Physics Letters B 788 (2019) 480–485

Contents lists available at ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb

A possible signature of extra-dimensions: The enhanced open string pair production

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ARTICLE INFO

Article history:
Received 11 September 2018
Received in revised form 28 November 2018
Accepted 30 November 2018
Available online 3 December 2018
Editor: M. Cvetič

ABSTRACT

In analogy to the Schwinger pair production in QED, there exists also the so-called open string pair production for a system of two Dp branes, placed parallel at a separation, with at least one brane carrying a worldvolume electric flux, in Type II string theories. There is however no such pair production if an isolated Dp brane carrying an electric flux is considered. The produced open strings are directly related to the brane separation, therefore to the extra-dimensions if taken from the viewpoint of a brane observer. This pair production can be greatly enhanced if one Dp brane carries also a magnetic flux. The largest pair production rate occurs for $p = 3$, i.e., the D3 brane system, with the same applied fluxes. A detection of this pair production by the brane observer as the charged particle/anti-charged particle pair one may signal the existence of extra-dimensions and therefore provides a potential means to test the underlying string theories.

Whether there exist extra-dimensions remains an important yet unanswered question. String theory, as a candidate of quantum gravity, has built-in extra-dimensions and various non-perturbative solitonic extended objects such as Dp branes. One therefore expects string theory to provide means for exploring this question. In this work, we discuss a special type of enhanced open string pair production, directly related to the extra-dimensions from the viewpoint of a brane observer, for a system of two D3 branes placed parallel at a separation and carrying certain worldvolume electric and magnetic fluxes. If we take one of D3 branes as our own 4-dimensional world, the brane observer, just like ourselves, knows about string theories but the observer can only, if all possible, detect the ends, not the whole, of the open strings so produced as charged particle/anti-charged particle pairs, in a fashion similar to the Schwinger pair production [1], for example, by measuring the corresponding current in a laboratory setup within the brane. If this is indeed possible and the measurements against the applied electric and magnetic fluxes agree with the stringy prediction of the pair production rate, this then will have an implication on the existence of extra-dimensions, also a potential test of the underlying string theories.

A static D3 brane in Type IIB superstring theory, being 1/2 Bogomol'ny–Prasad–Sommereld (BPS) vacuum-like object, is stable. Its dynamics can also be described by a perturbative oriented open string with its two ends stick to the D3 brane along the transverse directions [2] when the string coupling is small. This open string is charge-neutral, having zero net-charge with its two ends carrying charge $+1$ and -1, respectively. Just like the virtual electron/positron pair in quantum electrodynamics (QED) vacuum, we have here the pair of virtual open string/anti open string, created from the present vacuum at some instant, existing for a short period of time, then annihilating to the vacuum. An observer on the brane can only sense the open string ends, not its whole, as virtual charged or anti-charged particles. So the pair of virtual open string/anti open string appears to the observer with one pair of their two nearby ends as the first pair of virtual charged particle/anti-charged particle and the other pair of their two other nearby ends as the second pair of virtual anti-charged particle/charged particle. So the quantum fluctuations from the perspective of brane observer are quite different from those of QED vacuum.

Just like the Schwinger pair production [1], one would also expect to produce the charged particle/anti-charged particle or the open string pairs if a constant worldvolume electric field is applied to an isolated D3 brane, depending on whether the observer is a brane one or a 10 dimensional bulk one. However, in a sharp contrast, the stringy computations give a null result due to the open strings being charge-neutral and their ends experiencing the same electric field [3,4]. This is consistent with that a D3 carrying a constant electric field is a 1/2 BPS non-threshold bound state [5], therefore being stable rather than unstable. The other way to un-

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https://doi.org/10.1016/j.physletb.2018.11.057

derstand this is as follows. To have the open string/anti open string pair detectable, they have to be separated infinitely away. However, this is impossible since either of them experiences a zero net-force under the action of the constant applied electric field less than its critical value. In other words, there is no Schwinger-type pair production here.

In order to have the pair production, a possibility is to let the two ends of the charge-neutral open string experience different electric fluxes. A simple setup for this is to consider two Dp branes placed parallel at a separation with each carrying a different electric flux (we consider a general p with $p = 3$ as a special case). The open string pair production should then come from those virtual open strings with each connecting the two Dp branes along their transverse directions, therefore directly related to the extra-dimensions from the perspective of the brane observer. Stringy computations do give a non-zero but usually vanishingly small rate for realistic electric fluxes applied [6], due to the large string scale $M_s = 1/\sqrt{\alpha'}$ whose current constraint is from a few TeV upto the order of $10^{16} \sim 10^{17}$ GeV [7]. This rate can however be greatly enhanced if at least one such Dp carries also a magnetic flux [8,4]. This enhancement makes it possible to detect the pair production and therefore to have the potential to address the question on the existence of extra-dimensions raised at the outset.

