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The doubly charmed pseudoscalar tetraquarks $T^{++}_{cc;\bar{s}\bar{s}}$ and $T^{++}_{cc;\bar{d}\bar{s}}$

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Abstract

The mass and coupling of the doubly charmed $J^P = 0^-$ diquark–antidiquark states $T^{++}_{cc;\bar{s}\bar{s}}$ and $T^{++}_{cc;\bar{d}\bar{s}}$ that bear two units of the electric charge are calculated by means of QCD two-point sum rule method. Computations are carried out by taking into account vacuum condensates up to and including terms of tenth dimension. The dominant S-wave decays of these tetraquarks to a pair of conventional $D_s^+ D_{s0}^{*+}(2317)$ and $D^+ D_{s0}^{*+}(2317)$ mesons are explored using QCD three-point sum rule approach, and their widths are found. The obtained results $m_T = (4390 \pm 150)$ MeV and $\Gamma = (302 \pm 113$ MeV) for the mass and width of the state $T^{++}_{cc;\bar{s}\bar{s}}$, as well as spectroscopic parameters $\widetilde{m}_T = (4265 \pm 140)$ MeV and $\widetilde{\Gamma} = (171 \pm 52)$ MeV of the tetraquark $T^{++}_{cc;\bar{d}\bar{s}}$ may be useful in experimental studies of exotic resonances.

1. Introduction

The investigation of exotic mesons, i.e. particles either with unusual quantum numbers that are not accessible in the quark–antiquark $\overline{q}q$ model or built of four valence quarks (tetraquarks) remains among interesting and important topics in high energy physics. Existence of multiquark

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hadrons does not contradict to first principles of QCD and was theoretically predicted already by different authors [1–3]. But only after experimental discovery of the charmonium-like resonance $X(3872)$ by Belle Collaboration in 2003 [4] the exotic hadrons became an object of rapidly growing studies. In the years that followed, various collaborations reported about observation of similar resonances in exclusive and inclusive hadronic processes. Theoretical investigations also achieved remarkable successes in interpretation of exotic hadrons by adapting existing methods to a new situation and/or inventing new approaches for their studies. Valuable experimental data collected during fifteen years passed from the discovery of the $X(3872)$ resonance, as well as important theoretical works constitute now the physics of the exotic hadrons [5–9].

One of the main problems in experimental investigations of the charmonium-like resonances is separation of tetraquark's effects from contributions of the conventional charmonium and its numerous excited states. Indeed, it is natural to explain neutral resonances detected in an invariant mass distribution of final mesons as ordinary charmonia: only detailed analyses may reveal their exotic nature. But there are few classes of tetraquarks which can not be confused with the charmonium states. The first class of such particles are resonances that bear the electric charge: it is evident that $\overline{q}q$ mesons are neutral particles. The first charged tetraquarks $Z_c^{\pm}(4430)$ were observed in decays of the B meson $B \to K\psi'\pi^{\pm}$ as resonances in the $\psi'\pi^{\pm}$ invariant mass distribution [10]. Later other charged resonances such as $Z_c^{\pm}(3900)$ were discoreved, as well.

The next group are resonances composed of more than two quark flavors. The quark content of such states can be determined from analysis of their decay products. The prominent member of this group is the resonance $X^{\pm}(5568)$ which is presumably composed of four distinct quark flavors. It was first observed in the $B_s^0\pi^{\pm}$ invariant mass distribution in the B_s^0 meson hadronic decay mode, and confirmed later with the B_s^0 meson's semileptonic decays by the D0 Collaboration [11,12]. However, the LHCb and CMS collaborations could not provide an evidence for its existence from analysis of relevant experimental data [13,14]. Therefore, the experimental status of the $X(5568)$ resonance remains unclear and controversial.

Resonances carrying a double electric charge constitute another very interesting class of exotic states, because the doubly charged resonances cannot be explained as conventional mesons [15, 16]. The doubly charged particles may exist as doubly charmed tetraquarks composed of the heavy diquark cc and light antidiquarks $\bar{s}\bar{s}$ or $\bar{d}\bar{s}$. In other words, they can contain two or three quark flavors. The diquark bb and antidiquark $\overline{uu}$ can also bind to form the doubly charged resonance $T^{--}_{bb;\bar{u}\bar{u}}$ containing only two quark spices. The states built of four quarks of different flavors can carry a double charge, as well [17]. The doubly charged molecular compounds with the quark content $\overline{QQ}qq$, where Q is c or b-quark were analyzed in Ref. [15]. In this paper the authors used the heavy quark effective theory to derive interactions between heavy mesons and coupled channel Schrodinger equations to find the bound and/or resonant states with various quantum numbers. It was demonstrated that, for example, D and D^* mesons can form doubly charged P-wave bound state with $J^P = 0^-$.

The class of exotic states composed of heavy cc and bb diquarks and heavy or light antidiquarks attracted already interests of scientists. The four-quark systems $QQ\bar{Q}\bar{Q}$ and $QQ\bar{q}\bar{q}$ were studied in Ref. [3,18,19] by adopting the conventional potential model with additive pairwise interaction of color-octet exchange type. The goal was to find four-quark states which are stable against spontaneous dissociation into two mesons. It turned out that within this approach there are not stable mesons built of only heavy quarks. But the states $QQ\bar{q}\bar{q}$ may form the stable composites provided the ratio m_Q/m_q is large. The same conclusions were drawn from a more general analysis in Ref. [20], where the only assumption made about the confining potential was its finiteness when two particles come close together. In accordance with predictions of this paper the isoscalar $J^P = 1^+$ tetraquark $T^-_{bb;\bar{u}\bar{d}}$ lies below the two B-meson threshold and hence, can decay only weakly. The situation with of $T_{cc;\bar{q}\bar{q}'}$ and $T_{bc;\bar{q}\bar{q}'}$ is not quite clear, but they may exist as unstable bound states. The stability of the $QQ\overline{qq}$ compounds in the limit $m_Q \to \infty$ was studied in Ref. [21] as well.

Production mechanisms of the doubly charmed tetraquarks in the ion, proton–proton and electron–positron collisions, as well as their possible decay channels were also examined in the literature [22–25]. The chiral quark models, the dynamical and relativistic quark models were employed to investigate properties (mainly to compute masses) of these exotic mesons [26–29]. The similar problems were addressed in the context of QCD sum rule method as well. The masses of the axial-vector states $T_{QQ;\bar{u}\bar{d}}$ were extracted from the two-point sum rules in Ref. [30]. The mass of the tetraquark $T^-_{bb;\bar{u}\bar{d}}$ in accordance with this work amounts to 10.2 ± 0.3 GeV, and is below the open bottom threshold. Within the same framework masses of the $QQ\bar{q}\bar{q}$ states with the spin-parity $0^-,\ 0^+,\ 1^-$ and 1^+ were computed in Ref. [31].

