

Resonant state $D_0^*(2400)$ in the quasi-two-body B meson decaysWen-Fei Wang ^{a,b,*}^a Institute of Theoretical Physics, Shanxi University, Taiyuan, Shanxi 030006, China^b State Key Laboratory of Quantum Optics and Quantum Optics Devices, Shanxi University, Taiyuan, Shanxi 030006, China

ARTICLE INFO

Article history:

Received 14 November 2018

Received in revised form 23 November 2018

Accepted 30 November 2018

Available online 3 December 2018

Editor: J. Hisano

ABSTRACT

We study four quasi-two-body decay processes including $D_0^*(2400)$ as the intermediate state in the perturbative QCD (PQCD) approach. The branching fraction predicted in this work for the decay mode $B^- \rightarrow D_0^*(2400)^0 \pi^- \rightarrow D^+ \pi^- \pi^-$ agree with the data from Belle, BaBar and LHCb Collaborations. The PQCD prediction of the branching ratio for the decay $\bar{B}^0 \rightarrow D_0^{*+} K^- \rightarrow D^0 \pi^+ K^-$ is consistent with the value given by LHCb. For the decays $\bar{B}^0 \rightarrow D_0^{*+} \pi^- \rightarrow D^0 \pi^+ \pi^-$ and $B^- \rightarrow D_0^{*0} K^- \rightarrow D^+ \pi^- K^-$, the PQCD predicted branch ratios are $2.85_{-0.80}^{+1.23}(\omega_B)_{-0.81}^{+1.05}(\omega_{D\pi})_{-0.31}^{+0.33}(a_{D\pi})_{-0.05}^{+0.06}(\Gamma_{D_0^{*+}}) \times 10^{-4}$ and $4.65_{-1.30}^{+1.89}(\omega_B)_{-1.24}^{+1.51}(\omega_{D\pi})_{-0.38}^{+0.40}(a_{D\pi})_{-0.18}^{+0.22}(\Gamma_{D_0^{*0}}) \times 10^{-5}$, respectively. We analyze the experimental branching fractions using the ratios $R_{D_0^{*0}}$ and $R_{D_0^{*+}}$ which are related to the decays with the neutral and charged $D_0^*(2400)$, respectively. The available experimental results for the quasi-two-body decays including $D_0^*(2400)$ are not in agreement with the isospin relation and $SU(3)$ flavor symmetry.

© 2018 The Author(s). Published by Elsevier B.V. This is an open access article under the CC BY license (<http://creativecommons.org/licenses/by/4.0/>). Funded by SCOAP³.

Many excited open-charm states have been discovered by various experiments in recent years, see Ref. [1] for a review. One of them, the p -wave orbitally excited state $D_0^*(2400)$, with the light degree of freedom $j_q^p = \frac{1}{2}^+$ [2–4] and quantum number $J^P = 0^+$ [5], was first discovered by Belle Collaboration in the three-body decays $B^- \rightarrow D^+ \pi^- \pi^-$, with the mass $m_{D_0^{*0}} = 2308 \pm 17 \pm 15 \pm 28$ MeV and width $\Gamma_{D_0^{*0}} = 276 \pm 21 \pm 18 \pm 60$ MeV [6]. For simplicity, we adopt D_0^* to denote the $D_0^*(2400)$ state and the inclusion of charge-conjugate processes is implied throughout this work. The neutral resonant state D_0^{*0} has been confirmed by BaBar Collaboration in the same decay processes in [7], with the close but preciser values for its mass and width. While in the wideband photoproduction experiment, differ from those three-body decay processes, FOCUS Collaboration provided quite different values for the broad structure D_0^{*0} in [8]. One has $m_{D_0^{*0}} = 2407 \pm 21 \pm 35$ MeV and $\Gamma_{D_0^{*0}} = 240 \pm 55 \pm 59$ MeV in company with $m_{D_0^{*+}} = 2403 \pm 14 \pm 35$ MeV and $\Gamma_{D_0^{*+}} = 283 \pm 24 \pm 34$ MeV for a charged state D_0^{*+} from Ref. [8].

Unlike the charmed-strange state $D_{s0}^*(2317)$ [9–11], which lies just below DK threshold and mainly decays into the isospin break-

ing channel $D_s \pi$, the state D_0^* is expected to decay rapidly through the s -wave pion emission, the conservation of its angular momentum implies this resonance primarily couple to $D\pi$ and has a broad decay width [12] as revealed by experiments. While the discrepancy of its properties between the experimental results [6,7] and the predictions from the quark model [13,14] has triggered many studies on its true nature. The strong decays, radiative decays and/or the spectra have been studied extensively in Refs. [3, 4, 15–21] to explore the exact feature of the resonant state D_0^* . In Refs. [22,23], the possible four-quark structure of D_0^* was investigated, the authors pointed out that the four-quark structure is acceptable for the resonant state observed by Belle [6] and BaBar [7], but not for the cases observed by FOCUS [8]. While in Refs. [24–26], it was claimed that there exist two poles in D_0^* energy region. And a pole near the $D\pi$ threshold was obtained from lattice QCD in [27], which was said to share the similarities with the experimental resonance D_0^* . The resonant state D_0^* has also been explained as a mixture of two- and four-quark state [28] or the bound state of $D\pi$ [29].

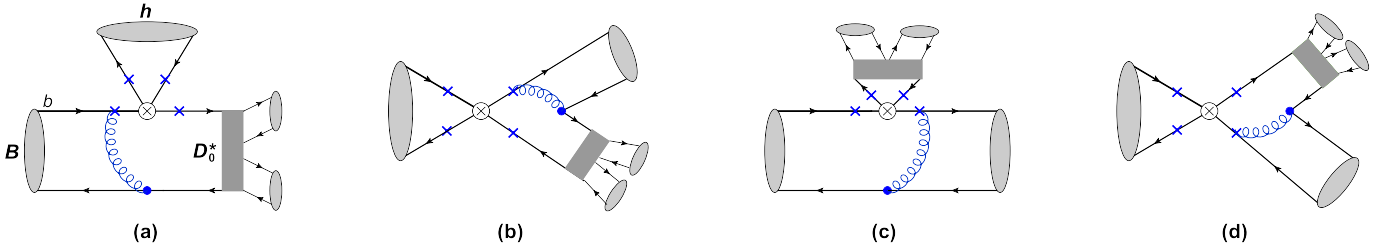
Semileptonic or hadronic B meson decays including a resonant state D_0^* shall yield clues to its properties. Employing constituent quark model or the light-cone sum rules to evaluate the $B \rightarrow D_0^*$ transition form factors, the decays of $B \rightarrow D_0^* l \nu$ have been studied in Refs. [30,31]. With the help of a chiral unitarity model, the ratio between the decay widths of $\bar{B}_s^0 \rightarrow D_{s0}^{*+}(2317) l \bar{\nu}_l$ and $\bar{B}^0 \rightarrow D_0^{*+} l \bar{\nu}_l$ was calculated in [32]. The model independent stud-

* Correspondence to: Institute of Theoretical Physics, Shanxi University, Taiyuan, Shanxi 030006, China.

E-mail address: wfwang@sxu.edu.cn.

Table 1Data for the quasi-two-body hadronic B meson decays including the D_0^* as the resonant state.

