Hindawi
Advances in High Energy Physics
Volume 2018, Article ID 4537058, 7 pages
https://doi.org/10.1155/2018/4537058

Research Article

A Covariant Canonical Quantization of General Relativity

Stuart Marongwe

Department of Physics and Astronomy, Botswana International University of Science and Technology, P. Bag 16, Palapye, Botswana

Correspondence should be addressed to Stuart Marongwe; stuartmarongwe@gmail.com

Received 13 June 2018; Accepted 8 November 2018; Published 19 December 2018

Guest Editor: Farook Rahaman

A Hamiltonian formulation of General Relativity within the context of the Nexus Paradigm of quantum gravity is presented. We show that the Ricci flow in a compact matter free manifold serves as the Hamiltonian density of the vacuum as well as a time evolution operator for the vacuum energy density. The metric tensor of GR is expressed in terms of the Bloch energy eigenstate functions of the quantum vacuum allowing an interpretation of GR in terms of the fundamental concepts of quantum mechanics.

1. Introduction

The gravitational field which is elegantly described by Einstein's field equations has so far eluded a quantum description. Much effort has been placed into formulating General Relativity (GR) in terms of Hamilton's equations since a Hamiltonian formulation of a classical field theory leads naturally to its quantization. The earliest such attempt is the ADM formalism [1], named for its authors Richard Arnowitt, Stanley Deser, and Charles W. Misner first published in 1959. This formalism starts from the assumption that space is foliated into a family of time slices Σ_t, labeled by their time coordinate t and with space coordinates on each slice given by x^k. The dynamic variables of this theory are then taken to be the metric tensor of three-dimensional spatial slices $\gamma_{ij}(t.x^k)$ and their conjugate momenta $\pi^{ij}(t.x^k)$. Using these variables it is possible to define a Hamiltonian and thereby write the equations of motion for GR in Hamilton's form. The time slices are then welded together using four Lagrange multipliers and components of a shift vector field. An extensive review of this formalism can be found in the literature notably in [2–5].

The ADM formalism was first applied by Bryce De Witt in 1967 [6] to quantize gravity which resulted in the Wheeler–De Witt equation of quantum gravity. It is a functional differential equation in which the three-dimensional spatial metrics have the form of an operator acting on a wave function. This wave function contains all of the information about the geometry and matter content of the universe of each time slice. However, the Hamiltonian no longer determines the evolution of the system and leads to the problem of timelessness [7, 8]. Hawking rightly points out that the very act of splitting space-time into space and time destroys the spirit of GR (general covariance) and therefore not much can be gained from this approach to quantization of gravity. Perturbative covariant approaches to the problem of quantum gravity have an inherent weakness in that they depend on a fine classical background. It is therefore difficult to obtain a self-consistent quantum theory of gravity with a classical background space-time. Steven Carlip [9, 10] and Claus Kiefer [11, 12] have made excellent and extensive reviews of the problems faced by current approaches to the problem of quantum gravity. Carlip [9] in particular singles out the lack of a firm conceptual understanding of the foundational concepts of quantum gravity as the source of much of the difficulty in understanding quantum gravity. These deep conceptual issues result in technical problems in the attempt to develop a consistent quantum theory of gravity.

In this paper we report a successful covariant canonical quantization of the gravitational field which preserves the success of GR while simultaneously explaining Dark Energy (DE) and Dark Matter (DM). This approach to quantization takes place in 4-space of metric signature (-1,1,1,1) in which the quanta are excitations of the quantum vacuum called Nexus gravitons. Though the Nexus Paradigm has been introduced in the following papers [13–15], the aim of this study is to explicitly express the Hamiltonian formulation of the theory using the Bloch eigenstate functions of the quantum vacuum. These wave functions contain information about the energy state of the quantum vacuum which in turn dictates the geometry of space-time.

2. Methods

Our first step towards a covariant canonical quantization begins with defining a quantized space-time and its quanta. We then modify Einstein's vacuum equations to be consistent with the quantized space-time followed by the defining of Hamilton's equations of the quantized space-time. This step is then followed by the Poisson brackets which provide the bridge between classical and quantum mechanics (QM). The covariant canonical quantization procedure is carried out within the context of the Nexus Paradigm of quantum gravity.

2.1. Quantization of 4-Space and the Nexus Graviton. The primary objective of physics is the study of functional relationships amongst measurable physical quantities. In particular, a unifying paradigm of physical phenomena should reveal the functional relationship between the fundamental physical quantities of 4-space and 4-momentum. Currently GR and QM offer the best predictions of the results of measurement of physical phenomena in their respective domains using different languages. GR describes gravitation in the language of geometry and thus far, it has been difficult to apply the the language of wave functions used in QM to give it a QM description. The problem of quantum gravity is therefore to interpret GR in terms of the wavefunctions of QM. Translating the language of measurement of GR into that of QM becomes the primary objective of this present attempt to resolve the problem of quantum gravity.