Recently interest to double-charm and double-bottom tetraquarks renewed after discovery of the doubly charmed baryon $\Xi^{++}_{cc} = ccu$ by the LHCb Collaboration [32]. Thus, in Ref. [33] the masses of the tetraquarks $T^-_{bb;\overline{u}\overline{d}}$ and $T^+_{cc;\overline{u}\overline{d}}$ were estimated in the context of a phenomenological model. The obtained prediction for $m = 10389 \pm 12$ MeV confirms that the isoscalar state $T^-_{bb\overline{u}\overline{d}}$ with spin-parity $J^P = 1^+$ is stable against strong and electromagnetic decays, whereas the tetraquark $T^+_{cc\overline{u}\overline{d}}$ lies above the open charm threshold D^0D^{*+} and can decay to these mesons. The various aspects of double- and fully-heavy tetraquarks were also considered in Refs. [34–46]. Works devoted to investigation of the hidden-charm (-bottom) tetraquarks containing $c\overline{c}$ ($b\overline{b}$) may also provide interesting information on properties of the heavy exotic states (see Ref. [47] and references therein).

The masses of the doubly charged exotic mesons built of four different quark flavors were extracted from QCD sum rules in Ref. [17]. The spectroscopic parameters and full width of the scalar, pseudoscalar and axial-vector doubly charged charm-strange tetraquarks $Z_{\bar{c}s} = [sd][\overline{uc}]$ were calculated in Ref. [48]. It was shown that width of these compounds evaluated using their strong decay channels ranges from $\Gamma_{\rm PS} = 38.10$ MeV in the case of the pseudoscalar resonance till $\Gamma_{\rm S} = 66.89$ MeV for the scalar state, which is typical for most of the diquark–antidiquark resonances.

In the present work we explore the pseudoscalar tetraquarks $T^{++}_{cc;\overline{s}\overline{s}}$ and $T^{++}_{cc;\overline{d}\overline{s}}$ that are doubly charmed and, at the same time doubly charged exotic mesons. Their masses and couplings are calculated using QCD two-point sum rules approach which is the powerful quantitative method to analyze properties of hadrons including exotic states [49,50]. Since the tetraquarks under discussion are not stable and can decay strongly in S-wave to $D^+_sD^{*+}_{s0}(2317)$ and $D^+D^{*+}_{s0}(2317)$ mesons we calculate also widths of these channels. To this end, we utilize QCD three-point sum rule method to compute the strong couplings G_s and G_d corresponding to the vertices $T^{++}_{cc;\overline{s}\overline{s}}D^+_sD^{*+}_{s0}(2317)$ and $T^{++}_{cc;\overline{d}\overline{s}}D^+D^{*+}_{s0}(2317)$, respectively. Obtained information on G_s and G_d, as well as spectroscopic parameters of the tetraquarks are applied as key ingredients to evaluate the partial decay widths $\Gamma[T^{++}_{cc;\overline{s}\overline{s}} \to D^+_sD^{*+}_{s0}(2317)]$ and $\widetilde{\Gamma}[T^{++}_{cc;\overline{d}\overline{s}} \to D^+D^{*+}_{s0}(2317)]$.

This work is organized in the following way: In the section 2 we calculate the masses and couplings of the pseudoscalar tetraquarks using the two-point sum rule method by including into analysis the quark, gluon and mixed condensates up to dimension ten. The spectroscopic parameters of these resonances are employed in Sec. 3 to evaluate strong couplings and widths of the $T^{++}_{cc;\overline{s}\overline{s}}$ and $T^{++}_{cc;\overline{d}\overline{s}}$ states' S-wave strong decays. The section 4 is reserved for analysis and our concluding remarks. The Appendix contains explicit expressions of the correlation functions used in calculations of the spectroscopic parameters and strong coupling of the tetraquark $T^{++}_{cc;\overline{s}\overline{s}}$.

2. The spectroscopy of the $J^P = 0^-$ tetraquarks $T^{++}_{cc;\overline{s}\overline{s}}$ and $T^{++}_{cc;\overline{d}\overline{s}}$

One of the effective tools to evaluate the masses and couplings of the tetraquarks $T^{++}_{cc;\overline{s}\overline{s}}$ and $T^{++}_{cc;\overline{d}\overline{s}}$ is QCD two-point sum rule method. In this section we present in a detailed form calculation of these parameters in the case of the diquark–antidiquark $T^{++}_{cc;\overline{s}\overline{s}}$ and provide only final results for the second state $T^{++}_{cc;\overline{d}\overline{s}}$.

The basic quantity in the sum rule calculations is the correlation function chosen in accordance with a problem under consideration. The best way to derive the sum rules for the mass and coupling is analysis of the two-point correlation function

$$\Pi(p) = i \int d^4x e^{ipx} \langle 0|\mathcal{T}\{J(x)J^{\dagger}(0)\}|0\rangle, \tag{1}$$

where $J(x)$ in the interpolating current for the isoscalar $J^P = 0^-$ state $T^{++}_{cc;\overline{s}\overline{s}}$. It can be defined in the following form [31]

$$J(x) = c_a^T(x)Cc_b(x)\left[\overline{s}_a(x)\gamma_5 C\overline{s}_b^T(x) + \overline{s}_b(x)\gamma_5 C\overline{s}_a^T(x)\right], \tag{2}$$

where C is the charge conjugation operator, a and b are color indices. The interpolating current for the isospinor tetraquark $T^{++}_{cc;\overline{d}\overline{s}}$ is given by the similar expression

$$\widetilde{J}(x) = c_a^T(x)Cc_b(x)\left[\overline{d}_a(x)\gamma_5 C\overline{s}_b^T(x) + \overline{d}_b(x)\gamma_5 C\overline{s}_a^T(x)\right]. \tag{3}$$

The currents $J(x)$ and $\widetilde{J}(x)$ have symmetric color structure $[\mathbf{6_c}]_{cc} \otimes [\overline{\mathbf{6}}_\mathbf{c}]_{\overline{s}\overline{s}}$ and are composed of the heavy pseudoscalar diquark and light scalar antidiquark. There are other interpolating currents with $J^P = 0^-$ but composed, for example, of the heavy scalar diquark and light pseudoscalar antidiquark [31]. To describe the tetraquarks $T^{++}_{cc;\overline{s}\overline{s}}$ and $T^{++}_{cc;\overline{d}\overline{s}}$ one can also use linear combinations of these currents. In general, different currents may modify the results for the spectroscopic parameters of the tetraquarks under consideration. In the present work we restrict our analysis by the interpolating currents $J(x)$ and $\widetilde{J}(x)$ bearing in mind that among various diquarks the scalar ones are most tightly bound states.

