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# Mass spectra of heavy mesons with instanton effects

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We investigate the mass spectra of ordinary heavy mesons, based on a nonrelativistic potential approach. The heavy–light quark potential contains the Coulomb-type potential arising from one-gluon exchange, the confining potential, and the instanton-induced nonperturbative local heavy–light quark potential. All parameters are theoretically constrained and fixed. We carefully examine the effects from the instanton vacuum. Within the present form of the local potential from the instanton vacuum, we conclude that the instanton effects are rather marginal on the charmed mesons.

Subject Index D32

## 1. Introduction

The structure of hadrons containing a heavy quark is systematically understood when the mass of the heavy quark is taken to infinity. This is valid, since the heavy-quark mass $m_Q$ is much larger than $\Lambda_{\mathrm{QCD}}$, i.e., $m_Q \gg \Lambda_{\mathrm{QCD}}$. Then a new type of symmetry arises: the physics is not changed by the exchange of the heavy-quark flavor. This is called heavy-quark flavor symmetry. In this limit, the spin of the heavy quark $\boldsymbol{S}_Q$ is conserved, which brings about the spin conservation of the light degrees of freedom $\boldsymbol{S}_L$. So, the spin of a heavy hadron is also conserved in this limit: $\boldsymbol{S} = \boldsymbol{S}_L + \boldsymbol{S}_Q$. This is often called heavy-quark spin symmetry [1–3]. The heavy quark is entirely decoupled from the internal dynamics of a heavy hadron in the limit of $m_Q \to \infty$ and the interaction between the light degrees of freedom becomes spin-independent. The infinitely heavy-quark mass limit allows one to use the inverse of the heavy-quark mass, $1/m_Q$, as an expansion parameter. The spin-dependent part of the interaction appears as the next-to-leading order in the $1/m_Q$ expansion, which is proportional to $1/m_Q$ and stems from the chromomagnetic moment of the quark (see, e.g., Refs. [4–9]).

In the limit of $m_Q \to \infty$, the classification of conventional heavy meson states $Q\bar{q}$ with a single heavy quark $Q$ is rather simple, where $\bar{q}$ denotes the light anti-quark constituting the heavy meson.

Since the heavy quark is decoupled in the $m_Q \to \infty$ limit, the flavor structure is solely governed by the light quarks. Thus the lowest-lying states of the heavy meson are classified as the anti-triplet meson $\overline{\mathbf{3}}$. Moreover, the mesons with spin $s = 0$ and those with $s = 1$ are found to be degenerate, so that the pseudoscalar and vector heavy mesons consist of doublets in the limit of $m_Q \to \infty$. This degeneracy is lifted by introducing the spin-dependent interactions coming from $1/m_Q$ order. Based on this heavy-quark flavor–spin symmetry, there has been a great deal of theoretical work on the properties of both the lowest-lying and excited heavy mesons: lattice quantum chromodynamics (QCD) [10–15], nonrelativistic and relativistic quark models [16–20], potential models [21–26], QCD sum rules [27–29], holographic QCD [30], and so on.

The potential models for heavy mesons are usually based on two important parts of physics: quark confinement and perturbative one-gluon exchange. While these two ingredients of the potentials successfully describe the properties of both quarkonia and heavy mesons, certain nonperturbative effects need to be considered. Diakonov et al. derived the central part of the heavy-quark potential from the instanton vacuum, using the Wilson loop [31]. The spin-dependent part can be easily constructed by employing the Eichten–Feinberg formalism [32]. The effects of the heavy-quark potential from the instanton were examined only very recently by computing the quarkonium spectra [33]. The results showed that the effects of the instanton on the quarkonium spectra turn out to be rather small. Chernyshev et al. investigated the effects of a random gas of instantons and anti-instantons on mesons and baryons containing one or several heavy quarks [34]. They first derived the *local* effective interactions from the random instanton-gas model (RIGM) and then employed them to estimate the heavy-hadron mass spectra within a simple variational method, including the harmonic oscillator potential as a simple expression of the quark confinement. They obtained results in qualitative agreement with the experimental data on the low-lying heavy mesons. However, it is of great importance to examine cautiously such nonperturbative effects on the heavy hadron spectra in a quantitative manner.