Measurements in GR take place in a local patch of a Reimannian manifold. This local patch can be considered as a flat Minkowski space. The line element in Minkowski space which is the subject of measurement can be computed through the inner product of the local coordinates as

$$\begin{aligned}\Delta x^{\mu}\Delta x_{\mu} &= \Delta x^2 + \Delta y^2 + \Delta z^2 - c^2\Delta t^2 \\ &= (A\Delta x + B\Delta y + C\Delta z + iDc\Delta t) \\ &\quad \cdot (A\Delta x + B\Delta y + C\Delta z + iDc\Delta t)\end{aligned} \tag{1}$$

On multiplying the right hand side we see that to get all the cross terms such as $\Delta x\Delta y$ to cancel out we must assume

$$\begin{aligned}AB + BA &= 0 \\ A^2 = B^2 &= \cdots = 1\end{aligned} \tag{2}$$

The above conditions therefore imply that the coefficients (A, B, C, D) generate a Clifford algebra and therefore must be matrices. We rewrite these coefficients in the 4-tuple form as $(\gamma^1, \gamma^2, \gamma^3, \gamma^0)$ which may be summarized using the Minkowski metric on space-time as follows:

$$\{\gamma^{\mu}, \gamma^{\nu}\} = 2\eta^{\mu\nu} \tag{3}$$

The gammas are of course the Dirac matrices. Thus in order to satisfy (1) we can express a displacement 4-vector as

$$\Delta x^{\mu} = a\gamma^{\mu} \tag{4}$$

where a is the amplitude of a displacement vector in Minkowski space. If we consider the Hubble diameter as the maximum dimension of the local patch of space then $a = r_{HS}$ where r_{HS} is the Hubble radius. This choice of a maximum amplitude is justified from the fact that we cannot physically interact with objects beyond the Hubble 4-radius. It is important to note that the line element is the square of the amplitude of the displacement 4-vector. Thus if we are to express GR in terms of the language of QM we must make the radical assumption that the displacement vectors in Minkowski space are pulses of 4-space which can be expressed in terms of Fourier functions as follows:

$$\begin{aligned}\Delta x_n^{\mu} &= \frac{2r_{HS}}{n\pi}\gamma^{\mu}\int_{-\infty}^{\infty} \operatorname{sinc}(kx)\, e^{ikx}dk \\ &= \gamma^{\mu}\int_{-\infty}^{\infty} a_{nk}\varphi_{(nkx)}dk\end{aligned} \tag{5}$$

$$\text{Where } \frac{2r_{HS}}{n\pi} = \sum_{k=-\infty}^{k=+\infty} a_{nk} \tag{6}$$

Here $\varphi_{(nkx)} = \operatorname{sinc}(kx)e^{ikx}$ are Bloch energy eigenstate functions. The Bloch functions can only allow the four wave vector to assume the following quantized values:

$$k^{\mu} = \frac{n\pi}{r_{HS}^{\mu}} \qquad n = \pm 1, \pm 2 \ldots 10^{60} \tag{7}$$

The minimum 4-radius in Minkowski space is the Planck 4-length since it is impossible to measure this length without forming a black hole. The 10^{60} states arise from the ratio of Hubble 4-radius to the Planck 4-length. The displacement 4-vectors in each eigenstate of space-time generate an infinite Bravais 4-lattice. Also, condition (7) transforms (5) to

$$\Delta x_n^{\mu} = \gamma^{\mu}\int_{-nk_1}^{nk_1} a_{nk}\varphi_{(n,k,x)}dk \tag{8}$$

The second assumption we make is that each displacement 4-vector is associated with a conjugate pulse of four-momentum which can also be expressed as a Fourier integral

$$\begin{aligned}\Delta p_{(n)\mu} &= \frac{2np_1}{\pi}\gamma_{\mu}\int_{-nk_1}^{nk_1} \varphi_{(n,k,x)}dk \\ &= \gamma_{\mu}\int_{-nk_1}^{nk_1} c_{nk}\varphi_{(n,k,x)}dk\end{aligned} \tag{9}$$

where $p_{(1)\mu}$ is the four-momentum of the ground state.

A displacement 4-vector and its conjugate 4-momentum satisfy the Heisenberg uncertainty relation

$$\Delta x_n \Delta p_n \geq \frac{\hbar}{2} \tag{10}$$

The Uncertainty Principle plays the important role of generating a vector bundle, out of the total uncertainty space E of trivial displacement 4-vectors from which a closed compact manifold X is formed, i.e., $(\pi : E \longrightarrow X)$. Each point on the manifold is associated with a vector which is along a normal to the manifold.