The QCD sum rule method implies calculation of the correlation function $\Pi(p)$ using the phenomenological parameters of the tetraquark $T^{++}_{cc;\overline{s}\overline{s}}$, i.e. its mass m_T and coupling f_T from one side, and computation of $\Pi(p)$ in terms of the quark propagators from another side. Equating expressions obtained by this way and invoking the quark-hadron duality it is possible to derive the sum rules to evaluate m_T and f_T.

We assume that the phenomenological side of the sum rules can be approximated by a single pole term. In the case of the multiquark systems this approach has to be used with some caution, because the physical side receives contribution also from two-hadron reducible terms. In fact, the relevant interpolating current couples not only to the tetraquark (pentaquark), but also to the two-hadron continuum lying below the mass of the multiquark system [51,52]. These terms can be either subtracted from the sum rules or included into parameters of the pole term. The first method was employed mainly in investigating the pentaquarks [52,53], whereas the second approach was used to study the tetraquarks [54]. It turns out that the contribution of the two-meson continuum generates the finite width $\Gamma(p^2)$ of the tetraquark and leads to the modification

$$\frac{1}{m_T^2 - p^2} \to \frac{1}{m_T^2 - p^2 - i\sqrt{p^2}\Gamma(p^2)}. \tag{4}$$

These effects, properly taken into account in the sum rules, rescale the coupling f_T and leave untouched the mass of the tetraquark m_T.

In all cases explored in Refs. [52–54] the two-hadron continuum effects were found small and negligible. Therefore, to derive the phenomenological side of the sum rules we use the zero-width single-pole approximation and demonstrate in the section 4 the self-consistency of the obtained results by explicit computations.

In the context of this approach the correlation function $\Pi^{\rm Phys}(p)$ takes a simple form

$$\Pi^{\rm Phys}(p) = \frac{\langle 0|J|T(p)\rangle\langle T(p)|J^{\dagger}|0\rangle}{m_T^2 - p^2} + \ldots, \tag{5}$$

where by dots we indicate contribution of higher resonances and continuum states. This formula can be simplified further by introducing the matrix element

$$\langle 0|J|T(p)\rangle = \frac{m_T^2 f_T}{\mathcal{M}}, \tag{6}$$

where $\mathcal{M} = 2(m_c + m_s)$. After some simple manipulations we get

$$\Pi^{\rm Phys}(p) = \frac{m_T^4 f_T^2}{\mathcal{M}^2}\frac{1}{m_T^2 - p^2} + \ldots. \tag{7}$$

It is seen that the Lorentz structure of the correlation function is trivial and there is only a term proportional to I. The invariant amplitude $\Pi^{\rm Phys}(p^2) = m_T^4 f_T^2/[\mathcal{M}^2(m_T^2 - p^2)]$ corresponding to this structure constitutes the physical side of the sum rule. In order to suppress effects coming from higher resonances and continuum states one has to apply to $\Pi^{\rm Phys}(p^2)$ the Borel transformation which leads to

$$\mathcal{B}\Pi^{\rm Phys}(p^2) \equiv \Pi^{\rm Phys}(M^2) = \frac{m_T^4 f_T^2 e^{-m_T^2/M^2}}{\mathcal{M}^2}$$

with M^2 being the Borel parameter.

The second side of the required equality $\Pi^{\rm OPE}(p)$ is accessible through computation of Eq. (1) using the explicit expression of the interpolating current (2) and contracting quark fields under the time ordering operator $\mathcal{T}$. The expression of $\Pi^{\rm OPE}(p)$ in terms of quarks' propagators is written down in the Appendix A. We employ the heavy c and light s-quark propagators, explicit expressions of which can be found in Ref. [55], for example. The calculations are carried out at the leading order of the perturbative QCD by taking into account quark, gluon and mixed condensates up to dimension ten.

The invariant amplitude $\Pi^{\rm OPE}(p^2)$ can be written down in terms of the spectral density $\rho(s)$

$$\Pi^{\rm OPE}(p^2) = \int_{\mathcal{M}^2}^{\infty} \frac{\rho(s)}{s - p^2} ds. \tag{8}$$

After equating $\Pi^{\rm Phys}(M^2)$ to the Borel transform of $\Pi^{\rm OPE}(p^2)$ and performing the continuum subtraction we get a first expression that can be used to derive the sum rules for the mass and coupling. The second equality can be obtained from the first one by applying the operator $d/d(-1/M^2)$. Then it is not difficult we find the sum rules for m_T and f_T

$$m_T^2 = \frac{\int_{\mathcal{M}^2}^{s_0} ds\rho(s) s e^{-s/M^2}}{\int_{\mathcal{M}^2}^{s_0} ds\rho(s) e^{-s/M^2}}, \tag{9}$$

and

$$f_T^2 = \frac{\mathcal{M}^2}{m_T^4} \int_{\mathcal{M}^2}^{s_0} ds\rho(s) e^{(m_T^2 - s)/M^2}. \tag{10}$$

In Eqs. (9) and (10) s_0 is the continuum threshold parameter introduced during the subtraction procedure: it separates the ground-state and continuum contributions.

The sum rules for the mass and coupling depend on numerous parameters, which should be fixed to carry out numerical analysis. Below we write down the quark, gluon and mixed condensates

$$\begin{aligned}
&\langle \bar{q}q \rangle = -(0.24 \pm 0.01)^3 \text{ GeV}^3, \ \langle \bar{s}s \rangle = 0.8 \ \langle \bar{q}q \rangle, \\
&m_0^2 = (0.8 \pm 0.1) \text{ GeV}^2, \ \langle \overline{q} g_s \sigma G q \rangle = m_0^2 \langle \overline{q} q \rangle, \\
&\langle \overline{s} g_s \sigma G s \rangle = m_0^2 \langle \bar{s}s \rangle, \\
&\langle \alpha_s G^2 \rangle = (6.35 \pm 0.35) \cdot 10^{-2} \text{ GeV}^4, \\
&\langle g_s^3 G^3 \rangle = (0.57 \pm 0.29) \text{ GeV}^6,
\end{aligned} \tag{11}$$

used in numerical computations. For the gluon condensate $\langle \alpha_s G^2 \rangle$ we employ its new average value presented recently in Ref. [56], whereas $\langle g_s^3 G^3 \rangle$ is borrowed from Ref. [57]. For the masses of the c and s-quarks

$$m_c = 1.275^{+0.025}_{-0.035} \text{ GeV}, \ m_s = 95^{+9}_{-3} \text{ MeV}, \tag{12}$$

we utilize the information from Ref. [58].

