Hindawi
Advances in High Energy Physics
Volume 2018, Article ID 7269657, 7 pages
https://doi.org/10.1155/2018/7269657

Research Article

Bound State of Heavy Quarks Using a General Polynomial Potential

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Received 3 October 2018; Revised 15 November 2018; Accepted 27 November 2018; Published 17 December 2018

Academic Editor: Sally Seidel

In the present work, the mass spectra of the bound states of heavy quarks $c\bar{c}$, $b\bar{b}$, and B_c meson are studied within the framework of the nonrelativistic Schrödinger's equation. First, we solve Schrödinger's equation with a general polynomial potential by Nikiforov-Uvarov (NU) method. The energy eigenvalues for any L- value is presented for a special case of the potential. The results obtained are in good agreement with the experimental data and are better than previous theoretical studies.

1. Introduction

The study of quarkonium systems provides a good understating of the quantitative description of quantum chromodynamics (QCD) theory, the standard model and particle physics [1–7]. The quarkonia with a heavy quark and antiquark and their interaction are well described by Schrödinger's equation. The solution of this equation with a spherically symmetric potentials is one of the most important problems in quarkonia systems [8–11]. These potentials should take into account the two important features of the strong interaction, namely, asymptotic freedom and quark confinement [2–6].

In the present work, an interaction potential in the quark-antiquark bound system is taken as a general polynomial to get the general eigenvalue solution. In the next step, we chose a specific potential according to the physical properties of the system. Several methods are used to solve Schrödinger's equation. One of them is the Nikiforov-Uvarov (NU) method [12–14], which gives asymptotic expressions for the eigenfunctions and eigenvalues of the Schrödinger's equation. Hence one can calculate the energy eigenstates for the spectrum of the quarkonia systems [12–15].

The paper is organized as follows: In Section 2, the Nikiforov-Uvarov (NU) method is briefly explained. In Section 3, the Schrödinger equation with a general polynomial potential is solved by the Nikiforov-Uvarov (NU) method. In Section 4, results and discussion are presented. In Section 5, the conclusion is given.

2. The Nikiforov-Uvarov (NU) Method [12–15]

The Nikiforov-Uvarov (NU) method is based on solving the hypergeometric-type second-order differential equation.

$$\ddot{\Psi}(s) + \frac{\tilde{\tau}(s)}{\sigma(s)}\dot{\Psi}(s) + \frac{\tilde{\sigma}(s)}{\sigma^2(s)}\Psi(s) = 0. \tag{1}$$

Here $\sigma(s)$ and $\tilde{\sigma}(s)$ are second-degree polynomials, $\tilde{\tau}(s)$ is a first-degree polynomial, and $\psi(s)$ is a function of the hypergeometric-type.

By taking $\Psi(s) = \varphi(s)\Upsilon(s)$ and substituting in equation (1), we get the following equation

$$\ddot{\Upsilon}(s) + \left[2\frac{\dot{\varphi}(s)}{\varphi(s)} + \frac{\tilde{\tau}(s)}{\sigma(s)}\right]\Upsilon(s) + \left[\frac{\ddot{\varphi}(s)}{\varphi(s)} + \frac{\dot{\varphi}(s)}{\varphi(s)}\frac{\tilde{\tau}(s)}{\sigma(s)} + \frac{\tilde{\sigma}(s)}{\sigma^2(s)}\right]\Upsilon(s) = 0. \tag{2}$$

By taking

$$2\frac{\dot{\varphi}(s)}{\varphi(s)}+\frac{\tilde{\tau}(s)}{\sigma(s)}=\frac{\tau(s)}{\sigma(s)},$$
$$\frac{\dot{\varphi}(s)}{\varphi(s)}=\frac{\pi(s)}{\sigma(s)} \tag{3}$$

we get

$$\tau(s)=\tilde{\tau}(s)+2\pi(s), \tag{4}$$

where both $\pi(s)$ and $\tau(s)$ are polynomials of degree at most one.