The wave packet described by (8) is essentially a particle of four-space. The spin of this particle can be determined from the fact that each component of the four-displacement vector transforms according to the law

$$\Delta x'^{\mu}_{n} = \exp\left(\frac{1}{8}\omega_{\mu\nu}\left[\gamma_\mu, \gamma_\nu\right]\right)\Delta x^{\mu}_{n} \quad (11)$$

where $\omega_{\mu\nu}$ is an antisymmetric 4x4 matrix parameterizing the transformation.

Therefore, each component of the 4-vector has a spin half. A summation of all the four half spins yields a total spin of *2*. We give the name the Nexus graviton to this particle of 4-space since the primary objective of quantum gravity is to find the nexus between the concepts of GR and QM.

From (7) the norm squared of the 4-momentum of the n-th state graviton is

$$(\hbar)^2 k^\mu k_\mu = \frac{E_n^2}{c^2} - \frac{3\left(nhH_0\right)^2}{c^2} = 0 \quad (12)$$

where H_0 is the Hubble constant (2.2 x 10^{-18} s^{-1}) and can be expressed in terms of the cosmological constant, Λ, as

$$\Lambda_n = \frac{E_n^2}{(hc)^2} = \frac{3k_n^2}{(2\pi)^2} = n^2\Lambda \quad (13)$$

We infer from (13) that the Nexus graviton (or displacement 4-vector) in the n-th quantum state forms a trivial vector bundle via the Uncertainty Principle which generates a compact Riemannian manifold of positive Ricci curvature that can be expressed in the form

$$G_{(nk)\mu\nu} = n^2\Lambda g_{(n,k)\mu\nu} \quad (14)$$

where $G_{(nk)\mu\nu}$ is the Einstein tensor of space-time in the n-th state. Equation (14) depicts a contracting geodesic ball and as explained in [13–15] this is DM which is an intrinsic compactification of the elements space-time in the n-th quantum state. This compactification is a result of the superposition of several plane waves as described by (8) to form an increasingly localized wave packet as more waves are added. Similary the converse is also true. The loss of harmonic waves expands the elements of space-time which gives rise to DE. Thus the DE arises from the emission of a ground state graviton such that (14) becomes

$$G_{(nk)\mu\nu} = \left(n^2 - 1\right)\Lambda g_{(n,k)\mu\nu} \quad (15)$$

These are Einstein's vacuum field equations in the quantized space-time. If the graviton field is perturbed by the presence of baryonic matter then (15) becomes

$$\begin{aligned} G_{(nk)\mu\nu} &= kT_{\mu\nu} + \left(n^2 - 1\right)\Lambda g_{(n,k)\mu\nu} \\ &= kT_{\mu\nu} + \left(n^2 - 1\right)k\rho_{DE} g_{(n,k)\mu\nu} \end{aligned} \quad (16)$$

where ρ_{DE} is the density of DE.

From [14] the static solution to (14) for a spherically symmetric Nexus graviton is computed as

$$\begin{aligned} ds^2 = &-\left(1-\left(\frac{2}{n^2}\right)\right)c^2dt^2 + \left(1-\left(\frac{2}{n^2}\right)\right)^{-1}dr^2 \\ &+ r^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right) \end{aligned} \quad (17)$$

There are no singularities in (17). At high energies, characterized by microcosmic scale wavelengths of the Nexus graviton and high values of n, space-time is flat and highly compact. This implies a continuous length contraction of the local coordinates via the addition of more waves resulting in an increase in localization. Also, from a world line perspective of a test particle, it implies that the deviation from a rectilinear trajectory due to uncertainities in its location in 4-space is small. Thus at microcosmic scales, in the realm of subatomic particles space-time is extremely flat. The world line begins to substantially deviate from a well defined rectilinear path at low energies as the uncertainities in its location are increased. That is, at low values of n, the world line becomes degenerate allowing a multiplicity of trajectories which in GR are averaged by the Ricci curvature tensor. Thus gravity is a low energy phenomenon which vanishes asymptotically at high energies.

The gravitational effects of the Nexus graviton manifest at large scales and for galaxies, these effects begin to manifest when the density of DE is equal to the density of baryonic matter as described by (16) or when the acceleration due to baryonic matter is equal and opposite to the acceleration due to the emission of the ground state graviton as described in [14]. A solution to eqn (16) from [14] in the weak field at galactic scale radii is

$$\frac{d^2r}{dt^2} = \frac{GM(r)}{r^2} + H_0 v_n - H_0 c \quad (18)$$

Here c is the speed of light.