Besides, the sum rules contain also the auxiliary parameters M^2 and s_0 which may be varied inside of some regions and must satisfy standard restrictions of the sum rules computations. The analysis demonstrates that the working windows

$$M^2 = (4.7, \ 7.0) \text{ GeV}^2, \ s_0 = (22, \ 24) \text{ GeV}^2, \tag{13}$$

meet constraints imposed on M^2 and s_0. Indeed, the pole contribution (PC) changes within limits 55%–22% when one varies M^2 from its minimal to maximal allowed values: the higher limit of the Borel parameter is fixed namely from exploration of the pole contribution. The lower bound for M^2 stems from the convergence of the operator product expansion (OPE)

$$R(M^2) = \frac{\Pi^{\text{Dim}(8+9+10)}(M^2, \ s_0)}{\Pi(M^2, \ s_0)} < 0.05, \tag{14}$$

where $\Pi(M^2, \ s_0)$ is the subtracted Borel transform of $\Pi^{\text{OPE}}(p^2)$, and $\Pi^{\text{Dim}(8+9+10)}(M^2, \ s_0)$ is contribution of the last three terms in expansion of the correlation function. At minimal M^2 the ratio R is equal to $R(4.7 \text{ GeV}^2) = 0.018$ which proves the nice convergence of the sum rules. Moreover, at $M^2 = 4.7 \text{ GeV}^2$ the perturbative contribution amounts to more than 88% of the full result and considerably exceeds the nonperturbative contributions.

The mass m_T and coupling f_T extracted from the sum rules should be stable under variation of the parameters M^2 and s_0. However in calculations these quantities show a sensitivity to the choice both of M^2 and s_0. Therefore, when choosing the intervals for M^2 and s_0 we demand

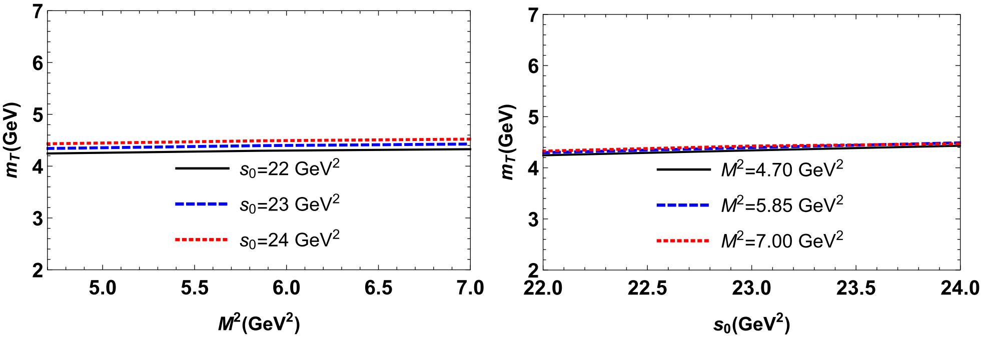

Fig. 1. The mass of the tetraquark $T^{++}_{cc;\overline{ss}}$ as a function of the Borel parameter (left), and continuum threshold parameter (right).

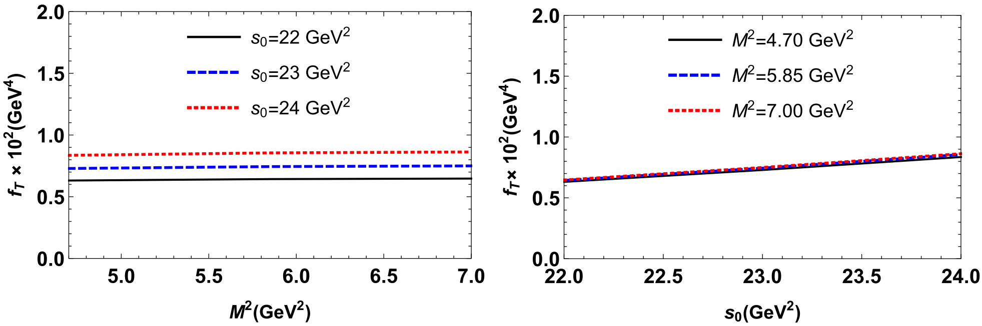

Fig. 2. The dependence of the coupling f_T on the Borel (left), and continuum threshold (right) parameters.

maximal stability of m_T and f_T on these parameters. As usual, the mass m_T of the tetraquark is more stable against variation of M^2 and s_0 which is seen from Figs. 1 and 2. This fact has simple explanation: the sum rule for the mass is given by Eq. (9) as the ratio of two integrals, therefore their uncertainties partly cancel each other smoothing dependence of m_T on the Borel and continuum threshold parameters. The coupling f_T is more sensitive to the choice of M^2 and s_0, nevertheless corresponding ambiguities do not exceed 20% staying within limits typical for sum rules calculations.

From performed analysis for the mass and coupling of the tetraquark $T^{++}_{cc;\overline{ss}}$ we find

$$m_T = (4390 \ \pm 150) \text{ MeV}, \quad f_T = (0.74 \pm 0.14) \cdot 10^{-2} \text{ GeV}^4. \tag{15}$$

The similar investigations of the $T^{++}_{cc;\overline{ds}}$ lead to predictions

$$\widetilde{m}_T = (4265 \pm 140\ ) \text{ MeV}, \quad \widetilde{f}_T = (0.62 \pm 0.10) \cdot 10^{-2} \text{ GeV}^4, \tag{16}$$

which have been obtained using the working regions

$$M^2 = (4.5,\ 6.5) \text{ GeV}^2,\ s_0 = (21,\ 23) \text{ GeV}^2. \tag{17}$$

Let us note that in calculations of $\widetilde{m}_T$ and $\widetilde{f}_T$ the pole contribution PC changes within limits 59%–27%. Contribution of the last three terms to the corresponding correlation function at the point $M^2 = 4.5\ \mathrm{GeV}^2$ amounts to 1.8% of the total result, which demonstrates convergence of the sum rules.