In the present work, we aim at exploring carefully the heavy–light quark potentials, which were derived from the RIGM, examining their effects on the mass spectra of the heavy mesons. For simplicity and convenience, we will use the nonrelativistic framework in dealing with the heavy–light quark interactions from the RIGM. In any potential model for describing quarkonia and heavy mesons, there are two essential components: the quark confinement and the one-gluon exchange contribution, which we want to introduce in addition to the interaction from the instantons. Instead of a simple variational method used in Ref. [34], we employ a more elaborate and sophisticated framework, i.e., the Gaussian expansion method (GEM), which is well known for the successful description of two- and few-body systems [35–38], so that we reduce the numerical uncertainties arising from the simple variational method. As will be shown in this work, the present form of the heavy–light quark interaction based on the RIGM has only marginal effects on the mass spectra of the heavy mesons. The quark potentials of one-gluon exchange and the quark confinement already approximately reproduce the experimental data on the spectra of the low-lying heavy mesons. However, since the heavy mesons contain a light quark, we still expect that certain nonperturbative effects will come into play. We will also discuss these in the present work.

This paper is organized as follows: In Sect. 2, we define the heavy–light quark potentials arising from one-gluon exchange and the quark confinement. We then introduce the effective potential coming from the nonperturbative heavy–light quark interactions based on the RIGM. In Sect. 3, we show how to solve the nonrelativistic Schrödinger equation with the heavy–light quark potential within the framework of the GEM. That will be the framework for the numerical calculations in the present work. In Sect. 4, we present the results and discuss them in comparison with the experimental data. The final section is devoted to the summary and conclusion. We also discuss a possible future outlook.

## 2. Heavy–light quark potential

The general structure of the heavy–light quark potentials is expressed as

$$V(r) = V_c(r) + V_{SS}(r)(\boldsymbol{S}_Q \cdot \boldsymbol{S}_q) + V_{LS}(r)(\boldsymbol{L} \cdot \boldsymbol{S}) + V_T(r)[3(\boldsymbol{S}_Q \cdot \hat{\boldsymbol{n}})(\boldsymbol{S}_q \cdot \hat{\boldsymbol{n}}) - \boldsymbol{S}_1 \cdot \boldsymbol{S}_2], \tag{1}$$

where $V_c$ is the central part of the potential. $V_{SS}$, $V_{LS}$, and $V_T$ are called, respectively, the spin–spin term, the $LS$ term that shows the coupling between the orbital angular momentum and the spin angular momentum, and the tensor term. Following Ref. [32], the spin-dependent potential is derived from the central potential. $\boldsymbol{S}_Q$ and $\boldsymbol{S}_q$ denote the spin operators for the heavy and light quarks, respectively. $\boldsymbol{L}$ and $\boldsymbol{S}$ represent, respectively, the operator of the relative orbital angular momentum and the total spin operator defined as $\boldsymbol{S} = \boldsymbol{S}_Q + \boldsymbol{S}_q$. In a nonrelativistic constitutent-quark potential model, the heavy–light quark potential consists of two different contributions: the confining linear potential

$$V_{\mathrm{conf}}(r) = \varkappa r \tag{2}$$

with the parameter of the string tension $\varkappa$ and the Coulomb-like interaction arising from one-gluon exchange

$$V_{\mathrm{Coul}}(r) = -\frac{4\alpha_{\mathrm{s}}}{3r}, \tag{3}$$

where $\alpha_{\mathrm{s}}$ is the strong running coupling constant at the one-loop level:

$$\alpha_{\mathrm{s}}(\mu) = \frac{1}{\beta_0}\frac{1}{\ln(\mu^2/\Lambda_{\mathrm{QCD}}^2)}. \tag{4}$$

The one-loop $\beta$ function is given as $\beta_0 = (33 - 2N_f)/(12\pi)$. The dimensional transmutation parameters are taken from the Particle Data Group (PDG) [40], i.e., $\Lambda_{\mathrm{QCD}} = 0.217$ GeV. Since we include the charmed quark, the number of flavor is given by $N_f = 4$. The scale parameter $\mu$ will be set equal to the mass of the charmed quark,

$$V_c(r) = V_{\mathrm{conf}}(r) + V_{\mathrm{Coul}}(r), \tag{5}$$

and the spin-dependent parts are generated from this central potential and are expressed as

$$\begin{aligned} V_{SS}(r) &= \frac{32\pi\alpha_{\mathrm{s}}}{9M_QM_q}\delta(\boldsymbol{r}), \\ V_{LS}(r) &= \frac{1}{2M_QM_q}\left(\frac{4\alpha_{\mathrm{s}}}{r^3} - \frac{\varkappa}{r}\right), \\ V_T(r) &= \frac{4\alpha_{\mathrm{s}}}{3M_QM_q}\frac{1}{r^3}, \end{aligned} \tag{6}$$

where $M_Q$ and $M_q$ are stand for the dynamical heavy and light quark masses, respectively, which will be discussed shortly.