We now compute this rate with the respective worldvolume dimensionless flux $\hat{F}$ and $\hat{F}'$, both being antisymmetric $(p+1)\times(p+1)$ matrices with the same structure. For the wanted enhancement, the non-vanishing components for $\hat{F}$ can be chosen, without loss of generality, to be

$$\hat{F}_{01} = -\hat{F}_{10} = -f, \qquad \hat{F}_{23} = -\hat{F}_{32} = -g, \tag{1}$$

with the electric flux $|f| < 1$ and the magnetic flux $|g| < \infty$. We have the same for $\hat{F}'$ but denoting the corresponding fluxes each with a prime. This choice of fluxes implies $p \geq 3$. To have this rate, we first need to have the open string annulus interaction amplitude between the two Dp in its integral representation. This was given recently by the present author in [9] as,

$$\Gamma = \frac{2^2 V_{p+1}|f-f'||g-g'|}{(8\pi^2\alpha')^{\frac{1+p}{2}}} \times \int_0^\infty \frac{dt}{t^{\frac{p-1}{2}}} e^{-\frac{y^2 t}{2\pi\alpha'}} \frac{(\cosh\pi\nu_0' t - \cos\pi\nu_0 t)^2}{\sin\pi\nu_0 t \sinh\pi\nu_0' t} Z(t), \tag{2}$$

where

$$Z(t) = \prod_{n=1}^\infty \frac{\prod_{i=1}^2\left[1-2|z|^{2n}e^{(-)^i\pi\nu_0' t}\cos\pi\nu_0 t + |z|^{4n}e^{(-)^i 2\pi\nu_0' t}\right]^2}{(1-|z|^{2n})^4\left(1-2|z|^{2n}\cos 2\pi\nu_0 t+|z|^{4n}\right)\left(1-2|z|^{2n}\cosh 2\pi\nu_0' t+|z|^{4n}\right)}. \tag{3}$$

In the above, $|z| = e^{-\pi t} < 1$, y is the brane separation, α' the Regge slope parameter, and the electric parameter $\nu_0 \in [0, \infty)$ and the magnetic one $\nu_0' \in [0, 1]$ are determined by the electric fluxes and magnetic ones, respectively, as

$$\tanh\pi\nu_0 = \frac{|f-f'|}{1-ff'}, \quad \tan\pi\nu_0' = \frac{|g-g'|}{1+gg'}. \tag{4}$$

The integrand in (2) has an infinite number of simple poles along the positive t-axis at $t_k = k/\nu_0$ with $k = 1, 2, \cdots$, for which $\sin\pi\nu_0 t_k = 0$. These poles actually give rise to the decay of the underlying system via the so-called open string pair production. The non-perturbative decay rate or usually also called pair production rate can be computed as the sum of the residues of the integrand at these poles times π per unit worldvolume following [3] as

$$\mathcal{W} = \frac{8|f-f'||g-g'|}{(8\pi^2\alpha')^{\frac{1+p}{2}}} \times \sum_{k=1}^\infty (-)^{k-1}\left(\frac{\nu_0}{k}\right)^{\frac{p-3}{2}} \frac{\left[\cosh\frac{\pi k\nu_0'}{\nu_0} - (-)^k\right]^2}{k\sinh\frac{\pi k\nu_0'}{\nu_0}} e^{-\frac{k y^2}{2\pi\alpha'\nu_0}} Z(t_k), \tag{5}$$

where $Z(t_k)$, given by $Z(t)$ in (3) with $t = t_k = k/\nu_0$, takes its explicit expression as

$$Z(t_k) = \prod_{n=1}^\infty \frac{\left[1-(-)^k e^{-\frac{2nk\pi}{\nu_0}\left(1-\frac{\nu_0'}{2n}\right)}\right]^4 \left[1-(-)^k e^{-\frac{2nk\pi}{\nu_0}\left(1+\frac{\nu_0'}{2n}\right)}\right]^4}{\left(1-e^{-\frac{2nk\pi}{\nu_0}}\right)^6 \left[1-e^{-\frac{2nk\pi}{\nu_0}(1-\nu_0'/n)}\right]\left[1-e^{-\frac{2nk\pi}{\nu_0}(1+\nu_0'/n)}\right]}. \tag{6}$$

Note that the odd and even k in (5) give their respective positive and negative contributions to the rate. For given electric and magnetic fluxes, this rate is highly suppressed by the brane separation y and the integer k. We can qualitatively understand this by noting that the mass for each produced open string is $k T_f y$ with $T_f = 1/(2\pi\alpha')$ the fundamental string tension. So the larger k or y or both are, the larger the mass is and therefore the more difficult the open string can be produced. For $f \neq f'$, one can check that the larger f or f' is, the larger ν_0 and $|f-f'|$ are and the larger the rate $\mathcal{W}$ is.