The spectroscopic parameters of the tetraquarks $T^{++}_{cc;\overline{s}\overline{s}}$ and $T^{++}_{cc;\overline{d}\overline{s}}$ obtained here will be utilized in the next section to determine width of their decay channels.

3. The decays $T^{++}_{cc;\overline{s}\overline{s}} \to D_s^+ D_{s0}^{*+}(2317)$ and $T^{++}_{cc;\overline{d}\overline{s}} \to D^+ D_{s0}^{*+}(2317)$

The masses of the tetraquarks $T^{++}_{cc;\overline{s}\overline{s}}$ and $T^{++}_{cc;\overline{d}\overline{s}}$ allow us to fix their possible decay channels. Thus, the tetraquark $T^{++}_{cc;\overline{s}\overline{s}}$ in S-wave decays to a pair of conventional mesons D_s^+ and $D_{s0}^{*+}(2317)$, whereas the process $T^{++}_{cc;\overline{d}\overline{s}} \to D^+ D_{s0}^{*+}(2317)$ is the main S-wave decay channel of $T^{++}_{cc;\overline{d}\overline{s}}$. In fact, the threshold for production of these particles can be easily calculated employing their masses (see, Table 1): for production of the mesons $D_s^+ D_{s0}^{*+}(2317)$ it equals to (4286.04 ± 0.60) MeV, and for $D^+ D_{s0}^{*+}(2317)$ amounts to (4187.35 ± 0.60) MeV. We see that the masses of the tetraquarks $T^{++}_{cc;\overline{s}\overline{s}}$ and $T^{++}_{cc;\overline{d}\overline{s}}$ are approximately 104 MeV and 78 MeV above these thresholds. There are also kinematically allowed P-wave decay modes of the tetraquarks $T^{++}_{cc;\overline{s}\overline{s}}$ and $T^{++}_{cc;\overline{d}\overline{s}}$. Thus, the tetraquark $T^{++}_{cc;\overline{s}\overline{s}}$ through P-wave can decay to the final state $D_s^+ D_s^{*+}$, whereas for $T^{++}_{cc;\overline{d}\overline{s}}$ these channels are $D^+ D_s^{*+}$ and $D^{*+} D_s^+$. In the present work we limit ourselves by considering only the S-wave decays of these tetraquarks.

In the present section we calculate the strong coupling form factor G_s of the vertex $T^{++}_{cc;\overline{s}\overline{s}} \to D_s^+ D_{s0}^{*+}(2317)$ and find the width of the corresponding decay channel $\Gamma[T^{++}_{cc;\overline{s}\overline{s}} \to D_s^+ D_{s0}^{*+}(2317)]$. We provide also our final predictions for G_d and $\widetilde{\Gamma}[T^{++}_{cc;\overline{d}\overline{s}} \to D^+ D_{s0}^{*+}(2317)]$ omitting details of calculations which can easily be reconstructed from analysis of the first process.

We use the three-point correlation function

$$\Pi(p, p') = i^2 \int d^4x d^4y e^{i(p'y - px)} \langle 0|\mathcal{T}\{J^{D_{s0}}(y) \times J^{D_s}(0) J^{\dagger}(x)\}|0\rangle, \tag{18}$$

to find the sum rule and extract the strong coupling G_s. Here $J^{D_s}(x)$ and $J^{D_{s0}}(x)$ are the interpolating currents for the mesons D_s^+ and $D_{s0}^{*+}(2317)$, respectively. The four-momenta of the tetraquark $T^{++}_{cc;\overline{s}\overline{s}}$ and meson $D_{s0}^{*+}(2317)$ are p and p': the momentum of the meson D_s^+ then equals to $q = p - p'$.

We define the interpolating currents of the mesons D_s^+ and $D_{s0}^{*+}(2317)$ in the following way

$$J^{D_s}(x) = \overline{s}^i(x) i\gamma_5 c^i(x), \quad J^{D_{s0}}(x) = \overline{s}^j(x) c^j(x). \tag{19}$$

By isolating the ground-state contribution to the correlation function, for $\Pi^{\mathrm{Phys}}(p, p')$ we get

$$\Pi^{\mathrm{Phys}}(p, p') = \frac{\langle 0|J^{D_{s0}}|D_{s0}^*(p')\rangle \langle 0|J^{D_s}|D_s(q)\rangle}{(p'^2 - m_{D_{s0}}^2)(q^2 - m_{D_s}^2)} \times \frac{\langle D_s(q) D_{s0}^*(p')|T(p)\rangle \langle T(p)|J^{\dagger}|0\rangle}{(p^2 - m_T^2)} + \ldots, \tag{20}$$

where the dots again stand for contributions of higher excited states and continuum.

The correlation function $\Pi^{\rm Phys}(p,p')$ can be further simplified by expressing matrix elements in terms of the mesons' physical parameters. To this end we introduce the matrix elements

$$\langle 0|J^{D_s}|D_s\rangle = \frac{m_{D_s}^2 f_{D_s}}{m_c+m_s},$$
$$\langle 0|J^{D_{s0}}|D_{s0}^{*}\rangle = m_{D_{s0}} f_{D_{s0}}, \tag{21}$$

where f_{D_s} and $f_{D_{s0}}$ are the decay constants of the mesons D_s^+ and $D_{s0}^{*+}(2317)$, respectively. We also use the following parametrization for the vertex

$$\langle D_s(q) D_{s0}^{*}(p')|T(p)\rangle = G_s\, p\cdot p' \tag{22}$$

After some calculations it is not difficult to show that

$$\Pi^{\rm Phys}(p,p') = G_s \frac{m_{D_{s0}} f_{D_{s0}} m_{D_s}^2 f_{D_s} m_T^2 f_T}{\mathcal{M}^2(p'^2-m_{D_{s0}}^2)(p^2-m_T^2)(q^2-m_{D_s}^2)} \times \left(m_T^2+m_{D_{s0}}^2-q^2\right)+\ldots. \tag{23}$$

Because the Lorentz structure of the $\Pi^{\rm Phys}(p,p')$ is proportional to I, the invariant amplitude $\Pi^{\rm Phys}(p^2,p'^2,q^2)$ is given exactly by Eq. (23). Its double Borel transformation over the variables p^2 and p'^2 with the parameters M_1^2 and M_2^2 constitutes the left side of the sum rule equality. Its right hand side is determined by the Borel transformation $\mathcal{B}\Pi^{\rm OPE}(p^2,p'^2,q^2)$, where $\Pi^{\rm OPE}(p^2,p'^2,q^2)$ is the invariant amplitude that corresponds to the structure $\sim I$ in $\Pi^{\rm OPE}(p,p')$. Explicit expression of the correlation function $\Pi^{\rm OPE}(p,p')$ in terms of the quark propagators is presented in the Appendix.