Mode	Unit	Branching fraction	Ref.
$B^- \rightarrow D_0^{*0} \pi^- \rightarrow D^+ \pi^- \pi^-$	(10^{-4})	$6.1 \pm 0.6 \pm 0.9 \pm 1.6$	Belle [6]
	(10^{-4})	$6.8 \pm 0.3 \pm 0.4 \pm 2.0$	BaBar [7]
	(10^{-4})	$5.78 \pm 0.08 \pm 0.06 \pm 0.09 \pm 0.39^1$	LHCb [42]
$\bar{B}^0 \rightarrow D_0^{*+} \pi^- \rightarrow D^0 \pi^+ \pi^-$	(10^{-4})	$0.60 \pm 0.13 \pm 0.15 \pm 0.22$	Belle [43]
	(10^{-5})	$7.7 \pm 0.5 \pm 0.3 \pm 0.3 \pm 0.4^2$	LHCb [44]
	(10^{-5})	$8.0 \pm 0.5 \pm 0.8 \pm 0.4 \pm 0.4^3$	LHCb [44]
$B^- \rightarrow D_0^{*0} K^- \rightarrow D^+ \pi^- K^-$	(10^{-6})	$6.1 \pm 1.9 \pm 0.5 \pm 1.4 \pm 0.4$	LHCb [45]
$B^0 \rightarrow D_0^{*+} K^+ \rightarrow \bar{D}^0 \pi^- K^+$	(10^{-5})	$1.77 \pm 0.26 \pm 0.19 \pm 0.67 \pm 0.20$	LHCb [46]

¹ Total S -wave $D^+ \pi^-$ contribution.² Isobar model.³ K -matrix model.**Fig. 1.** Typical Feynman diagrams for the quasi-two-body decays $B \rightarrow D_0^* h \rightarrow D \pi h$, $h = (\pi, K)$. The symbol \otimes stands for the weak vertex, \times denotes possible attachments of hard gluons, and the grey rectangle represents the intermediate states D_0^* .

ies of $B \rightarrow D^{**} l \nu$ have been performed within the standard model in [33] and beyond the standard model in [34] based on heavy quark symmetry. In [35], the branching fraction for the semileptonic decay $B \rightarrow D_0^* l \nu$ was predicted assuming the conventional quark-antiquark configuration for D_0^* state. And the hadronic matrix elements were evaluated in the Bethe–Salpeter approach for $B_c \rightarrow D_0^* \mu \bar{\mu}$ decays in [36]. Since Belle’s announcement [6], many works have been done on two-body hadronic B meson decays including the state D_0^* . For example, within the covariant light-front approach, the branching ratio of $B^- \rightarrow D_0^{*0} \pi^-$ was predicted to be 7.3×10^{-4} in [37]. In [38], the information on the Isgur–Wise functions at zero recoil was extracted from the $\bar{B}^0 \rightarrow D_0^{*+} \pi^-$ decay process. Using the improved version of the Isgur–Scora–Grinstein–Wise quark model for the $B \rightarrow D_0^*$ transition form factors, the decays $B^- \rightarrow D_0^{*0} \pi^-$ and $B^0 \rightarrow D_0^{*+} \pi^-$ have their branching ratios as 7.7×10^{-4} and 2.6×10^{-4} , respectively, in Ref. [39]. Two-body decays $B \rightarrow D_0^* \pi$ have also been discussed within factorization framework [40] and the perturbative QCD (PQCD) approach [41].

In Table 1, we have the data for the four quasi-two-body hadronic B meson decay modes including D_0^* from Belle, BaBar and LHCb Collaborations. In the processes $B \rightarrow D_0^* h \rightarrow D \pi h$, where h is a charged pion or kaon, the weak interaction point accompany the birth of the bachelor particle h , the intermediate state $D_0^{*0(+)}$ generated from the hadronization of c -quark plus $\bar{u}(\bar{d})$ -quark as demonstrated in Fig. 1. We stress that the resonance D_0^* is not necessary to be conventional quart-antiquark structure. In this letter, we shall analyze those four decay processes in a quasi-two-body framework based on the PQCD factorization approach [47–50]. In Refs. [51–55], the PQCD approach has been employed in the studies of the three-body B meson decays. With the help of the two-pion distribution amplitudes [56–58] and the experimental inputs for the time-like pion form factors, in Ref. [59], we calculated the decays $B \rightarrow K \rho(770)$, $K \rho'(1450) \rightarrow K \pi \pi$ in the quasi-two-body framework. The method used in [59] have been adopted for some other quasi-two-body B decays in Refs. [60–62]. In this work, we extend the previous studies to the $\bar{B}^0 \rightarrow D_0^{*+} h^- \rightarrow D^0 \pi^+ h^-$ and $B^- \rightarrow D_0^{*0} h^- \rightarrow D^+ \pi^- h^-$ decays.

Refer to the $K\pi$ system in Refs. [63,64], we define the scalar form factor $F_0^{D\pi}(s)$ for the final state $D^+ \pi^-$ decays from D_0^{*0} as

$$\langle D^+ \pi^- | \bar{c} u | 0 \rangle = \sqrt{2} B_0 F_0^{D\pi}(s), \quad (1)$$

with the constant

$$B_0 = \frac{m_D^2 - m_\pi^2}{\sqrt{2}(m_c - m_u)} \approx 1.93 \text{ GeV}, \quad (2)$$

where the $m_D(m_\pi)$ is the mass of $D(\pi)$ meson, the $m_c = 1.275 \text{ GeV}$ and $m_u = 2.2 \text{ MeV}$ (which could be neglected safely) for the mass of c and u quarks are adopted from [5]. Then we have

$$\begin{aligned} \langle D^+ \pi^- | \bar{c} u | 0 \rangle &\approx \langle D^+ \pi^- | D_0^{*0} \rangle \frac{1}{\mathcal{D}_{\text{BW}}} \langle D_0^{*0} | \bar{c} u | 0 \rangle \\ &= \Pi_{D_0^* D\pi}^{\text{BW}} \langle D_0^{*0} | \bar{c} u | 0 \rangle, \end{aligned} \quad (3)$$

and

$$\Pi_{D_0^* D\pi}^{\text{BW}} = \frac{g_{D_0^* D\pi}}{\mathcal{D}_{\text{BW}}} = \frac{\sqrt{2} B_0 F_0^{D\pi}(s)}{\langle D_0^{*0} | \bar{c} u | 0 \rangle} = \frac{\sqrt{2} B_0}{\bar{f}_{D_0^*} m_0} F_0^{D\pi}(s), \quad (4)$$

with $\bar{f}_{D_0^*} = \frac{m_0}{m_c(\mu) - m_u(\mu)} \cdot f_{D_0^*}$, and $f_{D_0^*}$ is the decay constant of D_0^* . One has different values from 78 MeV [65] to 148_{-46}^{+40} MeV [40] in different works for this decay constant, see [37,40,41,65–68], we support the moderate one $f_{D_0^*} = 0.13 \text{ GeV}$ which was adopted in the PQCD approach in [41]. The denominator $\mathcal{D}_{\text{BW}} = m_0^2 - s - im_0 \Gamma(s)$, the mass-dependent decay width $\Gamma(s)$ has its definition as $\Gamma(s) = \Gamma_0 \frac{q}{q_0} \frac{m_0}{\sqrt{s}}$, m_0 and Γ_0 are the pole mass and width of the resonant state D_0^* and s is the invariant mass square for the $D\pi$ pair in the final state. In the rest frame of the resonance D_0^* , its daughter D^+ or π^- has the magnitude of the momentum as

$$q = \frac{1}{2} \sqrt{[s - (m_D + m_\pi)^2][s - (m_D - m_\pi)^2]} / s, \quad (5)$$

and q_0 is the value of q at $s = m_0^2$. The coupling constant $g_{D_0^* D\pi}$ has its value from the relation [32,39]