In general, the presence of magnetic fluxes enhances this rate. We here consider two special cases to show explicitly this enhancement. The first is the case of $g = g' \neq 0$ and we have the enhancement from (5) and (4) as

$$\frac{\mathcal{W}_{g=g'\neq 0}}{\mathcal{W}_0} = 1 + g^2 > 1, \tag{7}$$

where the zero-magnetic flux rate is [6]

$$\mathcal{W}_0 = \frac{32\nu_0|f-f'|}{(8\pi^2\alpha')^{\frac{1+p}{2}}} \times \sum_{l=1}^\infty \frac{1}{(2l-1)^2}\left(\frac{\nu_0}{2l-1}\right)^{\frac{p-3}{2}} e^{-\frac{(2l-1)y^2}{2\pi\alpha'\nu_0}} \prod_{n=1}^\infty \left(\frac{1+e^{-\frac{2n(2l-1)\pi}{\nu_0}}}{1-e^{-\frac{2n(2l-1)\pi}{\nu_0}}}\right)^8. \tag{8}$$

A remark follows. From (8), one can check easily that $\mathcal{W}_0 = 0$ if we set identical f and f' (now $\nu_0 = 0$ from the first equation in (4)). This agrees with no Schwinger-type pair production of an isolated D3 brane carrying a constant electric flux mentioned earlier. So to have the expected pair production, we need to have a nearby D3 brane in the transverse directions, which may be invisible (hidden or dark) to our own D3 brane.

The second is the case of $\nu_0'/\nu_0 \gg 1$. This says $\nu_0 \ll 1$ since $\nu_0' \in (0, 1]$, implying $|f-f'| \ll 1$ from (4). For a fixed $\nu_0' \in (0, 1]$ and a very small ν_0, the rate (5) can be well approximated by its leading $k = 1$ term as

$$\mathcal{W} \approx \frac{4|f-f'||g-g'|}{(8\pi^2\alpha')^{\frac{1+p}{2}}} \nu_0^{\frac{p-3}{2}} e^{-\frac{y^2}{2\pi\alpha'\nu_0}} e^{\frac{\pi\nu_0'}{\nu_0}}. \tag{9}$$

The zero-magnetic flux rate (8) for the same small ν_0 is now

$$\mathcal{W}_0 \approx \frac{32\,\nu_0\,|f-f'|}{(8\pi^2\alpha')^{\frac{1+p}{2}}}\,\nu_0^{\frac{p-3}{2}}\,e^{-\frac{y^2}{2\pi\alpha'\nu_0}}. \tag{10}$$

The enhancement is then

$$\frac{\mathcal{W}}{\mathcal{W}_0} = \frac{|g-g'|}{8\,\nu_0}\,e^{\frac{\pi\nu_0'}{\nu_0}}, \tag{11}$$

which can be huge given that $\nu_0'/\nu_0 \gg 1$ and $\nu_0 \ll 1$. It has a value of 1.6×10^{35}, a very significant enhancement, for $\nu_0 = 0.02$ and $\nu_0' = 0.5$ for a moderate choice of $g = -g' = 1$, noting $g, g' \in (-\infty, \infty)$ for $p = 3$. Note that the rate for $p > 3$ from (9) is smaller than that for $p = 3$ by at least a factor of $(\nu_0/4\pi)^{1/2} \approx 0.04$ for the above sample case. One may wonder if further enhancement can be achieved when we add an extra magnetic flux with similar structure. For example, for $p = 5$, we add a flux $\hat{F}_{45} = -\hat{F}_{54} = -\tilde{g}$ in addition to those given in (1). It turns out that this diminishes rather than enhances the pair production rate. We will give a simple argument for this later on. The flux structure given in (1) actually gives the largest rate for each given $p \geq 3$ and moreover for the same applied fluxes the $p = 3$ rate is the largest among these $p \geq 3$. So this singles out the system of two D3 branes, therefore the 4-dimensional world. Curiously one of the D3 can be just our own world.

If we set now the dimensionless fluxes $f = 2\pi\alpha' qE$, $f' = 2\pi\alpha' qE'$, $g = 2\pi\alpha' qB$, $g' = 2\pi\alpha' qB'$ with E, E', B, B' the corresponding laboratory ones. The rate (5) for $p = 3$ now becomes

$$\mathcal{W} = \frac{2q^2|E-E'|\,|B-B'|}{(2\pi)^2} \times \sum_{k=1}^{\infty}(-)^{k-1}\frac{\left[\cosh\frac{\pi k\nu_0'}{\nu_0} - (-)^k\right]^2}{k\,\sinh\frac{\pi k\nu_0'}{\nu_0}}\,e^{-\frac{2k\pi\alpha' m^2}{\nu_0}}\,Z(t_k), \tag{12}$$

with $Z(t_k)$ given in (6). Note that for all $p \geq 3$, the string scale α' drops out, except for the exponential factor $\exp[-ky^2/(2\pi\alpha'\nu_0)]$, for the rate (5) in practice for which the fluxes f, f', g, g' are all very small (giving also very small ν_0 and ν_0' and so $Z(t_k) \approx 1$). If we define a scale, as for the above $p = 3$ case, $m = T_f\,y = y/(2\pi\alpha')$, the aforementioned exponential factor depends only on this scale and the α' also drops out. This can be checked easily.