Equating $\mathcal{B}\Pi^{\rm OPE}(p^2,p'^2,q^2)$ with the double Borel transformation of $\Pi^{\rm Phys}(p^2,p'^2,q^2)$ and performing continuum subtraction we get sum rule for the strong coupling G_s, which is a function of q^2 and depends also on the auxiliary parameters of calculations

$$G_s(\mathbf{M}^2,\ \mathbf{s}_0,\ q^2) = \frac{\mathcal{M}^2}{m_{D_{s0}} f_{D_{s0}} m_{D_s}^2 f_{D_s} m_T^2 f_T} \times \frac{q^2-m_{D_s}^2}{\left(m_T^2+m_{D_{s0}}^2-q^2\right)} \int_{\mathcal{M}^2}^{s_0} ds \int_{\widetilde{\mathcal{M}}^2}^{s_0'} ds' \rho_s(s,s',q^2) \times e^{(m_T^2-s)/M_1^2} e^{(m_{D_{s0}}^2-s')/M_2^2}, \tag{24}$$

where $\widetilde{\mathcal{M}}^2=\mathcal{M}^2/4$, and $\mathbf{M}^2=(M_1^2,\ M_2^2)$ and $\mathbf{s}_0=(s_0,\ s_0')$ are the Borel and continuum thresholds parameters, respectively.

One can see that the sum rule (24) is presented in terms of the spectral density $\rho_s(s,s',q^2)$ which is proportional to the imaginary part of $\Pi^{\rm OPE}(p,p')$. We calculate the correlation function $\Pi^{\rm OPE}(p,p')$ by including nonperturbative terms up to dimension six. But after double Borel transformation only s-quark and gluon vacuum condensates $\langle\bar{s}s\rangle$ and $\langle\alpha_s G^2/\pi\rangle$ contribute to spectral density $\rho_s(s,s',q^2)$, where, nevertheless, the perturbative component plays a dominant role.

The strong coupling form factor $G_s(\mathbf{M}^2,\ \mathbf{s}_0,\ q^2)$ can be calculated using the sum rule given by Eq. (24). The values of the masses and decay constants of the mesons that enter into this

Table 1
Parameters of the D-mesons used in numerical computations.

Parameters	Values (in MeV units)
m_D	1869.65 ± 0.05
f_D	211.9 ± 1.1
m_{D_s}	1968.34 ± 0.07
f_{D_s}	249.0 ± 1.2
$m_{D_{s0}}$	2317.7 ± 0.6
$f_{D_{s0}}$	201

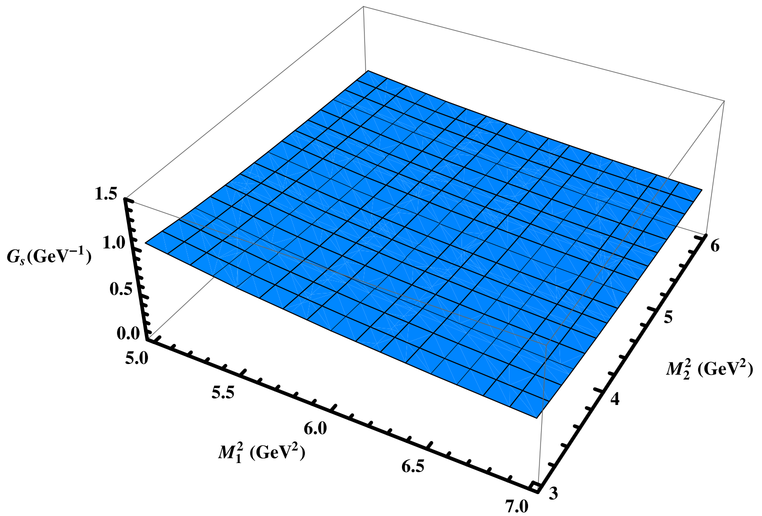

Fig. 3. The strong coupling form factor $G_s(Q^2)$ as a function of the Borel parameters $\mathbf{M}^2 = (M_1^2,\ M_2^2)$ at the middle point of the region $\mathbf{s}_0 = (s_0, s_0')$ and at fixed $Q^2 = 4\ \text{GeV}^2$.

expression are collected in Table 1. Requirements which should be satisfied by the auxiliary parameters $\mathbf{M}^2$ and $\mathbf{s}_0$ are similar to ones discussed in the previous section and are universal for all sum rules computations. Performed analysis demonstrates that the working regions

$$M_1^2 = (5,\ 7)\ \text{GeV}^2,\ s_0 = (22,\ 24)\ \text{GeV}^2,$$
$$M_2^2 = (3,\ 6)\ \text{GeV}^2,\ s_0' = (7,\ 9)\ \text{GeV}^2, \tag{25}$$

lead to stable results for the form factor $G_s(M^2,\ s_0,\ q^2)$, and therefore are appropriate for our purposes. In what follows we omit its dependence on the parameters and introduce $q^2 = -Q^2$ denoting the obtained form factor as $G_s(Q^2)$.

In order to visualize a stability of the sum rule calculations we depict in Fig. 3 the strong coupling $G_s(Q^2)$ as a function of the Borel parameters at fixed s_0 and Q^2. It is seen that there is a weak dependence of $G_s(Q^2)$ on M_1^2 and M_2^2. The dependence of $G_s(Q^2)$ on $\mathbf{M}^2$, and also its variations caused by the continuum threshold parameters are main sources of ambiguities in sum rule calculations, which should not exceed 30%.

For calculation of the decay width we need a value of the strong coupling at the D_s meson's mass shell, i.e. at $q^2 = m_{D_s}^2$ or at $Q^2 = -m_{D_s}^2$, where the sum rule method is not applicable.

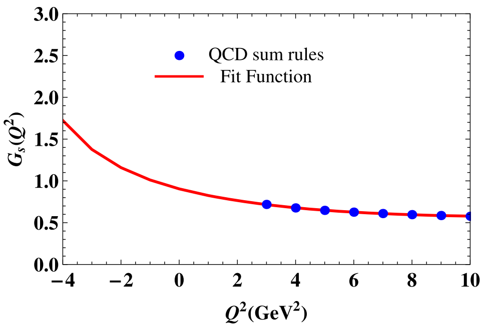

Fig. 4. The sum rule prediction and fit function for the strong coupling $G_s(Q^2)$.