$$g_{D_0^* D\pi} = \sqrt{\frac{8\pi m_0^2 \Gamma_0}{q_0}}. \quad (6)$$

We define

$$F_{D\pi}(s) = \frac{m_0^2}{m_0^2 - s - im_0 \Gamma(s)}, \quad (7)$$

then we have $F_{D\pi}^{D\pi}(s) = C_{D\pi} \cdot F_{D\pi}(s)$, with the parameter

$$C_{D\pi} = \frac{g_{D_0^* D\pi} \bar{f}_{D_0^*}}{\sqrt{2} B_0 m_0} \quad (8)$$

In the rest frame of the B meson, with m_B being its mass, we define the momentum $p = \frac{m_B}{\sqrt{2}}(1, \eta, 0)$ in the light-cone coordinates for the resonant state D_0^* and the $D\pi$ pair coming out from the resonance. Its easy to see $\eta = s/m_B^2$ with $s = p^2$. The light spectator quark comes from B meson and goes into D_0^* in the hadronization processes in Fig. 1 (a) got the momentum $k = (\frac{m_B}{\sqrt{2}}z, 0, k_T)$, z is the momentum fraction. The momenta p_B, p_3, k_B and k_3 for the B meson, bachelor meson h and the associated spectator quarks for B and h have their definitions as

$$p_B = \frac{m_B}{\sqrt{2}}(1, 1, 0_T), \quad p_3 = \frac{m_B}{\sqrt{2}}(0, 1 - \eta, 0_T),$$

$$k_B = \left(0, \frac{m_B}{\sqrt{2}}x_B, k_{BT}\right), \quad k_3 = \left(0, \frac{m_B}{\sqrt{2}}(1 - \eta)x_3, k_{3T}\right), \quad (9)$$

where x_B and x_3 are the corresponding momentum fractions.

The S -wave $D\pi$ system distribution amplitude could be collected into [41,54,59]

$$\Phi_{D\pi}^{S\text{-wave}} = \frac{1}{\sqrt{2}N_c} (\not{p} + \sqrt{s}) C_{D\pi} \phi_{D\pi}(z, b, s), \quad (10)$$

and the distribution amplitude

$$\phi_{D\pi}(z, b, s) = \frac{F_{D\pi}(s)}{2\sqrt{2}N_c} \left\{ 6z(1-z) \times \left[\frac{m_c(s) - m_u(s)}{\sqrt{s}} + a_{D\pi}(1-2z) \right] \right\} \exp(-\omega_{D\pi}^2 b^2/2), \quad (11)$$

the $a_{D\pi} = 0.40 \pm 0.10$ and $\omega_{D\pi} = 0.40 \pm 0.10$ GeV are adopted in the calculation in this work by catering to our numerical results to the data in Table 1 and considering the related parameters for the D and D^* mesons in the literature. The numbers for $\omega_{D\pi}$ and $a_{D\pi}$ in Ref. [41] have been considered as the references in this work, but we don't use the same values because of the different framework of the two-body and quasi-two-body decays and the different definitions of the distribution amplitudes. The distribution amplitudes for the pion, kaon and B meson are the same as those widely adopted in the PQCD approach to hadronic B meson decays, one can find their expressions and the relevant parameters in Ref. [69].

The decay amplitude \mathcal{A} for the quasi-two-body decay processes $\bar{B}^0 \rightarrow D_0^{*+} h^- \rightarrow D^0 \pi^+ h^-$ and $B^- \rightarrow D_0^{*0} h^- \rightarrow D^+ \pi^- h^-$ in the PQCD approach is given by [51,52,59]

$$\mathcal{A} = \phi_B \otimes H \otimes \phi_h \otimes \phi_{D\pi}, \quad (12)$$

where the symbol \otimes means convolutions in parton momenta, the hard kernel H contains only one hard gluon exchange at leading order in the strong coupling α_s as in the two-body formalism and the distribution amplitude ϕ_B ($\phi_h, \phi_{D\pi}$) absorbs nonperturbative dynamics in the decay processes. We then have the differential branching fraction (\mathcal{B}) [5]

$$\frac{d\mathcal{B}}{d\eta} = \tau_B \frac{q_h q B_0^2 C_{D\pi}^2}{32\pi^3 m_B m_0^2} \overline{|\mathcal{A}|^2}, \quad (13)$$

τ_B being the B meson mean lifetime, the magnitude momentum for bachelor h , in the center-of-mass frame of the $D\pi$ pair, as

$$q_h = \frac{1}{2} \sqrt{[(m_B^2 - m_h^2)^2 - 2(m_B^2 + m_h^2)s + s^2]}/s. \quad (14)$$

The m_h is the mass of the bachelor meson pion or kaon.

In the numerical calculation, we adopt $\Lambda_{\overline{MS}}^{(f=4)} = 0.25$ GeV. The decay constant $f_B = 0.19$ GeV for B meson comes from lattice QCD [70]. The masses and the mean lifetimes for the neutral and charged B meson, the pole masses and the widths of the neutral and charged D_0^* state, the Wolfenstein parameters, the masses of pion, kaon and D meson are all come from the Particle Data Group [5]. Utilizing the differential branching fraction Eq. (13) and the decay amplitudes collected in Appendix A, we obtain the branching fractions in Table 2 for the concerned quasi-two-body decay processes. The shape parameter uncertainty of the B meson, $\omega_B = 0.40 \pm 0.04$ GeV, contributes the largest error for the branching fractions in Table 2, the $\omega_{D\pi} = 0.40 \pm 0.10$ GeV for $D\pi$ system takes the second place and the $a_{D\pi} = 0.40 \pm 0.10$ in the Eq. (11) generates the third one. For the decay width of the resonance D_0^* , the charged state got $\Gamma_{D_0^{*+}} = 230 \pm 17$ MeV and neutral one has $\Gamma_{D_0^{*0}} = 267 \pm 40$ MeV [5], then we have the quite different weight of the error from decay width for those processes including charged and neutral D_0^* state, as shown in Table 2. There are other errors, which come from the uncertainties of the parameters in the distribution amplitudes for bachelor pion(kaon) [69] and the Wolfenstein parameters [5], are small and have been neglected.

The distributions of those four branching ratios in Table 2 in the $D\pi$ pair invariant mass $m_{D\pi}$ are shown in Fig. 2, with the curves for $B^- \rightarrow D_0^{*0} \pi^- \rightarrow D^+ \pi^- \pi^-$ (the dash line) and $\bar{B}^0 \rightarrow D_0^{*+} \pi^- \rightarrow D^0 \pi^+ \pi^-$ (the solid line) on the left, and the curves for $B^- \rightarrow D_0^{*0} K^- \rightarrow D^+ \pi^- K^-$ (the dash line) and $\bar{B}^0 \rightarrow D_0^{*+} K^- \rightarrow D^0 \pi^+ K^-$ (the solid line) at the right. The small mass difference of the charged and the neutral D_0^* exhibit the different peaks of the $m_{D\pi}$ dependence for the different decay modes. The main portion of each branching ratio lies obviously in the region around the pole mass of the resonant state D_0^* in the Fig. 2, the contributions from the energy region $m_{D\pi} > 3$ GeV can be safely omitted.