The first term on the right is the Newtonian gravitational acceleration, the second term is a radial acceleration due to space-time in the n-th quantum state, and the final term is acceleration due to DE. The dynamics becomes strongly non-Newtonian when

$$\frac{GM(r)}{r^2} = H_0 c = \frac{v_n^2}{r} \quad (19)$$

These are conditions in which the acceleration due to baryonic matter is annulled by that due to the DE. Under such conditions

$$r = \frac{v_n^2}{H_0 c} \quad (20)$$

Substituting for r in (19) yields

$$v_n^4 = GM(r)H_0 c \quad (21)$$

This is the Baryonic Tully–Fisher relation. The conditions permitting the DE to cancel out the acceleration due to baryonic matter leave quantum gravity as the unique source of gravity. Thus condition (19) reduces (18) to

$$\frac{d^2r}{dt^2} = \frac{dv_n}{dt} = H_0 v_n \quad (22)$$

from which we obtain the following equations of galactic and cosmic evolution:

$$r_n = \frac{1}{H_0}e^{(H_0 t)}\left(GM(r)H_0 c\right)^{1/4} \quad (23)$$

$$v_n = e^{(H_0 t)} \left(GM(r) H_0 c\right)^{1/4} \quad (24)$$

$$a_n = H_0 e^{(H_0 t)} \left(GM(r) H_0 c\right)^{1/4} \quad (25)$$

Here r_n is the radius of curvature of space-time in the n-th quantum state (which is also the radius of the n-th state nexus graviton), v_n the radial velocity of objects embedded in that space-time, and a_n their radial acceleration within it. The amplification of the radius of curvature with time explains the existence of ultra-diffuse galaxies and the spiral shapes of most galaxies. The increase in radial velocity with time explains why early type galaxies composed of population II stars are fast rotators. Equation (25) explains late time cosmic acceleration which began once condition (19) was satisfied or equivalently from (16), when the density of baryonic matter was at the same value as that of DE. Thus condition (19) also explains the Coincidence Problem.

2.2. Canonical Transformations in the Nexus Paradigm. In classical mechanics, a system is described by n independent coordinates $(q_1, q_2, \dots q_n)$ together with their conjugate momenta $(p_1, p_2, \dots p_n)$. In the Nexus Paradigm, the labeling q_n refers to a creation of a Nexus graviton in the n-th quantum state associated with a conjugate momentum p_n. The Hamiltonian equation

$$\dot{q}_n = \frac{\partial H}{\partial p_n} \quad (26)$$

refers to the rate of expansion or contraction of space-time generated by the addition or emission of harmonic waves to the Nexus graviton and

$$\dot{p}_n = -\frac{\partial H}{\partial q_n} \quad (27)$$

refers to the force field associated with the addition or emission of harmonic waves to the Nexus graviton. It is important to note that this force field generates an isotropic expansion or contraction of space-time within the spatiotemporal dimensions of the graviton.

We can also rewrite the Hamiltonian equations in terms of Poisson brackets which are invariant under canonical transformations as

$$\begin{aligned} \dot{q}_n &= \{q_n, H\}, \\ \dot{p}_n &= \{p_n, H\} \end{aligned} \quad (28)$$

The Poisson brackets provide the bridge between classical and QM and in QM, these brackets are written as

$$\begin{aligned} \dot{\widehat{q}}_n &= \left[\widehat{q}_n, \widehat{H}\right], \\ \dot{\widehat{p}}_n &= \left[\widehat{p}_n, \widehat{H}\right] \end{aligned} \quad (29)$$

and obey the following commutation rules

$$\begin{aligned} \left[\widehat{q}_n, \widehat{q}_s\right] &= 0, \\ \left[\widehat{p}_n, \widehat{p}_s\right] &= 0, \\ \left[\widehat{q}_n, \widehat{p}_s\right] &= \delta_{ns} \end{aligned} \quad (30)$$

2.3. The Hamiltonian Formulation for the Quantum Vacuum. The Nexus graviton is a pulse of 4-space which can only expand or contract and does not execute translational motion implying that the Hamiltonian density of the system is equal to the Lagrangian density.

$$H = L \quad (31)$$

GR is a metric field in which the energy density in four-space determines its value. Since the Bloch energy eigenstate functions determine the energy of space-time, we must seek to express the metric in terms of the Bloch wave functions. To this end we shall express the eigenstate four-space components of the Nexus graviton in the k-th band as

$$\Delta x^\mu_{n,k} = z^\mu_{n,k} = a_{nk} \gamma^\mu \operatorname{sinc}(kx)\, e^{ikx} \quad (32)$$

such that an infintesimal four-radius within the k-th band is computed as

$$dr_{n,k} = \frac{\partial z^\mu_{n,k}}{\partial k^\mu} dk^\mu = ixa_{nk} \gamma^\mu \operatorname{sinc}(kx)\, e^{ikx} dk \quad (33)$$