Therefore it is necessary to introduce a fit function $F(Q^2)$ that for the momenta $Q^2 > 0$ leads to the same results as the sum rule, but can be easily extended to the region of $Q^2 < 0$. It is convenient to model it in the form

$$F(Q^2) = \frac{f_0}{1 - a\left(Q^2/m_T^2\right) + b\left(Q^2/m_T^2\right)^2}, \tag{26}$$

where f_0, a and b are fitting parameters. The performed analysis allows us to fix these parameters as $f_0 = 0.91$ GeV^{-1}, $a = -1.94$ and $b = -1.65$. The fit function $F(Q^2)$ and sum rule results for $G_s(Q^2)$ are plotted in Fig. 4, where one can see a very nice agreement between them.

At the mass shell $Q^2 = -m_{D_s}^2$ the strong coupling is equal to

$$G_s(-m_{D_s}^2) = (1.67 \pm 0.43)\ \text{GeV}^{-1}. \tag{27}$$

The width of the decay $T_{cc;\overline{ss}}^{++} \to D_s^+ D_{s0}^{*+}(2317)$ is determined by the following formula

$$\Gamma[T_{cc;\overline{ss}}^{++} \to D_s^+ D_{s0}^{*+}(2317)] = \frac{G_s^2 m_{D_{s0}}^2}{8\pi}\lambda\left(1 + \frac{\lambda^2}{m_{D_{s0}}^2}\right), \tag{28}$$

where

$$\lambda = \lambda\left(m_T^2, m_{D_{s0}}^2, m_{D_s}^2\right) = \frac{1}{2m_T}\left[m_T^4 + m_{D_{s0}}^4 + m_{D_s}^4 - 2(m_T^2 m_{D_{s0}}^2 + m_T^2 m_{D_s}^2 + m_{D_{s0}}^2 m_{D_s}^2)\right]^{1/2}. \tag{29}$$

Our result for the decay width is:

$$\Gamma = (302 \pm 113)\ \text{MeV}. \tag{30}$$

In the similar calculations of the strong coupling $G_d(Q^2)$ for the Borel and threshold parameters M_1^2 and s_0 we have employed

$$M_1^2 = (4.7,\ 6.5)\ \text{GeV}^2,\ s_0 = (21,\ 23)\ \text{GeV}^2, \tag{31}$$

whereas M_2^2 and s_0' have been chosen as in Eq. (25). For the strong coupling we have got

$$|G_d(-m_D^2)| = (1.37 \pm 0.34)\ \text{GeV}^{-1}. \tag{32}$$

Then the width of the process $T_{cc;\overline{ds}}^{++} \to D^+ D_{s0}^{*+}(2317)$ is

$$\widetilde{\Gamma} = (171 \pm 52)\ \text{MeV}. \tag{33}$$

The predictions for the widths Γ and $\widetilde{\Gamma}$ are the final results of this section.

4. Analysis and concluding remarks

In the present work we have calculated the spectroscopic parameters of the doubly charmed $J^P = 0^-$ tetraquarks $T^{++}_{cc;\overline{s}\overline{s}}$ and $T^{++}_{cc;\overline{d}\overline{s}}$ using QCD two-point sum rule approach. Obtained results for mass of these resonances $m_T = (4390 \pm 150)$ MeV and $\widetilde{m}_T = (4265 \pm 140)$ MeV demonstrate that they are unstable particles and lie above open charm thresholds $D_s^+ D_{s0}^{*+}(2317)$ and $D^+ D_{s0}^{*+}(2317)$, respectively. The mass splitting between the tetraquarks

$$m_T - \widetilde{m}_T \sim 125\ \text{MeV} \tag{34}$$

is equal approximately to a half of mass difference between the ground-state particles from $[cs][\overline{cs}]$ and $[cq][\overline{cq}]$ or from $[cs][\overline{bs}]$ and $[cq][\overline{bq}]$ multiplets [59]. The quark content of these resonances differs from each other by a pair of quarks $s\overline{s}$ and $q\overline{q}$, whereas the tetraquark $T^{++}_{cc;\overline{d}\overline{s}}$ can be obtained from $T^{++}_{cc;\overline{s}\overline{s}}$ by only $\overline{s} \to \overline{d}$ replacement. In other words, the mass splitting caused by the s-quark equals to 125 MeV. It is interesting that in the conventional mesons s-quark's "mass" is lower and amounts to $D_s^+(c\overline{s}) - D^0(c\overline{u}) \approx 100$ MeV, whereas for baryons, for example $\Xi_c^+(usc) - \Lambda_c^+(udc) \approx 180$ MeV, it is higher than 125 MeV.

We have also evaluated the widths of the tetraquarks $T^{++}_{cc;\overline{s}\overline{s}}$ and $T^{++}_{cc;\overline{d}\overline{s}}$ through their dominant S-wave strong decays to the pair of $D_s^+ D_{s0}^{*+}(2317)$ and $D^+ D_{s0}^{*+}(2317)$ mesons. To this end we have employed QCD three-point sum rules approach and found the strong couplings G_s and G_d: they are key ingredients of computations. The widths $\Gamma = (302 \pm 113)$ MeV and $\widetilde{\Gamma} = (171 \pm 52)$ MeV show that the tetraquarks $T^{++}_{cc;\overline{s}\overline{s}}$ and $T^{++}_{cc;\overline{d}\overline{s}}$ can be classified as rather broad resonances.

We have evaluated the spectroscopic parameters of the tetraquarks $T^{++}_{cc;\overline{s}\overline{s}}$ and $T^{++}_{cc;\overline{d}\overline{s}}$ using the zero-width single-pole approximation. But, as it has been emphasized in the section 2, the interpolating currents (2) and (3) couple not only to the tetraquarks, but also to the two-meson continuum (in our case, to the states $D_s^+ D_{s0}^{*+}(2317)$ and $D^+ D_{s0}^{*+}(2317)$), and these effects may correct our predictions for m_T, f_T and $\widetilde{m}_T$, $\widetilde{f}_T$, respectively. The two-meson continuum contribution modifies the zero-width approximation (4) and in the case of the tetraquark $T^{++}_{cc;\overline{s}\overline{s}}$ leads to the following corrections [54]:

$$\lambda_T^2 e^{-m_T^2/M^2} \to \lambda_T^2 \int\limits_{(m_{D_s}+m_{D_{s0}})^2}^{s_0} ds W(s) e^{-s/M^2} \tag{35}$$

and

$$\lambda_T^2 m_T^2 e^{-m_T^2/M^2} \to \lambda_T^2 \int\limits_{(m_{D_s}+m_{D_{s0}})^2}^{s_0} ds W(s) s e^{-s/M^2}, \tag{36}$$

where $\lambda_T^2 = m_T^4 f_T^2 / \mathcal{M}^2$. In Eqs. (35) and (36) we have used

$$W(s) = \frac{1}{\pi} \frac{m_T \Gamma(s)}{\left(s - m_T^2\right)^2 + m_T^2 \Gamma^2(s)} \tag{37}$$

and

$$\Gamma(s)=\Gamma\frac{m_T}{s}\sqrt{\frac{s-(m_{D_s}+m_{D_{s0}})^2}{m_T^2-(m_{D_s}+m_{D_{s0}})^2}}. \tag{38}$$