Assuming the D_0^* state decays essentially into two-body modes, from the Clebsch-Gordan coefficients, we have $\mathcal{B}(D_0^{*0} \rightarrow D^+ \pi^-) = \mathcal{B}(D_0^{*+} \rightarrow D^0 \pi^+) = \frac{2}{3}$. Then we have the two-body results as $\mathcal{B}(B^- \rightarrow D_0^{*0} \pi^-) = 8.93_{-3.48}^{+4.71} \times 10^{-4}$ and $\mathcal{B}(\bar{B}^0 \rightarrow D_0^{*+} \pi^-) = 4.28_{-1.77}^{+2.48} \times 10^{-4}$ from Table 2. The two-body value for the $B^- \rightarrow D_0^{*0} \pi^-$ decay agree well with the results in Refs. [37–39,41]. The result $4.28_{-1.77}^{+2.48} \times 10^{-4}$ for the decay $B^- \rightarrow D_0^{*0} \pi^-$ is consistent with the results 2.6×10^{-4} in [39] and 3.1×10^{-4} in [40] within errors, but it's smaller than the corresponding results in Ref. [41].

Compare our numerical results in Table 2 with the corresponding data in Table 1, we find that the PQCD value of the branching fraction for the quasi-two-body decay process $B^- \rightarrow D_0^{*0} \pi^- \rightarrow D^+ \pi^- \pi^-$ in this work agree well with the values $(6.1 \pm 0.6 \pm 0.9 \pm 1.6) \times 10^{-4}$ taken from Belle [6] and $(6.8 \pm 0.3 \pm 0.4 \pm 2.0) \times 10^{-4}$ picked up from BaBar [7]. In Ref. [42], LHCb Collaboration presented a result $(5.78 \pm 0.08 \pm 0.06 \pm 0.09 \pm 0.39) \times 10^{-4}$ for the total S -wave $D\pi$ system, which should be supposed to mainly contributed by the D_0^* state, in the $B^- \rightarrow D^+ \pi^- \pi^-$ decays. For the decay $\bar{B}^0 \rightarrow D_0^{*+} K^- \rightarrow D^0 \pi^+ K^-$, the result $2.38_{-0.65-0.68-0.28-0.03}^{+0.95+0.85+0.30+0.04} \times 10^{-5}$ in Table 2 is consistent with the data $(1.77 \pm 0.26 \pm 0.19 \pm 0.67 \pm 0.20) \times 10^{-5}$ given by

Table 2

PQCD predictions for the concerned quasi-two-body decays including the D_0^* as the intermediate state.

Mode	Unit	Branching fraction
$B^- \rightarrow D_0^{*0} \pi^- \rightarrow D^+ \pi^- \pi^-$	(10^{-4})	$5.95_{-1.64}^{+2.37} (\omega_B)^{+1.97}_{-1.55} (\omega_{D\pi})^{+0.54}_{-0.49} (a_{D\pi})^{+0.29}_{-0.21} (\Gamma_{D_0^{*0}})$
$\bar{B}^0 \rightarrow D_0^{*+} \pi^- \rightarrow D^0 \pi^+ \pi^-$	(10^{-4})	$2.85_{-0.83}^{+1.23} (\omega_B)^{+1.05}_{-0.81} (\omega_{D\pi})^{+0.33}_{-0.31} (a_{D\pi})^{+0.06}_{-0.05} (\Gamma_{D_0^{*+}})$
$B^- \rightarrow D_0^{*0} K^- \rightarrow D^+ \pi^- K^-$	(10^{-5})	$4.65_{-1.30}^{+1.89} (\omega_B)^{+1.51}_{-1.24} (\omega_{D\pi})^{+0.40}_{-0.38} (a_{D\pi})^{+0.22}_{-0.18} (\Gamma_{D_0^{*0}})$
$\bar{B}^0 \rightarrow D_0^{*+} K^- \rightarrow D^0 \pi^+ K^-$	(10^{-5})	$2.38_{-0.65}^{+0.95} (\omega_B)^{+0.85}_{-0.68} (\omega_{D\pi})^{+0.30}_{-0.28} (a_{D\pi})^{+0.04}_{-0.03} (\Gamma_{D_0^{*+}})$

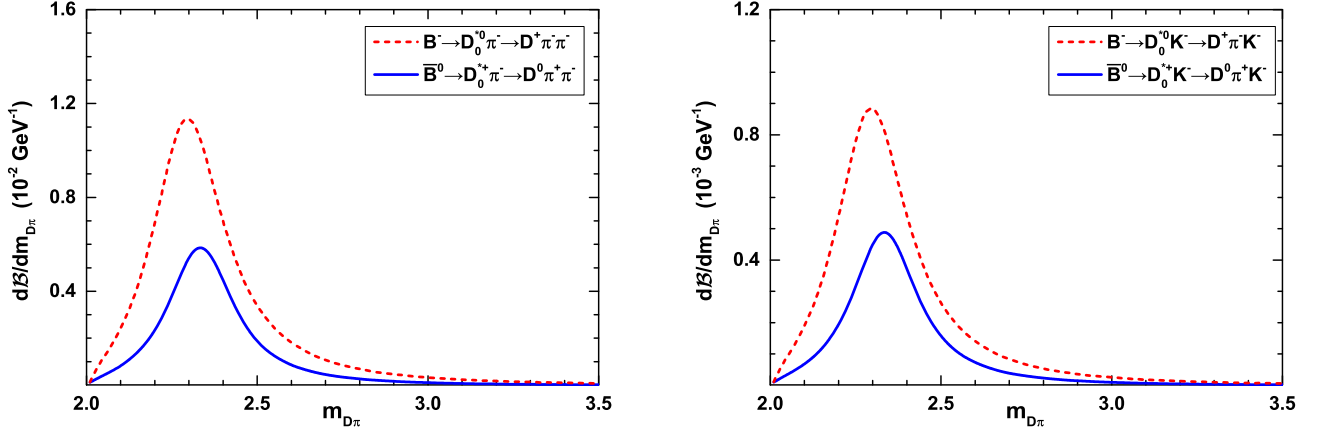


Fig. 2. The differential branching fractions for the decays $B^- \rightarrow D_0^{*0} \pi^- \rightarrow D^+ \pi^- \pi^-$ and $\bar{B}^0 \rightarrow D_0^{*+} \pi^- \rightarrow D^0 \pi^+ \pi^-$ (left), $B^- \rightarrow D_0^{*0} K^- \rightarrow D^+ \pi^- K^-$ and $\bar{B}^0 \rightarrow D_0^{*+} K^- \rightarrow D^0 \pi^+ K^-$ (right).

LHCb [46]. While for the other two decay modes, there are apparent inconsistencies for the branching ratios between the PQCD predictions and the results from Belle and LHCb Collaborations. The Belle's branching fraction [43] for the decay $\bar{B}^0 \rightarrow D_0^{*+} \pi^- \rightarrow D^0 \pi^+ \pi^-$ is only about 21% of the PQCD prediction in this work, other two values from LHCb [44] in Table 1 for this process are some larger, but still less than 30% of our result when considering only the central values. The data for the decay $B^- \rightarrow D_0^{*0} K^- \rightarrow D^+ \pi^- K^-$ selected from LHCb [45] is probably worse than the $\bar{B}^0 \rightarrow D_0^{*+} \pi^- \rightarrow D^0 \pi^+ \pi^-$ case, the branching fraction $\mathcal{B} = (6.1 \pm 1.9 \pm 0.5 \pm 1.4 \pm 0.4) \times 10^{-6}$ is about one order of magnitude smaller than the predicted value in Table 2.