In (33) the first-order derivative of the periodic sinc function is equal to zero for all integral values of n. The interval within the band is then computed as

$$\begin{aligned} ds^2 = dr^2_{n,k} &= \frac{\partial z^\mu_{n,k}}{\partial k^\mu} \frac{\partial z^\mu_{n,k}}{\partial k^\nu} dk^\mu dk^\nu \\ &= \alpha\beta \gamma^\mu \gamma^\nu \varphi_{(n,k,x)} \varphi_{(n,k,x)} dk^\mu dk^\nu \end{aligned} \quad (34)$$

Here the interval is described in terms of the reciprocal lattice with $\alpha = ix_\mu a_{nk}$ and $\beta = ix_\nu a_{nk}$. The metric tensor of four-space in the k-th band is therefore associated with the Bloch energy eigenstate functions of the quantum vacuum as follows:

$$g^{\mu\nu}_{(n,k)} = \gamma^\mu \gamma^\nu \varphi_{(n,k,x)} \varphi_{(n,k,x)} = \eta^{\mu\nu} \varphi_{(n,k,x)} \varphi_{(n,k,x)} \quad (35)$$

Equation (35) translates the geometric language of GR into the wave function language of QM.

We initiate the translation procedure of GR into QM by first finding the Lagrange density for the quantized vacuum from (15) which following Einstein and Hilbert is found to be

$$L_{EH} = k\left(R - 2\left(n^2 - 1\right)\Lambda\right) \quad (36)$$

Given that the Einstein tensor in a compact manifold is equal to the Ricci flow

$$-\partial_t g_{\mu\nu} = \Delta g_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = G_{\mu\nu} \quad (37)$$

therefore the equations of motion of the quantum vacuum obtained from (36) yield the following quantized field equations:

$$\begin{aligned} &-\partial_t \left(\gamma^\mu \varphi_{(n,k,x)} \gamma^\nu \varphi_{(n.k.x)}\right) \\ &\quad = \left(n^2 - 1\right) \Lambda \gamma^\mu \varphi_{(n,k,x)} \gamma^\nu \varphi_{(n,k,x)} \end{aligned} \quad (38)$$

which can be written as

$$\begin{aligned} &\partial_t \left(\gamma^\mu \varphi_{(n-1,k,x)} \gamma^\nu \varphi_{(n+1,k,x)}\right) \\ &\quad = \frac{-i^2}{(2\pi)^2} \nabla_\mu \nabla_\nu \gamma^\mu \varphi_{(n-1,k,x)} \gamma^\nu \varphi_{(n+1,k,x)} \\ &\quad = \frac{1}{4\pi^2} \nabla_\mu \nabla_\nu \gamma^\mu \varphi_{(n-1,k,x)} \gamma^\nu \varphi_{(n+1,k,x)} \end{aligned} \quad (39)$$

where

$$\varphi_{(n-1,k,x)} = \operatorname{sinc}\left((n-1) k_1 x\right) e^{i(n-1)k_1 x} \quad (40)$$

$$\varphi_{(n+1,k,x)} = \operatorname{sinc}\left((n+1)k_1 x\right) e^{i(n+1)k_1 x} \quad (41)$$

$$\frac{3k_1^2}{(2\pi)^2} = \Lambda \quad (42)$$

For large values of n the Bloch functions statisfy the condition

$$\varphi_{(n-1,k,x)} \approx \varphi_{(n,k,x)} \approx \varphi_{(n+1,k,x)} \quad (43)$$

The quantum vacuum can therefore be interpreted as a system in which there are a constant annihilation and creation of quanta as implied by (40) and (41) which causes the Nexus graviton to either expand or contract.

2.4. The Hamiltonian Formulation in the Presence of Matter Fields. We now seek to introduce matter fields into the quantum vacuum. If we compare the quantized metric of (17) describing the gravity within a Nexus graviton with the Schwarzschild metric describing the gravitational field around baryonic matter, we notice that we can describe the gravitational field around baryonic matter in terms of the quantum state of space-time through the relation

$$\frac{2}{n^2} = \frac{2GM}{c^2 r} \quad (44)$$

This yields a relationship between the quantum state of space-time and the amount of baryonic matter embedded within it as follows:

$$n^2 = \frac{c^2 r}{GM} = \frac{c^2}{v^2} \quad (45)$$

Equation (45) reveals a family of concentric stable circular orbits $r_n = n^2 GM/c^2$ with corresponding orbital speeds of $v_n = c/n$. Thus in the Nexus Paradigm, unlike in GR, the innermost stable circular orbit ocurrs at $n = 1$ or at half the Schwarzschild radius which implies that the event horizon predicted by the Nexus Paradigm is half the size predicted in GR. Also (45) reveals how the Nexus graviton in the n-th quantum state imitates DM if M is considered as the apparent mass of the DM. Through this comparision, we can also deduce that the deflection of light through gravitational lensing by space-time in the n-th quantum state is

$$\alpha = \frac{4}{n^2} \quad (46)$$

Thus gravitational lensing can be used to constrain the value of the quantum state n of space-time within a lensing system.

