3 - (d-4)(2d^2-157d+335)W_3^3$$
$$+ 8(6d-25)(10-d)W_4^3 - 2(10-d)(9-d)(6-d)W_5^3 - 6(d^2-46d+100)W_1^2W_3$$
$$- 40(10-d)W_1W_2^2 + 30(10-d)W_2^2W_3 + 40(10-d)W_2^2W_4 - 6(10-d)(9-d)W_2^2W_5$$

$$- 6(d^2-46d+100)(10-d)W_3W_4^2 + 15(10-d)(9-d)W_3W_5^2 + 40(10-d)(9-d)W_4W_5^2 + 2(50-3d)(10-d)(9-d)W_4^2W_5 + 4(50-3d)(10-d)W_1W_2W_4\big]. \quad \text{(A21)}$$

$$\begin{aligned} Y_4 &= X_{imim} - \frac{1}{9}(X_{mm} + X_{ii}) + \frac{1}{90}X \\ &= \frac{64}{5}\big[ - 4(7d-10)W_1^3 - 4(60-7d)W_2^3 - 2(d-1)(d^2-24d+50)W_3^3 \\ &\quad - 2(69d^2-690d+680)W_4^3 - 2(d^2+4d-90)(9-d)W_5^3 - 20(d-3)W_1^2W_2 - 20(7-d)W_1W_2^2 \\ &\quad + 6(d-1)(10-d)W_1^2W_3 + 10(11d-12)W_1^2W_4 + 10(98-11d)W_2^2W_4 + 6d(9-d)W_2^2W_5 \\ &\quad + 5(19d-48)(d-1)W_3^2W_4 + 2(d-1)(3d^2-50d+210)W_3W_4^2 \\ &\quad + 2(3d^2-10d+10)(9-d)W_4^2W_5 - 5(19d-142)(9-d)W_4W_5^2 + 60(d-1)W_1W_2W_3 \\ &\quad + 4(3d^2-30d+70)W_1W_2W_4 + 60(9-d)W_1W_2W_5 + 60(d-1)(9-d)W_3W_4W_5\big]. \end{aligned} \quad \text{(A22)}$$

$$\begin{aligned} Y_5 &= X_{mnmn} - \frac{1}{9}(X_{nn} + X_{mm}) + \frac{1}{90}X \\ &= \frac{64}{5}\big[ - 8dW_1^3 - 8(35-6d)W_2^3 - 2d(d-1)(d-4)W_3^3 + 8d(35-6d)W_4^3 \\ &\quad - (6 - d)(2d^2+117d-1035)W_5^3 - 40dW_1^2W_2 - 6d(d-1)W_1^2W_3 + 40dW_1^2W_4 \\ &\quad + 30dW_1^2W_5 - 6(d^2+26d-260)W_2^2W_5 + 40d(d-1)W_3^2W_4 + 2d(d-1)(3d+20)W_3W_4^2 \\ &\quad + 15d(d-1)W_3^2W_5 - 6d(d^2+26d-260)W_4^2W_5 + 4d(3d+20)W_1W_2W_4\big]. \end{aligned} \quad \text{(A23)}$$

Fifth, let us evaluate $D_cD_dY^c{}_{ab}{}^d$. Note that the nonzero components of the spin connection are $\omega_i{}^i{}_0 = H$ and $\omega_m{}^m{}_0 = G$, and terms with spatial derivatives become zero. Then $D_cD_dY^c{}_{ab}{}^d$ is simplified as follows:

$$\begin{aligned} D_cD_dY^c{}_{ab}{}^d &= \partial_c(\partial_dY^c{}_{ab}{}^d + \omega_d{}^c{}_eY^e{}_{ab}{}^d - \omega_d{}^e{}_aY^c{}_{eb}{}^d - \omega_d{}^e{}_bY^c{}_{ae}{}^d + \omega_d{}^d{}_eY^c{}_{ab}{}^e) \\ &\quad + \omega_c{}^c{}_e(\partial_dY^e{}_{ab}{}^d + \omega_d{}^e{}_fY^f{}_{ab}{}^d - \omega_d{}^f{}_aY^e{}_{fb}{}^d - \omega_d{}^f{}_bY^e{}_{af}{}^d + \omega_d{}^d{}_fY^e{}_{ab}{}^f) \\ &\quad - \omega_c{}^e{}_a(\partial_dY^c{}_{eb}{}^d + \omega_d{}^c{}_fY^f{}_{eb}{}^d - \omega_d{}^f{}_eY^c{}_{fb}{}^d - \omega_d{}^f{}_bY^c{}_{ef}{}^d + \omega_d{}^d{}_fY^c{}_{eb}{}^f) \\ &\quad - \omega_c{}^e{}_b(\partial_dY^c{}_{ae}{}^d + \omega_d{}^c{}_fY^f{}_{ae}{}^d - \omega_d{}^f{}_aY^c{}_{fe}{}^d - \omega_d{}^f{}_eY^c{}_{af}{}^d + \omega_d{}^d{}_fY^c{}_{ae}{}^f) \\ &= (\partial_0 + \omega_c{}^c{}_0)(\partial_0Y^0{}_{ab}{}^0 + \omega_d{}^0{}_dY^d{}_{ab}{}^d - \omega_d{}^d{}_aY^0{}_{db}{}^d - \omega_a{}^0{}_bY^0{}_{a0}{}^a + \omega_d{}^d{}_0Y^0{}_{ab}{}^0) \\ &\quad - \omega_b{}^0{}_a\partial_0Y^b{}_{0b}{}^0 + \omega_b{}^0{}_a\omega_d{}^d{}_0Y^b{}_{db}{}^d + \omega_b{}^0{}_a\omega_b{}^0{}_bY^b{}_{00}{}^b - \omega_b{}^0{}_a\omega_d{}^d{}_0Y^b{}_{0b}{}^0 \\ &\quad - \omega_c{}^c{}_b\partial_0Y^c{}_{ac}{}^0 + \omega_c{}^c{}_b\omega_d{}^d{}_aY^c{}_{dc}{}^d + \omega_c{}^c{}_b\omega_c{}^0{}_cY^c{}_{a0}{}^c - \omega_c{}^c{}_b\omega_d{}^d{}_0Y^c{}_{ac}{}^0. \end{aligned} \quad \text{(A24)}$$