In a practical calculation, the point-like spin–spin interaction needs to be smeared by using the exponential form

$$\delta_\sigma(r) = (\frac{\sigma}{\sqrt{\pi}})^3 e^{-\sigma^2 r^2}, \tag{7}$$

where $\sigma$ stands for the smearing factor. Thus, one has a given set of parameters $\varkappa$ and $\sigma$ that are fitted to the spectra of mesons. In order to reduce the number of free parameters in the present work, we fix the strong running coupling constant $\alpha_s = 0.4106$ defined in Eq. (4) at the the scale of the charmed quark mass: $\mu = M_Q = m_c^{\text{current}} + \Delta M_Q$ with $m_c^{\text{current}} = 1.275\,\text{GeV}$ and $\Delta M_Q = 0.086\,\text{GeV}$. Here $\Delta M_Q$ is the shift of the heavy-quark mass caused by the heavy–light quark interactions that arise from a random instanton gas of the QCD vacuum. Its numerical value used here is determined in Ref. [34] (see also discussions in Ref. [33]). The dynamical mass of the light quark arises from the spontaneous breakdown of chiral symmetry (SB$\chi$S). The QCD instanton vacuum explains quantitatively the mechanism of the $SB\chi$S [41] (see also Refs. [42,43]). In the present work, we take the value of $M_{\text{u,d}} = 340\,\text{MeV}$. The strange dynamical quark mass is taken to be $M_{\text{s}} = m_{\text{s}} + M_q = (150 + 340)\,\text{MeV} = 490\,\text{MeV}$.

Since the main purpose of the present work is to consider the contribution of the nonperturbative heavy–light quark interaction from the instanton vacuum, we will introduce the effective instanton-induced heavy–light quark potential. For simplicity, we follow Ref. [34], where the local effective interactions between the heavy and light quarks due to instantons were derived in terms of the heavy and light quark operators $Q$ and $q$:

$$\mathcal{L}_{qQ} = -\left(\frac{M_q \Delta M_Q}{2nN_c}\right)\left(\overline{Q}\frac{1+\gamma^0}{2}Q\overline{q}q + \frac{1}{4}\overline{Q}\frac{1+\gamma^0}{2}\lambda^a Q\overline{q}\lambda^a q\right),$$

$$\mathcal{L}_{qQ}^{\text{spin}} = -\left(\frac{M_q \Delta M_Q^{\text{spin}}}{2nN_c}\right)\frac{1}{4}\overline{Q}\frac{1+\gamma^0}{2}\lambda^a\sigma^{\mu\nu}Q\overline{q}\lambda^a\sigma_{\mu\nu}q. \tag{8}$$

The density parameter $n$ of the random instanton gas is defined by $N/2V_4N_c$, where $N/V_4 \sim 1\,\text{fm}^{-4}$ is the instanton density with the 4D volume $V_4$ and $N_c$ denotes the number of colors. $\Delta M_Q$ is the mass shift of the heavy quark caused by the instantons. $\Delta M_Q^{\text{spin}}$ arises from the $M_Q^{-1}$-order chromomagnetic interaction and, therefore, its value is different from that of $\Delta M_Q$. In Ref. [34], the numerical value of $\Delta M_Q^{\text{spin}}$ is determined to be 3 MeV for the charmed quark. Other standard quantities in the Lagrangian are the Gell-Mann matrices for color space and the combinations from the Dirac matrices. Consequently, the relevant two-body instanton-induced central and spin–spin potentials are expressed as

$$V_{\text{I}}^c(\boldsymbol{r}) = \left(\frac{M_q \Delta M_Q}{2nN_c}\right)\left(1 + \frac{1}{4}\lambda_q^a\lambda_Q^a\right)\delta^3(\boldsymbol{r}), \tag{9}$$

$$V_{\text{I}}^{\text{spin}}(\boldsymbol{r}) = -\left(\frac{M_q \Delta M_Q^{\text{spin}}}{2nN_c}\right)\boldsymbol{S}_q\cdot\boldsymbol{S}_Q\lambda_q^a\lambda_Q^a\delta^3(\boldsymbol{r}), \tag{10}$$

where $\boldsymbol{r}$ designates the relative coordinates $\boldsymbol{r} = \boldsymbol{r}_q - \boldsymbol{r}_Q$.