By utilizing the central values of the m_T and Γ, as well as $M^2=6$ GeV^2 and $s_0=23$ GeV^2 it is not difficult to find that

$$\lambda_T^2\rightarrow 0.86\lambda_T^2\rightarrow\frac{(0.927f_T)^2m_T^4}{\mathcal{M}^2}, \tag{39}$$

and

$$\lambda_T^2m_T^2\rightarrow 0.86\lambda_T^2m_T^2\rightarrow\frac{(0.927f_T)^2m_T^6}{\mathcal{M}^2}. \tag{40}$$

As is seen, in both cases the two-meson effects result in rescaling of the coupling $f_T\rightarrow 0.927f_T$, i.e., change it approximately by 7.3% and do not exceed the accuracy of the sum rule calculations which amounts to $\pm19\%$. The similar estimation $\widetilde{f}_T\rightarrow 0.945\widetilde{f}_T$ is valid for the coupling $\widetilde{f}_T$ as well.

The double-charmed tetraquarks investigated in the present work carry a double electric charge and may exist as diquark–antidiquarks. They are unstable resonances, but some of double-bottom tetraquarks may be stable against strong decays. Therefore theoretical and experimental studies of the double-heavy four-quark systems, their strong and weak decays remain in the agenda of high energy physics, and can provide valuable information on internal structure and properties of these exotic mesons.

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Appendix A. The correlation functions used in calculations

In this Appendix we have collected the explicit expressions of the correlation functions $\Pi^{\rm OPE}(p)$ and $\Pi^{\rm OPE}(p,p')$ used in the sections 2 and 3 to derive sum rules for calculation of the spectroscopic parameters of the tetraquark $T_{cc;\overline{ss}}^{++}$ and its decay width. The function $\Pi^{\rm OPE}(p)$ has the following expression in terms of the quark propagators:

$$\begin{aligned}
\Pi^{\rm OPE}(p)=-2i\int d^4xe^{ipx}\Big\{&\mathrm{Tr}\left[S_c^{bb'}(x)\widetilde{S}_c^{aa'}(x)\right]\mathrm{Tr}\left[\gamma_5\widetilde{S}_s^{a'b}(-x)\gamma_5S_s^{b'a}(-x)\right]\\
&+\mathrm{Tr}\left[S_c^{ba'}(x)\widetilde{S}_c^{ab'}(x)\right]\mathrm{Tr}\left[\gamma_5\widetilde{S}_s^{a'b}(-x)\gamma_5S_s^{b'a}(-x)\right]\\
&+\mathrm{Tr}\left[S_c^{bb'}(x)\widetilde{S}_c^{aa'}(x)\right]\mathrm{Tr}\left[\gamma_5\widetilde{S}_s^{b'b}(-x)\gamma_5S_s^{a'a}(-x)\right]\\
&+\mathrm{Tr}\left[S_c^{ba'}(x)\widetilde{S}_c^{ab'}(x)\right]\mathrm{Tr}\left[\gamma_5\widetilde{S}_s^{b'b}(-x)\gamma_5S_s^{a'a}(-x)\right]\\
&+\mathrm{Tr}\left[S_c^{bb'}(x)\widetilde{S}_c^{aa'}(x)\right]\mathrm{Tr}\left[\gamma_5\widetilde{S}_s^{a'a}(-x)\gamma_5S_s^{b'b}(-x)\right]\\
&+\mathrm{Tr}\left[S_c^{ba'}(x)\widetilde{S}_c^{ab'}(x)\right]\mathrm{Tr}\left[\gamma_5\widetilde{S}_s^{a'a}(-x)\gamma_5S_s^{b'b}(-x)\right]
\end{aligned}$$

$$+\,\mathrm{Tr}\left[S_c^{bb'}(x)\widetilde{S}_c^{aa'}(x)\right]\mathrm{Tr}\left[\gamma_5\widetilde{S}_s^{b'a}(-x)\gamma_5 S_s^{a'b}(-x)\right]$$
$$+\,\mathrm{Tr}\left[S_c^{ba'}(x)\widetilde{S}_c^{ab'}(x)\right]\mathrm{Tr}\left[\gamma_5\widetilde{S}_s^{b'a}(-x)\gamma_5 S_s^{a'b}(-x)\right]\Big\}, \tag{A.1}$$

where

$$\widetilde{S}_{c(s)}(x)=CS^{\mathrm{T}}_{c(s)}(x)C.$$

Here $S_{c(s)}(x)$ is the heavy c-quark (light s-quark) propagator.

The correlation function $\Pi^{\mathrm{OPE}}(p,p')$ is presented below

$$\Pi^{\mathrm{OPE}}(p,p')$$
$$=2i^2\int d^4xd^4ye^{-ipx}e^{ip'y}\Big\{\mathrm{Tr}\left[S_c^{jb'}(y-x)\widetilde{S}_c^{ia'}(-x)\gamma_5\widetilde{S}_s^{b'i}(x)\gamma_5 S_s^{a'j}(x-y)\right]$$
$$+\,\mathrm{Tr}\left[S_c^{ja'}(y-x)\widetilde{S}_c^{ib'}(-x)\gamma_5\widetilde{S}_s^{b'i}(x)\gamma_5 S_s^{a'j}(x-y)\right]+\mathrm{Tr}\left[S_c^{jb'}(y-x)\widetilde{S}_c^{ia'}(-x)\gamma_5\right.$$
$$\left.\times\widetilde{S}_s^{a'i}(x)\gamma_5 S_s^{b'j}(x-y)\right]+\mathrm{Tr}\left[S_c^{ja'}(y-x)\widetilde{S}_c^{ib'}(-x)\gamma_5\widetilde{S}_s^{a'i}(x)\gamma_5 S_s^{b'j}(x-y)\right]\Big\}. \tag{A.2}$$

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