Also we one can take

$$\frac{\ddot{\varphi}(s)}{\varphi(s)}+\frac{\dot{\varphi}(s)}{\varphi(s)}\frac{\tilde{\tau}(s)}{\sigma(s)}+\frac{\tilde{\sigma}(s)}{\sigma^2(s)}=\frac{\overline{\sigma}(s)}{\sigma^2(s)} \tag{5}$$

where

$$\frac{\ddot{\varphi}(s)}{\varphi(s)}=\left[\frac{\dot{\varphi}(s)}{\varphi(s)}\right]^{\cdot}+\left[\frac{\dot{\varphi}(s)}{\varphi(s)}\right]^2=\left[\frac{\pi(s)}{\sigma(s)}\right]^{\cdot}+\left[\frac{\pi(s)}{\sigma(s)}\right]^2 \tag{6}$$

And

$$\overline{\sigma}(s)=\tilde{\sigma}(s)+\pi^2(s)+\pi(s)\left[\tilde{\tau}(s)-\dot{\sigma}(s)\right]$$
$$+\dot{\pi}(s)\sigma(s). \tag{7}$$

So equation (2) becomes

$$\ddot{\Upsilon}(s)+\frac{\tau(s)}{\sigma(s)}\Upsilon(s)+\frac{\overline{\sigma}(s)}{\sigma^2(s)}\Upsilon(s)=0. \tag{8}$$

An algebraic transformation from equation (1) to equation (8) is systematic. Hence one can divide $\overline{\sigma}(s)$ by $\sigma(s)$ to obtain a constant λ; i.e.,

$$\overline{\sigma}(s)=\lambda\,\sigma(s). \tag{9}$$

Equation (8) can be reduced to a hypergeometric equation in the form

$$\sigma(s)\ddot{\Upsilon}(s)+\tau(s)\Upsilon(s)+\lambda\Upsilon(s)=0. \tag{10}$$

Substituting from equation (9) in equation (7) and solving the quadratic equation for $\pi(s)$, we obtain

$$\pi^2(s)+\pi(s)\left[\tilde{\tau}(s)-\dot{\sigma}(s)\right]+\tilde{\sigma}(s)-k\sigma(s)=0, \tag{11}$$

where

$$k=\lambda-\dot{\pi}(s). \tag{12}$$

$$\pi(s)=\frac{\dot{\sigma}(s)-\tilde{\tau}(s)}{2}$$
$$\pm\sqrt{\left(\frac{\dot{\sigma}(s)-\tilde{\tau}(s)}{2}\right)^2-\tilde{\sigma}(s)+k\sigma(s)}. \tag{13}$$

The possible solutions for $\pi(s)$depend on the parameter k according to the plus and minus signs of $\pi(s)$ [13]. Since $\pi(s)$ is a polynomial of degree at most one, the expression under the square root has to be the square of a polynomial. In this case, an equation of the quadratic form is available for the constant k. To determine the parameter k, one must set the discriminant of this quadratic expression to be equal to zero. After determining the values of k one can find the values of $\pi(s)$, λ *and* $\tau(s)$.

Applying the same systematic way for equation (10), we get

$$\lambda_n=-n\dot{\tau}(s)-\frac{n(n-1)}{2}\ddot{\sigma}(s), \tag{14}$$

where n is the principle quantum number.

By comparing equations (12) and (14), we get an equation for the energy eigenvalues.

3. The Schrödinger Equation with a General Polynomial Potential

The radial Schrödinger equation of a quark and antiquark system is

$$\frac{d^2Q}{dr^2}+\left[\frac{2\mu}{h^2}(E-V)-\frac{l(l+1)}{r^2}\right]Q=0. \tag{15}$$

We will use a generalized polynomial potential

$$V(r)=\sum_{m=0}^{m}A_{m-2}r^{m-2},\quad m=0,1,2,3,4,\ldots \tag{16}$$

By substituting in equation (15), we get

$$\frac{d^2Q}{dr^2}+\left[\frac{2\mu}{h^2}E-\frac{2\mu}{h^2}\sum_{m=0}^{m}A_{m-2}r^{m-2}-\frac{l(l+1)}{r^2}\right]Q=0. \tag{17}$$