In practice, we can apply electric and magnetic fluxes only to our own D3 brane and have no control over the other D3. This amounts to setting, for example, $E' = B' = 0$, in (12) (From now on, we will set $E' = B' = 0$.) We then have,

$$\mathcal{W} = \frac{2(qE)(qB)}{(2\pi)^2}\sum_{k=1}^{\infty}(-)^{k-1}\frac{\left[\cosh\frac{\pi k\nu_0'}{\nu_0} - (-)^k\right]^2}{k\,\sinh\frac{\pi k\nu_0'}{\nu_0}}\,e^{-\frac{2k\pi\alpha' m^2}{\nu_0}}\,Z(t_k). \tag{13}$$

According to [16], the above rate should be more properly interpreted as the decay rate of the underlying system while the pair production rate is just the leading $k = 1$ term in (13) since the higher k correspond to more massive open strings, not the fundamental one. So the open string pair production rate is

$$\mathcal{W}^{(1)} = \frac{2\,(qE)(qB)}{(2\pi)^2}\,\frac{\left[\cosh\frac{\pi\nu_0'}{\nu_0} + 1\right]^2}{\sinh\frac{\pi\nu_0'}{\nu_0}}\,e^{-\frac{2\pi\alpha' m^2}{\nu_0}}\,Z(t_1), \tag{14}$$

where $Z(t_1)$ is $Z(t_k)$ in (6) for $k = 1$. Note that we have now the parameters ν_0 and ν_0' determined, respectively, from (4) as

$$\tanh\pi\nu_0 = 2\pi\alpha' qE, \qquad \tan\pi\nu_0' = 2\pi\alpha' qB. \tag{15}$$

The above implies $\nu_0' \in [0, 1/2)$. Given the alternative sign appearing in the sum of (13), this rate looks more like the scalar QED one [1,10–13] than the spinor QED one [10,14,15]. We now come to compare the present rate (14) in the weak field limit with the QED spinor and scalar rates, respectively. In practice, $\alpha' qE \ll 1$ and $\alpha' qB \ll 1$. We have from (15) $\nu_0 = 2\alpha' qE \ll 1$, $\nu_0' = 2\alpha' qB \ll 1$ and $Z(t_k) \approx 1$. The rate (14) now becomes

$$\mathcal{W}^{(1)} = \frac{2(qE)(qB)}{(2\pi)^2}\,\frac{\left[\cosh\frac{\pi B}{E} + 1\right]^2}{\sinh\frac{\pi B}{E}}\,e^{-\frac{\pi m^2}{qE}}, \tag{16}$$

the spinor QED rate [16] is

$$\mathcal{W}^{(1)}_{\text{spinor}} = \frac{(qE)(qB)}{(2\pi)^2}\coth\left(\frac{\pi B}{E}\right)e^{-\frac{\pi m^2}{qE}}, \tag{17}$$

and the scalar QED one [16] is

$$\mathcal{W}^{(1)}_{\text{scalar}} = \frac{(qE)(qB)}{2(2\pi)^2}\text{csch}\left(\frac{\pi B}{E}\right)e^{-\frac{\pi m^2}{qE}}. \tag{18}$$

In order to make comparisons, we need to identify the present mass scale m with the corresponding one in QED. The present rate (16) has similarities with but also important differences from the other two rates. Let us focus first on the pure electric case. We have now

$$\mathcal{W}^{(1)} = 8\,\mathcal{W}^{(1)}_{\text{spinor}} = 16\,\mathcal{W}^{(1)}_{\text{scalar}} = \frac{8(qE)^2}{4\pi^3}\,e^{-\frac{\pi m^2}{qE}}. \tag{19}$$

The scalar QED rate is just half of the spinor QED one and this is due to the spinor factor $2S+1$ with S the corresponding spin. However, the present rate is 8 times of the spinor QED one and 16 times of the scalar QED one. These are due to the underlying lightest degrees of freedom for the produced open string in the weak field limit and we will come back to explain this later on.

When the magnetic flux is turned on, the present rate (16) is also always larger than the other two QED rates. This is evident since

$$\frac{\mathcal{W}^{(1)}}{\mathcal{W}^{(1)}_{\text{spinor}}} = \frac{2\left[1+\cosh\frac{\pi B}{E}\right]^2}{\cosh\frac{\pi B}{E}} > 1, \quad \text{or}$$
$$\frac{\mathcal{W}^{(1)}}{\mathcal{W}^{(1)}_{\text{scalar}}} = 4\left[1+\cosh\frac{qB}{E}\right]^2 > 1. \tag{20}$$

Especially when $B/E \gg 1$, the rate (16) is exponentially enhanced by the factor $\exp[\pi B/E]$ while the spinor rate (17) has no such enhancement and the scalar rate (18) on the contrast is exponentially suppressed by this factor. The exponential behavior given in (20) can be explained by the effective degrees of freedom for large B/E. Once again we will explain this later on.

We would like to stress that the above sharply different behavior between the rate (16) and the spinor rate (17) on magnetic flux lays a ground to distinguish the two when a detection of the underlying pair production becomes possible.

If the two D3 brane separation is due to, for example, the Standard model gauge symmetry breaking, the mass scale m should be naturally related to the W-boson mass. Let us see this and the previously unexplained issues in a bit detail in what follows.