Having obtained the relationship between the quantum state of space-time and the amount of baryonic matter embedded within it, the result of (45) is then added to (39) to yield the time evolution of the quantum vacuum in the presence of baryonic matter as

$$\begin{aligned}\partial_t &\left(\gamma^\mu \varphi_{(\boldsymbol{n-1,n,k,x})} \gamma^\nu \varphi_{(\boldsymbol{n+1,n,k,x})}\right) \\ &= \frac{1}{4\pi^2} \nabla_\mu \nabla_\nu \gamma^\mu \varphi_{(n-1,k,x)} \gamma^\nu \varphi_{(n+1,k,x)} \\ &\quad - n^2 \Lambda \gamma^\mu \varphi_{(n,k.x)} \gamma^\nu \varphi_{(n,k,x)}\end{aligned} \quad (47)$$

Thus the time evolution of the quantum vacuum in the presence of matter resembles thermal flow in the presence of a heat sink. The second term on the R.H.S. of (47) is an 8-cell or 4-cube that operates as a sinc filter with a four-wave cut-off of

$$k_c = nk_1 = 2\pi \sqrt{\frac{\Lambda}{3} \bullet \frac{c^2 r}{GM}} \quad (48)$$

The filtration of high frequencies from the vacuum lowers the quantum vacuum state and generates a gravitational field in much the same way as the Casimir Effect is generated. The physical process of filtration occurs as follows: A 4-cube with baryonic matter embedded with in it acquires inertia. The inertia gives the cell inductive impedance and becomes less reponsive to high frequency vibrations of the quantum vacuum. Thus the heavier the cell, the lower the cut-off frequency. Equation (48) shows that when the radius of the cell is equal to half the Schwarzschild radius, the only permited frequency is that of the ground state. Thus the inside of a black hole is in the ground state having a negative metric signature.

We now introduce a test particle of mass m, into the quantum vacuum perturbed by matter fields. The particle will flow along with the Ricci flow and the Hamiltonian of the system becomes

$$\begin{aligned}&\widehat{H}\left(\gamma^\mu \varphi_{(n-1,n,k,x)} \gamma^\nu \varphi_{(n+1,n,k,x)}\right) \\ &\quad = \frac{\widehat{\mathrm{P}}_\mu \widehat{\mathrm{P}}_\nu}{2m} \left(\gamma^\mu \varphi_{(n-1,k,x)} \gamma^\nu \varphi_{(n+1,k,x)}\right) \\ &\quad - V\left(\gamma^\mu \varphi_{(n,k,x)} \gamma^\nu \varphi_{(n,k,x)}\right)\end{aligned} \quad (49)$$

Here

$$V = n^2 \frac{\hbar^2}{2m} \Lambda = \frac{rc^2}{GM} \cdot \frac{3\hbar^2 k_1^2}{2m} \quad (50)$$

$$\widehat{P}_\mu = -i\hbar \nabla_\mu \quad (51)$$

$$\widehat{H} = -i\hbar \partial_t \quad (52)$$

Equation (49) is equivalent to (31) in which the Hamiltonian is equal to the Lagrangian.

$$\begin{aligned}&\widehat{H}\left(\gamma^\mu \varphi_{(n-1,n,k,x)} \gamma^\nu \varphi_{(n+1,n,k,x)}\right) \\ &\quad = \mathrm{L}\left(\gamma^\mu \varphi_{(n-1,n,k,x)} \gamma^\nu \varphi_{(n+1,n,k,x)}\right)\end{aligned} \quad (53)$$

The Lagrangian is not relativistic because the energy reference frames or space-time states involved in the transition dynamics $n-1$, n, and $n+1$ are separated by a small energy gap, $E = \sqrt{3} \bullet hH_0$. The exchanged quantum of energy between states is responsible for "welding" them together.

Equation (50) is the gravitational interaction in reciprocal space-time. The weakness of the gravitational interaction is due to the small value of the cosmological constant.