Let

$$\frac{2\mu}{h^2}A_{m-2}=a_{m-2},$$
$$b_{-2}=l(l+1)+a_{-2}, \tag{18}$$
$$b_0=a_0-\frac{2\mu}{h^2}E=a_0-\epsilon_0$$

and hence,

$$\frac{d^2Q}{dr^2}+\left[\sum_{m=0}^{m}A(a,b)_{m-2}r^{m-2}\right]Q=0, \tag{19}$$

where

$$\sum_{m=0}^{m}A(a,b)_{m-2}r^{m-2}$$
$$=-\left[b_{-2}r^{-2}+a_{-1}r^{-1}+b_0+a_1r+a_2r^2+a_3r^3+\cdots\right]. \tag{20}$$

Let r = 1/x; hence

$$\frac{d^2Q}{dr^2} = \frac{d}{dr}\left(\frac{dQ}{dr}\right) = \frac{d}{dx}\frac{dx}{dr}\left(\frac{dQ}{dr}\right) = \frac{d}{dx}\left(x^4\frac{dQ}{dx}\right) = 4x^3\frac{dQ}{dx} + x^4\frac{d^2Q}{dx^2} \quad (21)$$

And

$$\sum_{m=0}^{m} A(a,b)_{m-2}\left(\frac{1}{x}\right)^{m-2} = -\left[b_{-2}x^2 + a_{-1}x + b_0 + \sum_{m=1}^{m} a_m\left(\frac{1}{x}\right)^m\right]. \quad (22)$$

By substituting in equation (19), we get

$$x^4\frac{d^2Q}{dx^2} + 4x^3\frac{dQ}{dx} + \left[-b_{-2}x^2 - a_{-1}x - b_0 - \sum_{m=1}^{m} a_m\left(\frac{1}{x}\right)^m\right]Q = 0. \quad (23)$$

We propose the following approximation scheme on the term $a_m(1/x)^m$. Let us assume that there is a characteristic radius (residual radius) r_0 of the quark and antiquark system (which is the smallest distance between the two quarks where they cannot collide with each other). This scheme is based on the expansion of $a_m(1/x)^m$ in a power series around r_0, i.e., around $\delta = 1/r_0$ in the x-space, up to the second order, so that the a_m, dependent term, preserves the original form of equation (23). This is similar to Pekeris approximation [14, 15], which helps to deform the centrifugal potential such that the modified potential can be solved by the Nikiforov-Uvarov (NU) method. Setting $y = (x - \delta)$ around $y = 0$ (the singularity), one can expand into a power series as follows

$$\sum_{m=1}^{m} a_m\left(\frac{1}{x}\right)^m = \sum_{m=1}^{m}\frac{a_m}{(y+\delta)^m} = \sum_{m=1}^{m}\frac{a_m}{\delta^m}\left[1 + \frac{y}{\delta}\right]^{-m}. \quad (24)$$

$$\sum_{m=1}^{m} a_m\left(\frac{1}{x}\right)^m \approx \sum_{m=1}^{m}\left[\frac{a_m}{\delta^m} - \frac{ma_m}{\delta^{m+1}}x + \frac{m(m+1)a_m}{2\delta^{m+2}}x^2\right]. \quad (25)$$

By substituting from equation (25) in equation (23), dividing by x^4 where $x \neq 0$, and rearranging this equation, we get

$$\frac{d^2Q}{dx^2} + \frac{4x}{x^2}\frac{dQ}{dx} + \frac{1}{x^4}\left[-\left(b_0 + \sum_{m=1}^{m}\frac{a_m}{\delta^m}\right) + \left(\sum_{m=1}^{m}\frac{ma_m}{\delta^{m+1}} - a_{-1}\right)x - \left(b_{-2} + \sum_{m=1}^{m}\frac{m(m+1)a_m}{2\delta^{m+2}}\right)x^2\right]Q = 0. \quad (26)$$

We define

$$\left(b_0 + \sum_{m=1}^{m}\frac{a_m}{\delta^m}\right) = q,$$
$$\left(\sum_{m=1}^{m}\frac{ma_m}{\delta^{m+1}} - a_{-1}\right) = w, \quad (27)$$
$$\left(b_{-2} + \sum_{m=1}^{m}\frac{m(m+1)a_m}{2\delta^{m+2}}\right) = z$$

And, hence, equation (26) becomes

$$\frac{d^2Q}{dx^2} + \left(\frac{4x}{x^2}\right)\frac{dQ}{dx} + \frac{1}{x^4}\left[-q + wx - zx^2\right]Q = 0. \quad (28)$$