For the system of two D3 branes under consideration, when there are no fluxes present, the mass level for the open string connecting the two D3 branes is

$$\alpha' M^2 = -\alpha' p^2 = \begin{cases} \frac{y^2}{4\pi^2\alpha'} + N_{\text{R}} & (\text{R}-\text{sector}), \\ \frac{y^2}{4\pi^2\alpha'} + N_{\text{NS}} - \frac{1}{2} & (\text{NS}-\text{sector}), \end{cases} \tag{21}$$

where $p=(k,0)$ with k the momentum along the brane worldvolume directions, $N_{\rm R}$ and $N_{\rm NS}$ are the standard number operators in the R-sector and NS-sector, respectively, as

$$N_{\rm R}=\sum_{n=1}^{\infty}(\alpha_{-n}\cdot\alpha_n+nd_{-n}\cdot d_n),$$
$$N_{\rm NS}=\sum_{n=1}^{\infty}\alpha_{-n}\cdot\alpha_n+\sum_{r=1/2}^{\infty}rd_{-r}\cdot d_r. \tag{22}$$

When $y=0$, the supersymmetric open string has 16 massless modes, which are eight bosons ($8_{\rm B}$) and eight fermions ($8_{\rm F}$), in addition to an infinite tower of massive modes. In terms of D3 brane worldvolume description, these 16 massless modes correspond to one of U(2) generators of the 4-dimensional $N=4$ super Yang–Mills, which is broken when $y\neq 0$. The massless $8_{\rm B}$ consists of a massless vector and 6 massless scalars in four dimensions when $y=0$ and gives a massive vector, the W-boson, plus 5 massive scalars when one of 6 massless scalars takes a non-vanishing vacuum expectation, giving $y\neq 0$. The $8_{\rm F}$ gives four 4-dimensional spinors. We have now massive $8_{\rm B}+8_{\rm F}$, all with the same mass $T_f\,y=y/(2\pi\alpha')$ as given in (21). This is consistent with the fact that the underlying system is still 1/2 BPS as before.

If we now consider turning on different electric field on the two D3 branes, this will give rise to the open string pair production. In practice, the electric parameter $\nu_0\ll 1$ and so $Z(t_1)\approx 1$, implying the contribution to the pair production almost all coming from the lowest 16 ($8_{\rm B}+8_{\rm F}$) massive charged degrees of freedom. In particular, each of these 16 degrees of freedom gives equal contribution to the rate no matter it is bosonic or fermionic. For a 4-dimensional massive charged particle with its spin S, we expect the pair production rate ratio of the open string with this charged particle to be

$$\frac{\mathcal{W}^{(1)}}{\mathcal{W}_{\rm QED}}=\frac{16}{2S+1}, \tag{23}$$

if the corresponding mass is the same. This gives an explanation to the factor 8 for QED spinor ($S=1/2$) and 16 for QED scalar ($S=0$) given in (19).

If we now consider instead to turn on only the magnetic fluxes, the energy in the R-sector is strictly positive and the R-sector vacuum does not give rise to a tachyonic shift, just like the QED case ($S=1/2, g_S=2$) [18,19]. In the so-called weak field limit, i.e., $\alpha' qB\ll 1$, this can also be understood easily from the following energy spectrum for a particle with spin S, charge q, gyromagnetic ratio g_S and mass m_S^2 in a constant magnetic field B (see [18], for example) as

$$E_N^2=(2N+1)qB-g_S qB\cdot S+m_S^2, \tag{24}$$

where $N\geq 0$ denotes the Landau level. So it is clear that $E_{\rm N}^2>0$ provided $g_S S\leq 1$. For charged open string states, $g_S=2$ [17]. The $8_{\rm F}$ from the R-sector vacuum gives 4 four-dimensional Majorana spinors, each of which has spin $S=1/2$. So we have $g_S=1/S$, the so-called "minimal coupling", and $E_N^2>0$ if $y\neq 0$. The NS-sector has a different story, however. The energy in the NS-sector, following [18,19], is now

$$\alpha' E_{\rm NS}^2=(2N+1)\frac{\nu_0'}{2}-\nu_0' S+\alpha' M_{\rm NS}^2 \tag{25}$$

where the magnetic parameter ν_0' is defined in (4), the mass $M_{\rm NS}$ is given in (21) in the NS-sector, the Landau level $N=b_0^+b_0$, and the spin operator in the 34-direction is

$$S=\sum_{n=1}^{\infty}(a_n^+a_n-b_n^+b_n)+\sum_{r=1/2}^{\infty}(d_r^+d_r-\tilde d_r^+\tilde d_r). \tag{26}$$