The $B \rightarrow D_0^*(\rightarrow D\pi)\pi$ processes can be decomposed in terms of two isospin amplitudes, $A_{1/2}$ and $A_{3/2}$, as have been done for the $\bar{B}^0 \rightarrow D^+ \pi^-$ and $B^- \rightarrow D^0 \pi^-$ decays in Ref. [72]. With the absolute value $|A_{1/2}/\sqrt{2}A_{3/2}|$ and the relative strong phase between $A_{1/2}$ and $A_{3/2}$ in [72], we have the ratio $R \approx 0.59$, which is close to the result $R \approx 0.54$ from Table 3, between the branching fractions of decays $\bar{B}^0 \rightarrow D^+ \pi^-$ and $B^- \rightarrow D^0 \pi^-$. It is reasonable to expect that the ratio between the branching fractions of the decays $\bar{B}^0 \rightarrow D_0^{*+}(\rightarrow D^0 \pi^+) \pi^-$ and $B^- \rightarrow D_0^{*0}(\rightarrow D^+ \pi^-) \pi^-$ is not far from the 0.54. Our results in this work provide $R \approx 0.48$ for the corresponding two decays with pion as the bachelor particle, while the value for R from the data in Table 1 is just slightly larger than 0.1. So one could conclude that the data in Table 1 for the decays $\bar{B}^0 \rightarrow D_0^{*+}(\rightarrow D^0 \pi^+) \pi^-$ and $B^- \rightarrow D_0^{*0}(\rightarrow D^+ \pi^-) \pi^-$ are probably inconsistent with the isospin relation.

For the decay processes $B^- \rightarrow D_0^{*0} \pi^- \rightarrow D^+ \pi^- \pi^-$ and $B^- \rightarrow D_0^{*0} K^- \rightarrow D^+ \pi^- K^-$, we have an identical step $D_0^{*0} \rightarrow D^+ \pi^-$, the difference of the two decay modes originated from the bachelor particles pion and kaon. Within the $SU(3)$ flavor symmetry, we have a straightforward ratio $R_{D_0^{*0}}$ for the branching fractions of these two decays as

$$R_{D_0^{*0}} = \frac{\mathcal{B}(B^- \rightarrow D_0^{*0} K^- \rightarrow D^+ \pi^- K^-)}{\mathcal{B}(B^- \rightarrow D_0^{*0} \pi^- \rightarrow D^+ \pi^- \pi^-)} \approx \left| \frac{V_{us}}{V_{ud}} \right|^2 \cdot \frac{f_K^2}{f_\pi^2}, \quad (15)$$

with

$$\left| \frac{V_{us}}{V_{ud}} \right| \frac{f_{K^+}}{f_{\pi^+}} = 0.276 \quad (16)$$

from *Review of Particle Physics* [5], then we have $R_{D_0^{*0}} \approx 0.076$. It's easy to obtain a similar ratio $R_{D_0^{*+}} \approx R_{D_0^{*0}}$,

$$R_{D_0^{*+}} = \frac{\mathcal{B}(\bar{B}^0 \rightarrow D_0^{*+} K^- \rightarrow D^0 \pi^+ K^-)}{\mathcal{B}(\bar{B}^0 \rightarrow D_0^{*+} \pi^- \rightarrow D^0 \pi^+ \pi^-)} \approx \left| \frac{V_{us}}{V_{ud}} \right|^2 \cdot \frac{f_K^2}{f_\pi^2} \quad (17)$$

for the decay modes $\bar{B}^0 \rightarrow D_0^{*+} K^- \rightarrow D^0 \pi^+ K^-$ and $\bar{B}^0 \rightarrow D_0^{*+} \pi^- \rightarrow D^0 \pi^+ \pi^-$. The energy dependent curves of the $R_{D_0^{*0}}$ and $R_{D_0^{*+}}$ predicted by PQCD are shown in Fig. 3, from which one can find that there is little variation for the $R_{D_0^{*0}}$ or $R_{D_0^{*+}}$ as $m_{D\pi}$ runs from its threshold to 3.5 GeV. There are similar patterns for the ratios of the related branching fractions for the decay modes including a pseudoscalar D or a vector D^* rather than D_0^* as listed in the Table 3. If we accept the average value $\mathcal{B}(B^- \rightarrow D_0^{*0} \pi^-) \times \mathcal{B}(D_0^{*0} \rightarrow D^+ \pi^-) = (6.4 \pm 1.4) \times 10^{-4}$ in Ref. [5], the branching fraction $\mathcal{B} = (4.86 \pm 1.06) \times 10^{-5}$, which agree well with the PQCD prediction in Table 2, for the decay process $B^- \rightarrow D_0^{*0} K^- \rightarrow D^+ \pi^- K^-$ could be derived from Eq. (15). If we believe the result $\mathcal{B} = (1.77 \pm 0.77) \times 10^{-5}$ given by LHCb [46] for the decay $B^0 \rightarrow D_0^{*-} K^+ \rightarrow \bar{D}^0 \pi^- K^+$, then the three values listed in Table 1 for the decay $\bar{B}^0 \rightarrow D_0^{*+} \pi^- \rightarrow D^0 \pi^+ \pi^-$ announced by Belle and LHCb are simply not credible when considering the Eq. (17). In fact, there is a *preliminary result* from the Dalitz plot analysis of the $B^0 \rightarrow \bar{D}^0 \pi^+ \pi^-$ decay processes in Ref. [71] announced by BaBar as

$$\begin{aligned} \mathcal{B}(B^0 \rightarrow D_0^{*-} \pi^+) \times \mathcal{B}(D_0^{*-} \rightarrow \bar{D}^0 \pi^-) \\ = (2.18 \pm 0.23 \pm 0.33 \pm 1.15 \pm 0.03) \times 10^{-4}. \end{aligned} \quad (18)$$

Table 3
Data for the concerned decays from *Review of Particle Physics* [5] and the ratios for the related branching fractions.

Mode	\mathcal{B}	Mode	\mathcal{B}	$R_{D^{(*)}}$
$B^- \rightarrow D^0 K^-$	$(3.63 \pm 0.12) \times 10^{-4}$	$B^- \rightarrow D^0 \pi^-$	$(4.68 \pm 0.13) \times 10^{-3}$	0.078 ± 0.003
$B^- \rightarrow D^*(2007)^0 K^-$	$(3.97^{+0.31}_{-0.28}) \times 10^{-4}$	$B^- \rightarrow D^*(2007)^0 \pi^-$	$(4.90 \pm 0.17) \times 10^{-3}$	$0.081^{+0.007}_{-0.006}$
$\bar{B}^0 \rightarrow D^+ K^-$	$(1.86 \pm 0.20) \times 10^{-4}$	$\bar{B}^0 \rightarrow D^+ \pi^-$	$(2.52 \pm 0.13) \times 10^{-3}$	0.074 ± 0.009
$\bar{B}^0 \rightarrow D^*(2010)^+ K^-$	$(2.12 \pm 0.15) \times 10^{-4}$	$\bar{B}^0 \rightarrow D^*(2010)^+ \pi^-$	$(2.74 \pm 0.13) \times 10^{-3}$	0.077 ± 0.007

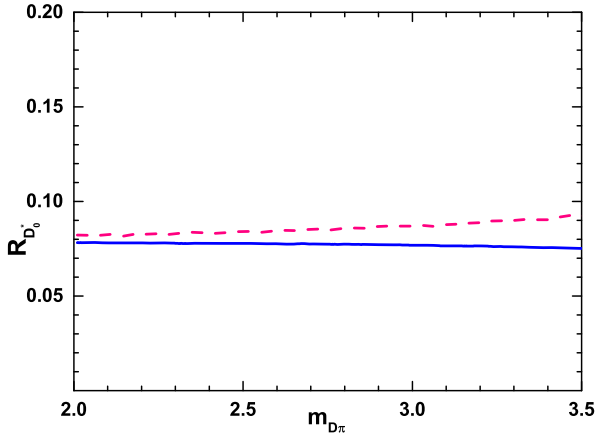


Fig. 3. Energy dependent ratios for the branching fractions between the decays $B^- \rightarrow D_0^{*0} K^- \rightarrow D^+ \pi^- K^-$ and $B^- \rightarrow D_0^{*0} \pi^- \rightarrow D^+ \pi^- \pi^-$ (the solid curve), $\bar{B}^0 \rightarrow D_0^{*+} K^- \rightarrow D^0 \pi^+ K^-$ and $\bar{B}^0 \rightarrow D_0^{*+} \pi^- \rightarrow D^0 \pi^+ \pi^-$ (the dash curve).