Comparing with equation (1), we get

$$\tilde{\tau} = 4x,$$
$$\sigma = x^2, \quad (29)$$
$$\tilde{\sigma} = -q + wx - zx^2$$

And, by substituting in equation (13), we get

$$\pi(x) = -x \pm \sqrt{(1 + k + z)x^2 - wx + q}. \quad (30)$$

Now one can obtain the value of the parameter k, by knowing that $\pi(x)$ is a polynomial of degree at most one and by putting the discriminant of this expression under the square root equal to zero.

$$w^2 - 4(1 + k + z)q = 0 \longrightarrow k = \frac{w^2}{4q} - z - 1 \quad (31)$$

By substituting in equation (30) and taking the negative value of $\pi(x)$, for bound state solutions, one finds that the solution is in agreement with the free hydrogen atom spectrum, because of the Coulomb term

$$\pi(x) = -x - \frac{w}{2\sqrt{q}}x + \sqrt{q}. \quad (32)$$

Hence, by substituting in equation (4), we get the following.

$$\tau(x) = \left[2 - \frac{w}{\sqrt{q}}\right]x + 2\sqrt{q}., \quad \text{where } \left(2 - \frac{w}{\sqrt{q}}\right) < 0 \quad (33)$$

Substituting in equation (12), we obtain

$$\lambda = k + \dot{\pi}(x) = \frac{w^2}{4q} - \frac{w}{2\sqrt{q}} - z - 2. \quad (34)$$

Using equation (14), we obtain

$$\lambda_n = -n\left[2 - \frac{w}{\sqrt{q}}\right] - n(n-1) = -2n + \frac{w}{\sqrt{q}}n - n^2 + n.[5pt] \quad (35)$$

Equalizing equations (34) and (35), we get

$$q = \frac{w^2}{4\left[\sqrt{z + 9/4} + (n + 1/2)\right]^2}. \quad (36)$$

By substituting the values of $(q, w, z$ and $\delta)$ in equation (36), we get

$$\epsilon_0 = a_0 + \sum_{m=1}^{m} a_m r_0^{\ m} - \frac{\left(\sum_{m=1}^{m} m a_m r_0^{\ m+1} - a_{-1}\right)^2}{4\left[\sqrt{\left(l(l+1) + a_{-2} + (1/2)\sum_{m=1}^{m} m(m+1) a_m r_0^{\ m+2}\right) + 9/4} + (n + 1/2)\right]^2}. \quad (37)$$

Equation (37) is the desired equation of the energy eigenvalues in spherical symmetric coordinates with a general polynomial radial potential using the Nikiforov-Uvarov (NU) method.

A special case of the above potential was chosen to describe the $q\bar{q}$ interaction, namely,

$$V(r) = \frac{b}{r} + ar + dr^2 + pr^4 \quad (38)$$

where the first term is the Coulomb potential because the two quarks are charged and the second term is the linear term in r which means that $V(r)$ continues growing as $r \longrightarrow \infty$. It is this linear term that leads to quark confinement. One of the striking properties of QCD asymptotic freedom is that the interaction strength between quarks becomes smaller as the distance between them gets shorter.

The third term is a harmonic term and the fourth is an anharmonic term and they are responsible also for quark confinement.

For the above chosen potential, we put $a_{-2} = a_0 = 0$ and $m = 1, 2, 4$, and hence

$$\epsilon_0 = a_1 r_0 + a_2 r_0^{\ 2} + a_4 r_0^{\ 4} - \frac{\left(a_1 r_0^{\ 2} + 2a_2 r_0^{\ 3} + 4a_4 r_0^{\ 5} - a_{-1}\right)^2}{4\left[\sqrt{l(l+1) + a_1 r_0^{\ 3} + 3a_2 r_0^{\ 4} + 10 a_4 r_0^{\ 6} + 9/4} + (n + 1/2)\right]^2}. \quad (39)$$