The only states which have the potential to be tachyonic [18], in particular for $y=0$, belong to the first Regge trajectory, given by

$$(a_1^+)^{\tilde n}d_{1/2}^+|0\rangle_{\rm NS}, \tag{27}$$

where $|0\rangle_{\rm NS}$ is the NS-sector vacuum, a tachyonic one projected out in the superstring case. For these states, we have

$$\alpha' E_{\rm NS}^2=-\frac{\nu_0'}{2}+(1-\nu_0')(S-1)+\frac{y^2}{4\pi^2\alpha'}, \tag{28}$$

where $S=\tilde n+1\geq 1$. In the absence of magnetic fluxes, the above gives the mass in the NS-sector as

$$\alpha' M_{\rm NS}^2=\frac{y^2}{4\pi^2\alpha'}+S-1, \tag{29}$$

which becomes massless for the spin $S=1$ state $d_{1/2}^+|0\rangle_{\rm NS}$ when $y=0$ and is massive for $y\neq 0$. In other words, the state $d_{1/2}^+|0\rangle_{\rm NS}$ gives rise to a tachyonic shift $-\nu_0'/2$ in the presence of magnetic flux.[1] A finite brane separation y for the spin $S=1$ state $d_{1/2}^+|0\rangle_{\rm NS}$ corresponds to the gauge symmetry breaking $U(2)\to U(1)\times U(1)$, with the W-boson mass $M_W=y/(2\pi\alpha')$, as mentioned earlier.

From (28) and (29), the lowest mass is for the spin $S=1$ state and the magnetic flux gives a tachyonic shift $-\nu_0'/2$ in the energy. This same tachyonic shift can also be seen from the integrand of the open string annulus amplitude (2) for the $t\to\infty$ limit, which gives a blowing-up factor $e^{-2\pi t[-\frac{\nu_0'}{2}+\frac{y^2}{4\pi^2\alpha'}]}$ for $y<\pi\sqrt{2\alpha'\nu_0'}$. It is clear that this is due to the appearance of a tachyon mode in the presence of the magnetic flux. Once this happens, we will have a phase transition via the so-called tachyon condensation [18]. If we take the weak field limit, i.e., $\alpha' qB\ll 1$, giving $\nu_0'\approx 2\alpha' qB\ll 1$, from (4) with $B'=0$, the corresponding instability on the D3 brane worldvolume is the Nielsen–Olesen one for the non-abelian gauge theory of the 4-dimensional $N=4$ U(2) super Yang–Mills [20].[2]

As mentioned earlier, we have $\tan\pi\nu_0'=|g|=2\pi\alpha' qB$ from (15) with $\nu_0'\in[0,1/2)$. To valid the computations of the annulus amplitude (2) and possibly the decay rate (5), we need to have $y\geq\pi\sqrt{2\alpha'\nu_0'}$, i.e., before the onset of tachyonic instability. If $y\geq y_c=\pi\sqrt{\alpha'}$, there is no tachyonic instability to occur no matter how strong the applied magnetic field B is [19]. This is quite

[1] Adding an additional magnetic flux, say, along 56-direction with the corresponding magnetic parameter ν_0'', without loss of generality assuming $\nu_0''\leq\nu_0'$, will actually reduce rather than increase the shift to $-(\nu_0'-\nu_0'')/2$. So this will diminish rather than increase the rate as mentioned earlier.

[2] We thank our anonymous referee for bringing this and the related to our attention. In the electroweak theory, the W-bosons each has $g_S=2$, implied by the "non-minimal" electromagnetic coupling of the W-bosons from the corresponding non-abelian gauge theory (now $g_S\neq 1/S$), and has the energy in a constant magnetic field, from (24), as $E_{N=0}^2=-qB+m_{\rm W}^2$. Here a tachyonic instability, called the Nielsen–Olsen instability [20], will be developed if $B>m_{\rm W}^2/q$ and the W and Z will increase with time. This will cause the W and Z to condensate and to give the gauge symmetry restoration [21,22]. For the system of two D3 under consideration, adding different magnetic flux to each brane will cause the system from preserving 1/2 spacetime supersymmetry to preserving no supersymmetry. As such, there is a net attractive force acting between the two D3, which can be seen from the annulus amplitude (2) if we turn off the electric flux (note that $\Gamma>0$ gives an attractive force for our conventions), therefore pulling the two D3 toward each other. Once the tachyonic instability develops, the attractive force blows up and the tachyon condensation occurs in addition to the existing open string pair production, making the two D3 quickly become coincidental and at the same time giving away the excess energy of the system, therefore restoring the $U(2)$ gauge symmetry on the brane and the supersymmetry of the system as well. It is nice to see a consistent picture from either description.

different from the Nielsen-Olsen instability for electroweak theory which occurs whenever the magnetic field exceeds m_W^2/q. We now come to compare the open string pair production rate with the corresponding W-boson pair rate in non-abelian gauge theory in the presence of the same applied electric and magnetic fields.