This result is consistent with the prediction $2.85^{+1.23+1.05+0.33+0.06}_{-0.80-0.81-0.31-0.05} \times 10^{-4}$ within errors.

To sum up, we studied the quasi-two-body decays $B^- \rightarrow D_0^{*0} \pi^- \rightarrow D^+ \pi^- \pi^-$, $\bar{B}^0 \rightarrow D_0^{*+} \pi^- \rightarrow D^0 \pi^+ \pi^-$, $B^- \rightarrow D_0^{*0} K^- \rightarrow D^+ \pi^- K^-$ and $\bar{B}^0 \rightarrow D_0^{*+} K^- \rightarrow D^0 \pi^+ K^-$ in the PQCD approach in this work. The branching fraction predicted by PQCD for the decay process $B^- \rightarrow D_0^{*0} \pi^- \rightarrow D^+ \pi^- \pi^-$ agree well with the data from Belle, BaBar and LHCb Collaborations. The result for the $\bar{B}^0 \rightarrow D_0^{*+} K^- \rightarrow D^0 \pi^+ K^-$ in this work is consistent with the data $(1.77 \pm 0.26 \pm 0.19 \pm 0.67 \pm 0.20) \times 10^{-5}$ given by LHCb. For the other two decays, we analyzed the experimental results using the ratio relations between the decay branching fractions including D_0^{*0} or D_0^{*+} . From $R_{D_0^{*0}}$ and $R_{D_0^{*+}}$, we argued that the experimental results for the decays $\bar{B}^0 \rightarrow D_0^{*+} \pi^- \rightarrow D^0 \pi^+ \pi^-$ and $B^- \rightarrow D_0^{*0} K^- \rightarrow D^+ \pi^- K^-$ are probably questionable. The PQCD predictions for these two decay modes, in this work, are $2.85^{+1.23}_{-0.80}(\omega_B)^{+1.05}_{-0.81}(\omega_{D\pi})^{+0.33}_{-0.31}(a_{D\pi})^{+0.06}_{-0.05}(\Gamma_{D_0^{*+}}) \times 10^{-4}$ and $4.65^{+1.89}_{-1.30}(\omega_B)^{+1.51}_{-1.24}(\omega_{D\pi})^{+0.40}_{-0.38}(a_{D\pi})^{+0.22}_{-0.18}(\Gamma_{D_0^{*0}}) \times 10^{-5}$, respectively. We concluded that the available experimental results for the four decays including $D_0^*(2400)$ are not in agreement with the isospin relation and $SU(3)$ flavor symmetry.

Acknowledgements

This work was supported in part by National Science Foundation of China under Grant No. 11547038.

Appendix A. Decay amplitudes

The concerned quasi-two-body decay amplitudes are given, in the PQCD approach, by

$$\begin{aligned} \mathcal{A}(B^- \rightarrow \pi^- [D_0^{*0} \rightarrow] D^+ \pi^-) \\ = \frac{G_F}{\sqrt{2}} V_{cb} V_{ud}^* \left[\left(\frac{c_1}{3} + c_2 \right) F_{TD_0^*} + c_1 M_{TD_0^*} + \left(c_1 + \frac{c_2}{3} \right) F_{T\pi} \right. \\ \left. + c_2 M_{T\pi} \right], \end{aligned} \quad (\text{A.1})$$

$$\begin{aligned} \mathcal{A}(B^- \rightarrow K^- [D_0^{*0} \rightarrow] D^+ \pi^-) \\ = \frac{G_F}{\sqrt{2}} V_{cb} V_{us}^* \left[\left(\frac{c_1}{3} + c_2 \right) F_{TD_0^*} + c_1 M_{TD_0^*} + \left(c_1 + \frac{c_2}{3} \right) F_{TK} \right. \\ \left. + c_2 M_{TK} \right], \end{aligned} \quad (\text{A.2})$$

$$\begin{aligned} \mathcal{A}(\bar{B}^0 \rightarrow \pi^- [D_0^{*+} \rightarrow] D^0 \pi^+) \\ = \frac{G_F}{\sqrt{2}} V_{cb} V_{ud}^* \left[\left(\frac{c_1}{3} + c_2 \right) F_{TD_0^*} + c_1 M_{TD_0^*} + \left(c_1 + \frac{c_2}{3} \right) F_{A\pi} \right. \\ \left. + c_2 M_{A\pi} \right], \end{aligned} \quad (\text{A.3})$$

$$\begin{aligned} \mathcal{A}(\bar{B}^0 \rightarrow K^- [D_0^{*+} \rightarrow] D^0 \pi^+) \\ = \frac{G_F}{\sqrt{2}} V_{cb} V_{us}^* \left[\left(\frac{c_1}{3} + c_2 \right) F_{TD_0^*} + c_1 M_{TD_0^*} \right], \end{aligned} \quad (\text{A.4})$$

in which G_F is the Fermi coupling constant, V 's are the CKM matrix elements. And it should be understood that the Wilson coefficients c_1 and c_2 appear in convolutions in momentum fractions and impact parameters b .

The amplitudes from Fig. 1 are written as

$$\begin{aligned} F_{TD_0^*} &= 8\pi C_F m_B^4 f_{\pi(K)} (\eta - 1) \\ &\times \int dx_B dz \int b_B db_B b db \phi_B(x_B, b_B) \phi_{D\pi}(z, b, s) \\ &\times \left\{ [\sqrt{\eta}(2z-1) - 1 - z] E_a^{(1)}(t_e^{(1)}) h(x_B, z, b_B, b) \right. \\ &\left. + (2\sqrt{\eta}(r_c - 1) + \eta - r_c) E_a^{(2)}(t_e^{(2)}) h(z, x_B, b, b_B) \right\}, \end{aligned} \quad (\text{A.5})$$

$$\begin{aligned} M_{TD_0^*} &= 32\pi C_F m_B^4 / \sqrt{2N_c} (\eta - 1) \\ &\times \int dx_B dz dx_3 \int b_B db_B b_3 db_3 \phi_B(x_B, b_B) \phi_{D\pi}(z, b, s) \phi^A \\ &\times \left\{ [\eta(1-z-x_3) + z\sqrt{\eta} + (x_B + x_3 - 1)] \right. \\ &\times E_b(t_b^{(1)}) h_b^{(1)}(x_i, b_i) + [x_3(1-\eta) + z(1-\sqrt{\eta}) - x_B] \\ &\left. \times E_b(t_b^{(2)}) h_b^{(2)}(x_i, b_i) \right\}, \end{aligned} \quad (\text{A.6})$$