Now, we can rewrite equation (39) in a different form which depends on the parameters of the potential as follows

$$E = ar_0 + dr_0^{\ 2} + pr_0^{\ 4} - \frac{\left(2\mu/h^2\right)\left(ar_0^{\ 2} + 2dr_0^{\ 3} + 4pr_0^{\ 5} - b\right)^2}{4\left[\sqrt{\left(2\mu a/h^2\right) r_0^{\ 3} + \left(6\mu d/h^2\right) r_0^{\ 4} + \left(20\mu p/h^2\right) r_0^{\ 6} + l(l+1) + 9/4} + (n + 1/2)\right]^2}. \quad (40)$$

4. Results and Discussion

In this section, we will calculate the spectra for the bound states of heavy quarks such as charmonium, bottomonium, and B_C meson. To determine the mass spectra in three dimensions, we use the following relation.

$$M = m_q + m_{\bar{q}} + E \quad (41)$$

By substituting in equation (40), we get

$$M = 2m_q + ar_0 + dr_0^{\ 2} + pr_0^{\ 4} - \frac{\left(2\mu/h^2\right)\left(ar_0^{\ 2} + 2dr_0^{\ 3} + 4pr_0^{\ 5} - b\right)^2}{4\left[\sqrt{\left(2\mu a/h^2\right) r_0^{\ 3} + \left(6\mu d/h^2\right) r_0^{\ 4} + \left(20\mu p/h^2\right) r_0^{\ 6} + l(l+1) + 9/4} + (n + 1/2)\right]^2}. \quad (42)$$

It is clear that equation (42) depends on the potential parameters $(a, b, d, p \text{ and } r_0)$ which will be obtained from the experimental data.

In the case of charmonium $[\Psi = c\bar{c}]$, the rest mass equation is as follows.

$$M = 2m_c + ar_0 + dr_0^2 + pr_0^4 - \frac{32.743 * \left(ar_0^2 + 2dr_0^3 + 4pr_0^5 - b\right)^2}{4\left[\sqrt{32.743ar_0^3 + 98.229dr_0^4 + 327.43pr_0^6 + l(l+1) + 9/4} + (n + 1/2)\right]^2} \quad (43)$$

In the case of bottomonium $[\Upsilon = b\bar{b}]$, the rest mass equation is as follows.

$$M = 2m_b + ar_0 + dr_0^2 + pr_0^4 - \frac{107.86 * \left(ar_0^2 + 2dr_0^3 + 4pr_0^5 - b\right)^2}{4\left[\sqrt{107.86 * ar_0^3 + 323.58 * dr_0^4 + 1078.6 * pr_0^6 + l(l+1) + 9/4} + (n + 1/2)\right]^2} \quad (44)$$

And in the case of the meson $[B_c = b\bar{c}]$, the rest mass equation is as follows.

$$M = m_c + m_b + ar_0 + dr_0^2 + pr_0^4 - \frac{50.24 * \left(ar_0^2 + 2dr_0^3 + 4pr_0^5 - b\right)^2}{4\left[\sqrt{50.24 * ar_0^3 + 150.72 * dr_0^4 + 502.4 * pr_0^6 + l(l+1) + 9/4} + (n + 1/2)\right]^2} \quad (45)$$

Comparing our theoretical work with the experimental data, we found that the maximum errors are 0.229% for the charmonium, 0.0742% for the bottomonium, and 0.00123% for the B_C meson. These may be due to errors in the measurements of the device. The spin can also be taken into account if one uses relativistic corrections and the appropriate relativistic Schrödinger's equation. Our results are shown in Tables 1, 2, and 3, with a comparison between our results and those obtained in previous calculations in the literature. In the charmonium system, the maximum distance where a quark and antiquark can approach each other is $r_0 = 0.8043$ *fm*. Similarly the maximum distances in the cases of the bottomonium system and B_C meson system are $r_0 = 0.47$ *fm* and $r_0 = 0.4256$ *fm*, respectively. The positive and negative signs of the coefficient of the harmonic potential refer to the direction of motion of the oscillation. The negative sign of the coefficient of the Coulomb potential refers to the charges of the two quarks, but the positive sign refers to the existence of another negative contribution. The positive and negative signs of the coefficient of the anharmonic potential give a correction to the linear potential.