The W-boson pair production rate can be read from the general one for a massive vector given a while ago in [23] (taking $g = 2$, $\sigma = 0$ there) as,

$$\mathcal{W}^{(1)}_{\text{vector}} = \frac{(qE)(qB)}{2(2\pi)^2}\frac{2\cosh\frac{2\pi B}{E}+1}{\sinh\frac{\pi B}{E}}\,e^{-\frac{\pi m^2}{qE}}. \tag{30}$$

If we set the mass scale $m = T_f\, y = m_W$, the rate (14) and (30) are obviously different when ν_0 and ν'_0 are not small for the same E and B in the two cases. Let us compare the weak-field open string rate (16) with the W-boson rate (30). It is clear that the two rates with $m = m_W$ are still quite different for a general B/E, since

$$\frac{\mathcal{W}^{(1)}}{\mathcal{W}^{(1)}_{\text{w-boson}}} = \frac{4\left[\cosh\frac{\pi B}{E}+1\right]^2}{2\cosh\frac{2\pi B}{E}+1}, \tag{31}$$

which decreases from 16/3 for $B/E = 0$ to 1 when $B/E \to \infty$. The factor 16/3 can be easily understood from the reason given for (23) for $B = 0$ and $\nu_0 \ll 1$ for the weak electric field. This is due to that a massive vector has only 3 degrees of freedom while the string rate counts 16 degrees of freedom. Given what has been said, that the two rates become identical for large B/E can also be understood easily. For very small ν_0, the first simple pole $t_1 = 1/\nu_0 \gg 1$ gives already a very large t. As such, as discussed earlier, the open string rate is mostly due to its 16 lowest-mass modes. Among these 16 ones, only the spin $S = 1$, the W-boson, has a tachyonic shift[3] $-\nu'_0/2$ which will give a large enhancement to the rate. Using (24), we can see that the contributions from the 5 scalars, 4 spinors and one W-boson, respectively, among the 16 modes, in the weak field limit, should satisfy the following relation for large B/E as $5 : 4e^{\frac{\pi B}{E}} : e^{\frac{2\pi B}{E}}$. In other words, for large B/E, the open string rate in the weak field limit is just given by the spin S = 1 mode and this explains the limit of (31) for large B/E. By a similar token, this also explains the large B/E behavior of (20) for a single spinor and a single scalar, respectively.

In general, the weak-field open string rate (16) is still quite different from the W-boson rate (30) in terms of their dependence on the applied E and B. This feature can be used to distinguish the two or falsify either if a detection of the corresponding rate is possible. This same applies also to the charged scalar or the charged spinor case as discussed earlier. Given what has been described so far, we understand the physical reason for their distinction for each case discussed. It is due to different dynamical degrees of freedom involved for the respective rate.

If the weak-field open string rate (16) can indeed be confirmed by detection, i.e., the dependence of the rate on the applied E and B follows indeed (16), what lesson can we draw from this? The first implication should be the existence of supersymmetry since this rate is expected to be the same as the one for the 4-dimensional $N = 4$ U(2) super Yang–Mills via $U(2) \to U(1) \times U(1)$ (or the like). Only when the gauge group has its origin from the two D3 branes, we can have the interpretation of the rate detection as the existence of extra-dimension(s).

However, if the open string rate (14) can be detected by a brane observer, for example, the leading behavior of $Z(t_1)$ on ν_0 or ν'_0 or both with the applied E and B can be confirmed, this may indicate the existence of the extra-dimension(s) and provide a potential test of the underlying string theories.

In the absence of electric and magnetic fluxes E and B, the system of the two D3 branes under consideration remains as 1/2 BPS even if $y \neq 0$. In other words, the supersymmetry of the system remains unbroken even though its gauge symmetry breaks from $U(2) \to U(1) \times U(1)$ when the brane separation y changes from $y = 0 \to y \neq 0$. The consequence of this is that the 16 $(8_B + 8_F)$ massless modes when $y = 0$ become massive all with the same mass $m = y/(2\pi\alpha')$ when $y \neq 0$. All the other open string massive states have their masses shifted by the same amount of $y/(2\pi\alpha')$ as given in (21). So each of the supersymmetric charged degrees of freedom at each given mass level contributes the same amount to the pair production rate (14) in the presence of electric field E. In the weak-field limit, only the 16 charged degrees of freedom with the same lowest mass contribute to the rate (16). If a magnetic field B is also added, the rate is in general enhanced but the effective degrees of freedom will become less when we increase B. For large B/E, the weak-field rate is almost identical to the W-boson one as demonstrated earlier. Note that turning on the electric field E or magnetic field B or both breaks all the supersymmetries of the underlying system of the two D3 branes but the mass of each mass level of the open string remains unchanged, still given by (21). With this understanding plus the relation (23), we expect the following identify in the weak field limit if we identify the scalar mass, the spinor mass and the vector mass the same as the mass for the 16 stringy modes,

$$\mathcal{W}^{(1)} = 5\,\mathcal{W}^{(1)}_{\text{scalar}} + 4\,\mathcal{W}^{(1)}_{\text{spinor}} + \mathcal{W}^{(1)}_{\text{vector}}, \tag{32}$$

where the charged scalar rate $\mathcal{W}^{(1)}_{\text{scalar}}$, the charged spinor rate $\mathcal{W}^{(1)}_{\text{spinor}}$ and the charged vector rate $\mathcal{W}^{(1)}_{\text{vector}}$ are given in (18), (17) and (30), respectively. One can check that this holds indeed. It is very satisfying to see this given that each of these rates is computed complete independently and using different approaches. We expect that the above feature of the rate will hold true for certain supersymmetric extensions of Standard model and may be used to test the existence of supersymmetry as mentioned earlier.