3. Line Elements and Information in K-Space

A close inspection of (34) reveals the relationship between a line element in 4-space and its conjugate line in K-space

$$ds^2 = \alpha\beta d\kappa^2 \quad (54)$$

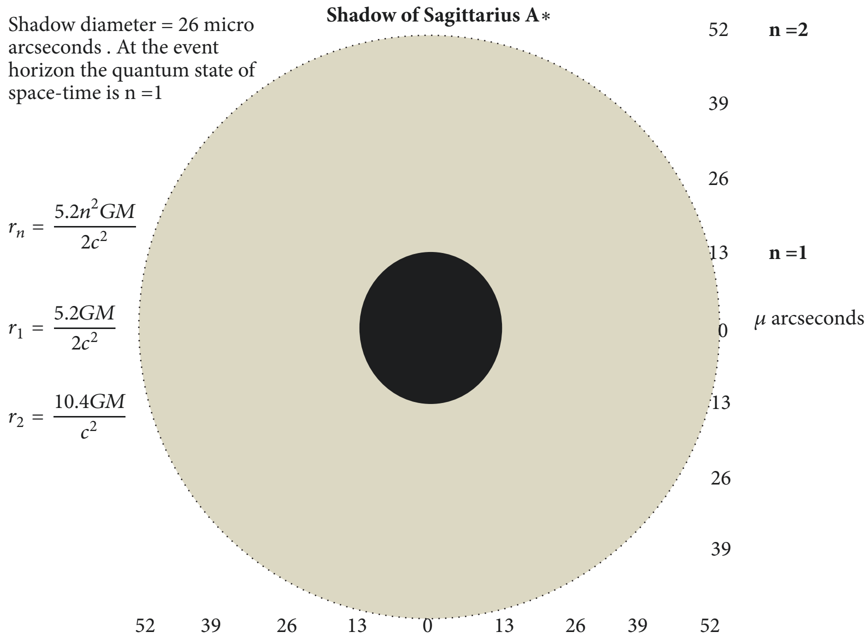

FIGURE 1: Shadow of Sagittarius A∗.

where

$$d\kappa^2 = \gamma^{\mu}\varphi_{(n,k,x)}\gamma^{\nu}\varphi_{(n,k,x)}dk^{\mu}dk^{\nu} \quad (55)$$

The term $\gamma^{\mu}\varphi_{(n,k,x)}\gamma^{\nu}\varphi_{(n,k,x)}$ refers to the normalized information flux density passing through an elementary surface $dk^{\mu}dk^{\nu}$ in K-space. The gradient of the flux density yields the baseband bandwidth $k_{(n)\mu}$.

$$\partial_{\mu}\gamma^{\mu}\varphi_{(n,k,x)}\gamma^{\nu}\varphi_{(n,k,x)} = k_{(n)\mu}\gamma^{\mu}\varphi_{(n,k,x)}\gamma^{\nu}\varphi_{(n,k,x)} \quad (56)$$

This gradient is also a measure of the acutance or sharpness of the information. A large bandwidth provides detailed information while a small bandwidth provides diffuse information. Equation (49) describes the diffusion of information as it flows into a gravitational well. Unitarity is preserved if the flux is summed over the total diffusion surface.

$$\iint_0^{\Sigma} \boldsymbol{\gamma}^{\boldsymbol{\mu}}\varphi_{(n,k,x)}\boldsymbol{\gamma}^{\boldsymbol{\nu}}\varphi_{(n,k,x)}dk_{\mu}dk_{\nu} = 1 \quad (57)$$

Here the integral implies that one bit of information is found on a surface $\Sigma = k_n^2$ in K-space which corresponds to an area $A_n = 4\pi^2/k_n^2$ in 4-space. The k_1 bandwidth permitted within a black hole generates the smallest elementary surface or pixel in K-space of k_1^2 suggesting that information inside a blackhole is diluted to one bit per area equivalent to the square of the Hubble 4-radius in 4-space or from (48) the information surface density is $k_1^2 = 4\pi^2\Lambda/3$. Thus the cosmological constant can also be interpreted as a unit of information surface density. The ratio of the Planck surface in K-space to the smallest elementary surface in K-space yields 10^{120}. This huge number represents the maximum number of pixels that can fit on the largest surface in K-space. Hence the expression $W = k_{\mathrm{p}}^2/k_{\mathrm{n}}^2$ represents the number of empty pixel slots available in the n-th quantum state or the number of degenerate energy levels.

$$|\varphi\rangle_n = c_1|\psi_1\rangle_n + c_2|\psi_2\rangle_n \ldots c_W|\psi_W\rangle_n. \quad (58)$$

The entropy of the n-state therefore becomes

$$S = k_B \ln W = k_B \ln \frac{k_{\mathrm{p}}^2}{k_{\mathrm{n}}^2} = k_B \ln \frac{A_n}{l_{\mathrm{p}}^2} = k_B \ln \frac{c^3 A_n}{G\hbar} \quad (59)$$

Here we observe that objects fall into a gravitational field because it leads to an increase in entropy. In strong fields the information is delocalized. A black hole does not annihilate information; it simply diffuses or dilutes it by increasing the entropy. It can also be interpreted as a low pass filter that permits one bit of information to pass per Hubble time. Since information is delocalized close to a black hole, then we do not expect the Event Horizon Telescope to observe a silhouette surrounded by a well defined accretion disc but a circular silhouette surrounded by a cloud of degenerate (delocalized) matter (grey area) as depicted in Figure 1.