5. Conclusion

The mass spectra of the quarkonia (charmonium, bottomonium, and B_C meson) using our potential were studied within the framework of the nonrelativistic Schrödinger's equation by using the Nikiforov-Uvarov (NU) method. In [16, 17], the authors used an iteration method and the same potential when $p = 0$. It is found that adding the anharmonic potential gives a good accuracy and our work is comparable with them. In [18, 19], the authors used also the same potential when $a = p = 0$. We found that our work gives better results in comparison with experimental data. In [20, 21, 28, 29], the authors used the same method and the same potential when $p = 0$. We noticed that our work is in better agreement with the experimental data. In [22, 23], the authors used the Cornell potential only and used the same method. It is found that their results are comparable with our work. In [24, 25], the authors used the same potential when $p = 0$ and the same method used in the present work. We found that our work is comparable with them. In [22, 23, 30–37] for the mass spectra of *the* B_C meson, there is

TABLE 1: Mass spectra of charmonium in comparison with other works [$r_0 = 0.8043\,fm$, $a = 3.03857\,GeV/fm$, $d = -0.7054\,GeV/(fm)^2$, $b = -0.49842\,GeV.fm$, $P = -0.2379\,GeV/(fm)^4$].

Type	present work	[16, 17]	[18, 19]	[20, 21]	[22, 23]	[24, 25]	EXP [26, 27]
1s	3.0969	3.078	3.096	3.096	3.096	3.078	3.0969
1p	-	3.415	3.433	3.433	3.255	3.415	-
2s	-	4.187	3.686	3.686	3.686	3.581	-
1d	3.6861	3.752	3.676	3.770	3.504	3.749	3.6861
2p	3.7702	4.143	3.910	4.023	3.779	3.917	3.773
3s	-	5.297	3.984	4.040	4.040	4.085	-
4s	-	6.407	4.150	4.358	4.269	4.589	-
2d	-	-	-	4.096	-	3.078	-
4d	4.160	-	-	-	-	-	4.159
1g	4.039	-	-	-	-	-	4.039
6d	4.263	-	-	-	-	-	4.263

TABLE 2: Mass spectra of bottomonium in comparison with other works [$r_0 = 0.47\,fm$, $a = 10.7\,GeV/fm$, $d = -4.95\,GeV/(fm)^2$, $b = 6.39286\,GeV.fm$, $P = 7.1\,GeV/(fm)^4$].

Type	present work	[16, 17]	[18, 19]	[28, 29]	[22, 23]	[24, 25]	EXP [26, 27]
1s	9.4600	9.510	9.460	9.460	9.460	9.510	9.4601
1p	-	9.612	9.840	9.811	9.916	9.862	-
2s	-	10.627	10.023	10.023	10.023	10.038	-
1d	-	10.214	10.140	10.161	9.864	10.214	-
2p	-	10.944	10.160	10.374	10.114	10.390	-
3s	-	11.726	10.280	10.355	10.355	10.655	-
2d	10.02	-	-	-	-	-	10.023
4s	10.571	12.834	10.420	10.655	10.567	11.094	10.579
3d	10.358	-	-	-	-	-	10.355
6S	11.0198	-	-	-	-	-	11.019
5d	10.873	-	-	-	-	-	10.876

TABLE 3: Mass spectra of B_C *meson* in comparison with other works [$r_0 = 0.4256\,fm$, $a = 4.196\,GeV/fm$, $d = -2.6064\,GeV/(fm)^2$, $b = 9.5675\,GeV.fm$, $P = 6.0631\,GeV/(fm)^4$].

Type	present work	[30–32]	[33, 34]	[35–37]	[22, 23]	EXP [26, 27]
1s	6.227	6.349	6.264	6.270	6.278	6.277
1p	6.287	6.715	6.700	6.699	6.486	-
2s	6.714	6.821	6.856	6.853	6.866	-
1d	6.398	-	-	-	6.772	-
2p	6.759	7.102	7.108	7.091	6.973	-
3s	7.09	7.175	7.244	7.193	7.181	-
4s	7.386	-	-	-	7.369	-
2d	6.85	-	-	-	7.128	-
4d	7.469	-	-	-	-	-
1g	6.728	-	-	-	-	-

no enough experimental data to compare with. In conclusion, comparing with the experimental data, we found that our results are better than those given by previous theoretical estimates.

Data Availability

The information given in our tables is available for readers in the original references listed in our work.

Disclosure

Hesham Mansour is a Fellow of the Institute of Physics (FInstP).

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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