Note that $\partial_0 = \partial_t$. Nonzero components of $D_cD_dY^c{}_{ab}{}^d$ are calculated as follows:

$$\begin{aligned} D_cD_dY^c{}_{00}{}^d &= \omega_c{}^c{}_0\partial_0Y_{c0c0} + \omega_c{}^c{}_0\omega_d{}^d{}_0Y_{cdcd} - (\omega_c{}^c{}_0)^2Y_{0c0c} + \omega_c{}^c{}_0\omega_d{}^d{}_0Y_{c0c0} \\ &= dH\dot{Y}_1 + (10-d)G\dot{Y}_2 + d(d-1)H^2(Y_1 + Y_3) \\ &\quad + (10-d)(9-d)G^2(Y_2 + Y_5) + d(10-d)HG(Y_1 + Y_2 + 2Y_4), \\ D_cD_dY^c{}_{ii}{}^d &= (\partial_0 + \omega_c{}^c{}_0)(-\partial_0Y_{0i0i} - \omega_d{}^0{}_dY_{idid} + \omega_i{}^0{}_iY_{0i0i} - \omega_d{}^d{}_0Y_{0i0i}) \\ &\quad + \omega_i{}^0{}_i\partial_0Y_{i0i0} + \omega_i{}^0{}_i\omega_d{}^d{}_0Y_{idid} - (\omega_i{}^0{}_i)^2Y_{0i0i} + \omega_i{}^0{}_i\omega_d{}^d{}_0Y_{i0i0} \quad \text{(A25)} \\ &= \left\{\frac{d}{dt} + (d-1)H + (10-d)G\right\}\{-\dot{Y}_1 - (d-1)H(Y_1 + Y_3) - (10-d)G(Y_1 + Y_4)\}, \end{aligned}$$

$$D_c D_d Y^c{}_{mm}{}^d = (\partial_0 + \omega_c{}^c{}_0)(-\partial_0 Y_{0m0m} - \omega_d{}^0{}_d Y_{mdmd} + \omega_m{}^0{}_m Y_{0m0m} - \omega_d{}^d{}_0 Y_{0m0m})$$
$$+ \omega_m{}^0{}_m \partial_0 Y_{m0m0} + \omega_m{}^0{}_m \omega_d{}^d{}_0 Y_{mdmd} - (\omega_m{}^0{}_m)^2 Y_{0m0m} + \omega_m{}^0{}_m \omega_d{}^d{}_0 Y_{m0m0}$$
$$= \left\{\frac{d}{dt} + dH + (9-d)G\right\}\{-\dot{Y}_2 - dH(Y_2 + Y_4) - (9-d)G(Y_2 + Y_5)\}.$$

Sixth, components of $R_{cdea}Y^{cde}{}_b = 2R_{eaea}Y^{eae}{}_b$ are evaluated as follows:

$$R_{cde0}Y^{cde}{}_0 = -2R_{e0e0}Y_{e0e0} = 2d(\dot{H} + H^2)Y_1 + 2(10-d)(\dot{G} + G^2)Y_2,$$
$$R_{cdei}Y^{cde}{}_i = 2R_{eiei}Y_{eiei} = -2(\dot{H} + H^2)Y_1 + 2(d-1)H^2 Y_3 + 2(10-d)HGY_4,$$
$$R_{cdem}Y^{cde}{}_m = 2R_{emem}Y_{emem} = -2(\dot{G} + G^2)Y_2 + 2dHGY_4 + 2(9-d)G^2 Y_5. \tag{A26}$$

Finally, combining Eqs. (A7), (A25), and (A26), we obtain the $\Gamma$-dependent part in Eq. (8).

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