The radii are calculated from (45) and the magnification factor, 5.2/2, arises from gravitational lensing. The silhouette is half the size predicted in GR.

4. Discussion

A successful covariant canonical quantization of the gravitational field has been presented in which we find that gravity is akin to a thermal flow of space-time and that space-time can also be described in terms of a reciprocal lattice. The presence of matter creates an impure lattice and a potential well arises in the region of perturbation through filtration of high frequencies from the quantum vacuum. A test particle flows along with the quantum vacuum and the presence of a potential well will cause it to flow towards the sink. This formulation will find important applications in high energy lattice gauge field theories where it may help expand the standard model of particle physics by eliminating divergent terms in the current theory and predicting hitherto unknown phenomena.

Data Availability

The results of this theoretical research will be confirmed by data from the Event Horizon Collaboration on the size and shape of the shadow of Sgr A∗.

Conflicts of Interest

The author declares no conflicts of interest.

Acknowledgments

The author gratefully appreciates the funding and support from the Department of Physics and Astronomy at the Botswana International University of Science and Technology (BIUST).

References

[1] R. Arnowitt, S. Deser, and C. W. Misner, "Dynamical structure and definition of energy in general relativity," *Physical Review A: Atomic, Molecular and Optical Physics*, vol. 116, no. 5, pp. 1322–1330, 1959.

[2] Y. Sendouda, N. Deruelle, M. Sasaki, and D. Yamauchi, "Higher curvature theories of gravity in the adm canonical formalism," *International Journal of Modern Physics: Conference Series*, vol. 01, pp. 297–302, 2011.

[3] E. Gourgoulhon, "3+1 formalism and bases for numerical relativity," 2007, https://arxiv.org/abs/gr-qc/0703035.

[4] N. Kiriushcheva and S. Kuzmin, "The Hamiltonian formulation of general relativity: myths and reality," *Open Physics*, vol. 9, no. 3, pp. 576–615, 2011.

[5] E. Anderson, "Problem of time in quantum gravity," *Annalen der Physik*, vol. 524, no. 12, pp. 757–786, 2012.

[6] B. S. DeWitt, "Quantum theory of gravity. I. The canonical theory," *Physical Review A: Atomic, Molecular and Optical Physics*, vol. 160, article 1113, 1967.

[7] M. Bojowald and P. A. Hohn, "An Effective approach to the problem of time," *Classical and Quantum Gravity*, vol. 28, no. 3, Article ID 035006, 2011.

[8] C. Kiefer, "Does time exist in Quantum Gravity?" in *Towards a Theory of Spacetime Theories. Einstein Studies*, D. Lehmkuhl, G. Schiemamm, and E. Scholz, Eds., vol. 13, pp. 287–295, Birkhauser, New York, NY, USA, 2017.

[9] S. Carlip, "Quantum gravity: a progress report," *Reports on Progress in Physics*, vol. 64, no. 8, pp. 885–942, 2001.

[10] S. Carlip, D.-W. Chiou, W.-T. Ni, and R. Woodard, "Quantum gravity: a brief history of ideas and some prospects," *International Journal of Modern Physics D: Gravitation, Astrophysics, Cosmology*, vol. 24, no. 11, Article ID 1530028, 23 pages, 2015.

[11] C. Kiefer, "Quantum gravity: general introduction and recent developments," *Annalen der Physik*, vol. 15, no. 1-2, pp. 129–148, 2006.

[12] C. Kiefer, "Conceptual problems in quantum gravity and quantum cosmology," *ISRN Mathematical Physics*, vol. 2013, Article ID 509316, 17 pages, 2013.

[13] S. Marongwe, "The Nexus graviton: A quantum of Dark Energy and Dark Matter," *International Journal of Geometric Methods in Modern Physics*, vol. 11, no. 06, Article ID 1450059, 2014.

[14] S. Marongwe, "The Schwarzschild solution to the Nexus graviton field," *International Journal of Geometric Methods in Modern Physics*, vol. 12, no. 4, Article ID 1550042, 10 pages, 2015.

[15] S. Marongwe, "The electromagnetic signature of gravitational wave interaction with the quantum vacuum," *International Journal of Modern Physics D: Gravitation, Astrophysics, Cosmology*, vol. 26, no. 3, Article ID 1750020, 15 pages, 2017.