$$\begin{aligned} F_{T\pi(K)} &= 8\pi C_F m_B^4 F_{D\pi}(s) \int dx_B dx_3 \int b_B db_B b_3 db_3 \phi_B(x_B, b_B) \\ &\times \left\{ [\phi^A(1-\eta)(x_3(\eta-1) - 1) \right. \\ &\left. - r_0[\phi^P(\eta+1+2(\eta-1)x_3) + \phi^T(\eta-1)(2x_3-1)]] \right. \\ &\times E_c^{(1)}(t_i^{(1)}) h(x_B, x_3(1-\eta), b_B, b_3) \\ &\left. + [2r_0\phi^P(\eta(1+x_B)-1) + \eta(\eta-1)x_B\phi^A] \right. \\ &\left. \times E_c^{(2)}(t_i^{(2)}) h(x_3, x_B(1-\eta), b_3, b_B) \right\}, \end{aligned} \quad (\text{A.7})$$

$$\begin{aligned} M_{T\pi(K)} &= 32\pi C_F m_B^4 / \sqrt{2N_c} \\ &\times \int dx_B dz dx_3 \int b_B db_B b db \phi_B(x_B, b_B) \phi_{D\pi}(z, b, s) \end{aligned}$$

$$\begin{aligned}
& \times \left\{ \left[\phi^A (1 - \eta) (\sqrt{\eta} r_c + (1 + \eta)(1 - x_B - z)) \right. \right. \\
& + r_0 \phi^P (\eta (x_B + z + x_3 - 2) - 4\sqrt{\eta} r_c - x_3) \\
& + r_0 \phi^T (\eta (x_B + z - x_3) + x_3) \left. \right] E_d(t_d^{(1)}) h_d^{(1)}(x_i, b_i) \\
& + [(\eta - 1)[z - x_B + (1 - \eta)x_3] \phi^A \\
& + r_0 x_3 (1 - \eta) (\phi^P + \phi^T) + r_0 \eta (x_B - z) (\phi^T - \phi^P) \left. \right] \\
& \times E_d(t_d^{(2)}) h_d^{(2)}(x_i, b_i) \left. \right\}, \tag{A.8}
\end{aligned}$$

$$\begin{aligned}
F_{A\pi} &= 8\pi C_F m_B^4 f_B \int dz dx_3 \int b db b_3 db_3 \phi_{D\pi}(z, b, s) \\
& \times \left\{ [(\eta - 1)[(1 + 2\sqrt{\eta} r_c + (\eta - 1)x_3) \phi^A \right. \\
& + r_0 (r_c + 2x_3 \sqrt{\eta}) \phi^T] \\
& - r_0 [(1 + \eta)r_c + 2\sqrt{\eta}(x_3(\eta - 1) + 2)] \phi^P \left. \right] \\
& \times E_e^{(1)}(t_a^{(1)}) h_a(z, x_3(1 - \eta), b, b_3) \\
& + [(1 - \eta)z \phi^A + 2r_0 \sqrt{\eta}(1 - \eta + z) \phi^P] \\
& \times E_e^{(2)}(t_a^{(2)}) h_a(x_3, z(1 - \eta), b_3, b) \left. \right\}, \tag{A.9}
\end{aligned}$$

$$\begin{aligned}
M_{A\pi} &= 32\pi C_F m_B^4 / \sqrt{2N_c} \\
& \times \int dx_B dz dx_3 \int b_B db_B b_3 db_3 \phi_B(x_B, b_B) \phi_{D\pi}(z, b, s) \\
& \times \left\{ [(\eta - 1)(\eta(x_B + z - 1) + z + x_B) \phi^A \right. \\
& + r_0 \sqrt{\eta} [(\eta - 1)(1 - x_3) (\phi^P + \phi^T) \\
& + (z + x_B) (\phi^T - \phi^P) - 2\phi^P] \\
& \times E_f(t_f^{(1)}) h_f^{(1)}(x_i, b_i) + [(1 - \eta)[\eta(z - x_B) \\
& + (1 - x_3)(1 - \eta)] \phi^A + r_0 \sqrt{\eta} [(\eta - 1)(x_3 - 1) \\
& \times (\phi^P - \phi^T) + (z - x_B) (\phi^P + \phi^T)] \left. \right] E_f(t_f^{(2)}) h_f^{(2)}(x_i, b_i) \left. \right\}. \tag{A.10}
\end{aligned}$$

The evolution factors in the above factorization formulas are given by

$$E_a^{(1)}(t) = \alpha_s(t) \exp[-S_B(t) - S_C(t)] S_t(z),$$

$$E_a^{(2)}(t) = \alpha_s(t) \exp[-S_B(t) - S_C(t)] S_t(x_B), \tag{A.11}$$

$$E_b(t) = \alpha_s(t) \exp[-S_B(t) - S_C(t) - S_P(t)]|_{b=b_B}, \tag{A.12}$$

$$E_c^{(1)}(t) = \alpha_s(t) \exp[-S_B(t) - S_P(t)] S_t(x_3),$$

$$E_c^{(2)}(t) = \alpha_s(t) \exp[-S_B(t) - S_P(t)] S_t(x_B), \tag{A.13}$$

$$E_d(t) = \alpha_s(t) \exp[-S_B(t) - S_C(t) - S_P(t)]|_{b_3=b_B}, \tag{A.14}$$

$$E_e^{(1)}(t) = \alpha_s(t) \exp[-S_C(t) - S_P(t)] S_t(x_3),$$

$$E_e^{(2)}(t) = \alpha_s(t) \exp[-S_C(t) - S_P(t)] S_t(z), \tag{A.15}$$

$$E_f(t) = \alpha_s(t) \exp[-S_B(t) - S_C(t) - S_P(t)]|_{b_3=b}, \tag{A.16}$$

in which $S_{(B,C,P)}(t)$ are in the Appendix of [73], the hard functions $h, h_a, h_{(b,d,f)}^{(1,2)}$ and the hard scales $t_{(e,b,i,d,a,f)}^{(1,2)}$ have their explicit expressions in the Ref. [73]. We need to stress that, because of the different definitions of the momenta for the initial and final states, the concerned expressions in [73] could be employed in this work only after the replacements $\{x_1 \rightarrow x_B, b_1 \rightarrow b_B, x_2 \rightarrow z, b_2 \rightarrow b, r^2 \rightarrow \eta\}$. The parameter c in the Eq. (A1) of [73] is adopt to be 0.4 in this work according to the Refs. [74,75].