The above discussion provides also a guidance for us to look for a possible test of the open string pair production rate (14) or (16). The supersymmetry is broken by the electromagnetic fluxes added but is preserved (or in general at least partially preserved) during the spontaneously gauge symmetry breaking. Given what we know about the current status of supersymmetry, its breaking scale m should be at least on the order of TeV. So it is hard to perform such a test in an earthbound laboratory. So the hope is from our Universe at its relatively early stage, for example, after its inflation and before electroweak symmetry breaking, or a slight possibility from LHC. The benefit of such a test, if feasible, is that the supersymmetry scale m may be close to the string scale M_s and then the stringy rate (14) can be tested. If confirmed, its implication is the existence of extra-dimension(s) and a test of underlying string theories.

If we want to test against the Standard model, we have two choices here. In the first case, we identify the massive vector as the Standard model W-boson, i.e., with the mass scale $m = y/(2\pi\alpha') = m_W \sim 80$GeV. To put the actual test in practice, we need either $qE_W \sim m_W^2 \sim 6.4\times 10^3\,\text{GeV}^2$ or[4] $qB \le qB_W \sim m_W^2/\pi \sim 2.0\times 10^3\,\text{GeV}^2$. For either case, the weak field condition can be satisfied since $\alpha' qE$ or $\alpha' qB \sim (m_W/M_s)^2 < 10^{-4} \ll 1$ if the smallest

[3] To be precise, for W^+-boson, the tachyonic shift comes from the spin-down mode while for W^--boson, this shift comes from the spin-up mode.

[4] To validate the computation of rate (14) or (16), we need to avoid the onset of tachyonic instability which requires $qB \le m^2/\pi$ with $m = m_W$.

string scale M_s is a few TeV [7]. In the former case, we need only a small tunable B so that the dependence of the rate (16) on the applied B and $E \sim E_W$ can be verified against the W-boson rate (30). In the latter case, we need to have a small tunable E such that $qE \sim m_W^2 - \pi qB$ with $B \leq B_W$ to verify the rate. We expect the former case to be more contrast than the latter one against the W-boson rate (30).

To perform such a test, we need from the above $E_W \sim 10^{28}\, V/m$ and $B_W \sim 10^{19}$ Tesla, each of which far exceeds the current earth-bound laboratory limit for a constant electric/magnetic field. For example, the current laboratory limit for a constant electric field is $\sim 10^{10}\, V/m$, which is 18 orders of magnitude smaller than the E_W. Further, we need to have tunable E and B in a controllable way. This makes LHC and the early universe environment, which can produce large electric and magnetic fields needed, un-suitable for this purpose. All we can say here is: If the W-boson rate (30) can be tested, so can the weak-field open string rate (16) since the latter is in general larger than the former for the same E and B.

The second choice is that if we interpret the other nearby D3 as invisible (hidden or dark) to our own D3, we usually don't have a priori knowledge of the mass scale m for the rate (16). If it happens to be on the order of electron mass, we then can test this rate with a tunable magnetic flux against the Schwinger pair production when the latter detection becomes feasible which requires $E \sim 10^{18}\,\mathrm{V/m}$. The large enhancement with the presence of magnetic flux for our rate (16) can loosen this large field requirement to certain extent and may set such a detection sooner rather than later. This can be even more true if the scale is smaller than that of electron mass.

For either of these two choices, we have to admit that there are serious issues remained unanswered in the present simple setup. In the first case, for example, the charged fermions and scalars all have mass $\sim m_W \sim 80\,\mathrm{GeV}$, due to the underlying supersymmetry, if we take the W-boson here as the one in Standard model. In the second case, the other charged fermions except for the one identified with the electron, charged scalars and the vector all have their mass $\sim m_e \sim 0.51\,\mathrm{MeV}$. They are certainly not the Standard model particles. Resolving this may need to consider more sophisticated setup rather than the present simple one. Supersymmetry breaking may also need to be considered properly. Addressing either of these is beyond the scope of the present paper.

Given the simple setup considered in this paper, we may be able to test the enhanced pair production rate in some condensed matter systems where analog situation or analog supersymmetry can be realized.

Given what has been said, if we assume various issues mentioned above can be resolved and if a detection of the rate (16) from the perspective of a brane observer is indeed possible and the behavior of the rate against the applied tunable electric and magnetic fluxes is confirmed, the first implication shall be the existence of supersymmetry. If we further assume that the gauge symmetry has its origin from D3 branes, the rate confirmation will imply the existence of extra dimension(s) and moreover this also gives a possible test of the underlying string theories.

Acknowledgements

The author would like to express his sincere thanks to the anonymous referee for various questions raised which help to sharpen his understanding of the present work. He would like also to thank Daoneng Gao for fruitful discussion and acknowledges support by grants from the NSF of China with Grant No: 11235010 and 11775212.

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