References

- [1] H.X. Chen, W. Chen, X. Liu, Y.R. Liu, S.L. Zhu, Rep. Prog. Phys. 80 (2017) 076201.
- [2] A. Datta, P.J. O'Donnell, Phys. Lett. B 572 (2003) 164.
- [3] S. Godfrey, Phys. Rev. D 72 (2005) 054029.
- [4] S. Godfrey, K. Moats, Phys. Rev. D 93 (2016) 034035.
- [5] M. Tanabashi, et al., Particle Data Group, Phys. Rev. D 98 (2018) 030001.
- [6] K. Abe, et al., Belle Collaboration, Phys. Rev. D 69 (2004) 112002.
- [7] B. Aubert, et al., BaBar Collaboration, Phys. Rev. D 79 (2009) 112004.
- [8] J.M. Link, et al., FOCUS Collaboration, Phys. Lett. B 586 (2004) 11.
- [9] B. Aubert, et al., BaBar Collaboration, Phys. Rev. Lett. 90 (2003) 242001.
- [10] D. Besson, et al., CLEO Collaboration, Phys. Rev. D 68 (2003) 032002, Erratum: Phys. Rev. D 75 (2007) 119908.
- [11] P. Krokovny, et al., Belle Collaboration, Phys. Rev. Lett. 91 (2003) 262002.
- [12] E.S. Swanson, Phys. Rep. 429 (2006) 243.
- [13] S. Godfrey, N. Isgur, Phys. Rev. D 32 (1985) 189.
- [14] S. Godfrey, R. Kokoski, Phys. Rev. D 43 (1991) 1679.
- [15] F.E. Close, E.S. Swanson, Phys. Rev. D 72 (2005) 094004.
- [16] X.H. Zhong, Q. Zhao, Phys. Rev. D 78 (2008) 014029.
- [17] D. Mohler, S. Prelovsek, R.M. Woloshyn, Phys. Rev. D 87 (2013) 034501.
- [18] Q.T. Song, D.Y. Chen, X. Liu, T. Matsuki, Phys. Rev. D 92 (2015) 074011.
- [19] D. Ebert, R.N. Faustov, V.O. Galkin, Eur. Phys. J. C 66 (2010) 197.
- [20] J.B. Liu, C.D. Lü, Eur. Phys. J. C 77 (2017) 312.
- [21] X.Z. Tan, et al., Eur. Phys. J. C 78 (2018) 583.
- [22] M.E. Bracco, A. Lozea, R.D. Matheus, F.S. Navarra, M. Nielsen, Phys. Lett. B 624 (2005) 217.
- [23] M. Nielsen, R.D. Matheus, F.S. Navarra, M.E. Bracco, A. Lozea, Nucl. Phys. B 161 (2006) 193.
- [24] F.K. Guo, P.N. Shen, H.C. Chiang, R.G. Ping, B.S. Zou, Phys. Lett. B 641 (2006) 278.
- [25] M. Albaladejo, P. Fernandez-Soler, F.K. Guo, J. Nieves, Phys. Lett. B 767 (2017) 465.
- [26] F.K. Guo, et al., Rev. Mod. Phys. 90 (2018) 015004.
- [27] G. Moir, M. Peardon, S.M. Ryan, C.E. Thomas, D.J. Wilson, J. High Energy Phys. 1610 (2016) 011.
- [28] J. Vijande, F. Fernández, A. Valcarce, Phys. Rev. D 73 (2006) 034002, Erratum: Phys. Rev. D 74 (2006) 059903.
- [29] D. Gamermann, E. Oset, D. Strottman, M.J. Vicente Vacas, Phys. Rev. D 76 (2007) 074016.
- [30] M. De Vito, P. Santorelli, Eur. Phys. J. C 48 (2006) 441.
- [31] Y.L. Shen, Z.J. Yang, X. Yu, Phys. Rev. D 90 (2014) 114015.
- [32] F.S. Navarra, M. Nielsen, E. Oset, T. Sekihara, Phys. Rev. D 92 (2015) 014031.
- [33] F.U. Bernlochner, Z. Ligeti, Phys. Rev. D 95 (2017) 014022.
- [34] F.U. Bernlochner, Z. Ligeti, D.J. Robinson, Phys. Rev. D 97 (2018) 075011.
- [35] X.W. Kang, T. Luo, Y. Zhang, L.Y. Dai, C. Wang, arXiv:1808.02432 [hep-ph].
- [36] W.L. Ju, G.L. Wang, H.F. Fu, Z.H. Wang, Y. Li, J. High Energy Phys. 1509 (2015) 171.
- [37] H.Y. Cheng, C.K. Chua, C.W. Hwang, Phys. Rev. D 69 (2004) 074025.
- [38] F. Jugeau, A. Le Yaouanc, L. Oliver, J.C. Raynal, Phys. Rev. D 72 (2005) 094010.
- [39] H.Y. Cheng, Phys. Rev. D 68 (2003) 094005.
- [40] H.Y. Cheng, C.K. Chua, Phys. Rev. D 74 (2006) 034020.
- [41] C.H. Chen, Phys. Rev. D 68 (2003) 114008.
- [42] R. Aaij, et al., LHCb collaboration, Phys. Rev. D 94 (2016) 072001.
- [43] A. Kuzmin, et al., Belle Collaboration, Phys. Rev. D 76 (2007) 012006.
- [44] R. Aaij, et al., LHCb collaboration, Phys. Rev. D 92 (2015) 032002.
- [45] R. Aaij, et al., LHCb collaboration, Phys. Rev. D 91 (2015) 092002, Erratum: Phys. Rev. D 93 (2016) 119901.
- [46] R. Aaij, et al., LHCb collaboration, Phys. Rev. D 92 (2015) 012012.
- [47] Y.Y. Keum, H.n. Li, A.I. Sanda, Phys. Lett. B 504 (2001) 6.
- [48] Y.Y. Keum, H.n. Li, A.I. Sanda, Phys. Rev. D 63 (2001) 054008.
- [49] C.D. Lü, K. Ukai, M.Z. Yang, Phys. Rev. D 63 (2001) 074009.
- [50] H.n. Li, Prog. Part. Nucl. Phys. 51 (2003) 85.
- [51] C.H. Chen, H.n. Li, Phys. Lett. B 561 (2003) 258.
- [52] C.H. Chen, H.n. Li, Phys. Rev. D 70 (2004) 054006.
- [53] W.F. Wang, H.C. Hu, H.n. Li, C.D. Lü, Phys. Rev. D 89 (2014) 074031.
- [54] W.F. Wang, H.n. Li, W. Wang, C.D. Lü, Phys. Rev. D 91 (2015) 094024.
- [55] C. Wang, J.B. Liu, H.n. Li, C.D. Lü, Phys. Rev. D 97 (2018) 034033.
- [56] D. Müller, D. Robaschik, B. Geyer, F.M. Dittes, J. Hořejší, Fortschr. Phys. 42 (1994) 101.
- [57] M. Diehl, T. Gousset, B. Pire, O. Teryaev, Phys. Rev. Lett. 81 (1998) 1782.
- [58] M.V. Polyakov, Nucl. Phys. B 555 (1999) 231.
- [59] W.F. Wang, H.n. Li, Phys. Lett. B 763 (2016) 29.
- [60] Y. Li, A.J. Ma, W.F. Wang, Z.J. Xiao, Phys. Rev. D 95 (2017) 056008; Phys. Rev. D 96 (2017) 036014.
- [61] Y. Li, W.F. Wang, A.J. Ma, Z.J. Xiao, arXiv:1809.09816 [hep-ph].
- [62] A.J. Ma, Y. Li, W.F. Wang, Z.J. Xiao, Nucl. Phys. B 923 (2017) 54; Phys. Rev. D 96 (2017) 093011.
- [63] M. Jamin, J.A. Oller, A. Pich, Nucl. Phys. B 622 (2002) 279.
- [64] D.R. Boito, R. Escribano, Phys. Rev. D 80 (2009) 054007.

- [65] C.W. Hwang, Phys. Rev. D 86 (2012) 094031.
- [66] S. Veseli, I. Dunietz, Phys. Rev. D 54 (1996) 6803.
- [67] G.L. Wang, Phys. Lett. B 650 (2007) 15.
- [68] R.C. Verma, J. Phys. G 39 (2012) 025005.
- [69] W.F. Wang, Z.J. Xiao, Phys. Rev. D 86 (2012) 114025.
- [70] A. Bazavov, et al., FNAL and MILC Collaborations, Phys. Rev. D 98 (2018) 074512.
- [71] P. del Amo Sanchez, et al., BaBar Collaboration, arXiv:1007.4464 [hep-ex].
- [72] S.H. Zhou, et al., Phys. Rev. D 92 (2015) 094016.
- [73] R.H. Li, C.D. Lü, H. Zou, Phys. Rev. D 78 (2008) 014018.
- [74] T. Kurimoto, H.n. Li, A.I. Sanda, Phys. Rev. D 65 (2001) 014007.
- [75] H.n. Li, S. Mishima, Phys. Rev. D 80 (2009) 074024.