Yet more spin-dependent potentials [32] are derived from the central potential from the instanton vacuum as follows:

$$V_{SS}^I(r) = \frac{1}{3M_QM_q}\nabla^2 V_I(r),$$

$$V_{LS}^{I}(r) = \frac{1}{2M_Q M_q}\frac{1}{r}\frac{dV_I(r)}{dr},$$
$$V_T^I(r) = \frac{1}{3M_Q M_q}\left(\frac{1}{r}\frac{dV_I(r)}{dr} - \frac{d^2V_I(r)}{dr^2}\right). \tag{11}$$

Since the central and spin–spin potentials are given as the Dirac delta functions, we also need to introduce here a smearing function to remove any divergence that would be caused by them. So, we introduce a Gaussian type of smearing function,

$$\delta_{\sigma_I}(r) = \left(\frac{\sigma_I}{\sqrt{\pi}}\right)^3 e^{-\sigma_I^2 r^2}, \tag{12}$$

in both central and spin–spin potentials. Here $\sigma_I$ stands for another smearing factor, the numerical value of which will not be much changed from that of $\sigma$ to avoid any additional uncertainty. The explicit forms of the spin-dependent potentials are obtained as

$$V_{SS}^I(r) = \left(\frac{\Delta M_Q}{6nN_cM_Q}\right)\left(1+\frac{1}{4}\lambda_Q^a\lambda_{\bar{q}}^a\right)\left(-6\sigma_I^2 + 4\sigma_I^4 r^2\right)\delta_{\sigma_I}(r),$$
$$V_{LS}^I(r) = \left(\frac{\Delta M_Q}{4nN_cM_Q}\right)\left(1+\frac{1}{4}\lambda_Q^a\lambda_{\bar{q}}^a\right)(-2\sigma_I^2)\delta_{\sigma_I}(r),$$
$$V_T^I(r) = \left(\frac{\Delta M_Q}{6nN_cM_Q}\right)\left(1+\frac{1}{4}\lambda_Q^a\lambda_{\bar{q}}^a\right)(-4\sigma_I^4 r^2)\delta_{\sigma_I}(r). \tag{13}$$

The total potential can be constructed by combining the potentials from the instanton vacuum given in Eqs. (9), (10), and (13) with those from the confining and Coulomb-like potentials in Eqs. (5) and (6):

$$V_{Q\bar{q}}(r) = V(r) + V_I(r), \tag{14}$$

where $V_I(r)$ is defined as

$$V_I(r) = V_{\mathrm{I}}^c(r) + V_I^{\mathrm{spin}}(r) + V_{SS}^I(r)(\boldsymbol{S}_Q\cdot\boldsymbol{S}_q) + V_{LS}^I(r)(\boldsymbol{L}\cdot\boldsymbol{S}) + V_T^I(r)[3(\boldsymbol{S}_Q\cdot\hat{\boldsymbol{n}})(\boldsymbol{S}_q\cdot\hat{\boldsymbol{n}}) - \boldsymbol{S}_1\cdot\boldsymbol{S}_2]. \tag{15}$$

The matrix element of the potential in the ${}^{2S+1}L_J$ basis is given by

$$\langle{}^{2S+1}L_J|V_{Q\bar{q}}(\boldsymbol{r})|{}^{2S+1}L_J\rangle = \tilde{V}_c(r) + \left[\frac{1}{2}S(S+1) - \frac{3}{4}\right]\tilde{V}_{SS}(r) + \frac{1}{2}\langle\boldsymbol{L}\cdot\boldsymbol{S}\rangle\tilde{V}_{LS}(r)$$
$$+\left[-\frac{2\langle\boldsymbol{L}\cdot\boldsymbol{S}\rangle(2\langle\boldsymbol{L}\cdot\boldsymbol{S}\rangle+1)}{4(2L-1)(2L+3)} + \frac{S(S+1)L(L+1)}{3(2L-1)(2L+3)}\right]\tilde{V}_T(r), \tag{16}$$

where

$$\langle\boldsymbol{L}\cdot\boldsymbol{S}\rangle = [J(J+1) - L(L+1) - S(S+1)]/2. \tag{17}$$

Here we have taken the conventional spectroscopic notation ${}^{2S+1}L_J$ given in terms of the total spin $S$, the orbital angular momentum $L$, and the total angular momentum $J$ with the addition of the angular momenta, $\boldsymbol{J} = \boldsymbol{L} + \boldsymbol{S}$. The corresponding terms $\tilde{V}_c(r)$, $\tilde{V}_{SS}(r)$, $\tilde{V}_{LS}(r)$, and $\tilde{V}_T(r)$ denote generically the central, spin–spin, spin–orbit, and tensor parts of the total potential.

## 3. Calculations and results

In Ref. [34], the mass spectra of the heavy mesons were already studied with a simple variational method, the potential from the instanton vacuum and the potential of the simple harmonic oscillator being combined. The results from Ref. [34] were in qualitative agreement with the experimental data. However, it is essential to consider more realistic contributions such as the confining potential and the Coulomb-like potential from one-gluon exchange in order to understand the effects of the instantons on the mass spectra of the heavy mesons in a quantitative manner. In the present work, we will include all the potentials mentioned in the previous section.

A nonrelativistic potential approach for a heavy–light quark system is represented by the time-independent Schrödinger equation with the static potential $V_{Q\bar{q}}(\boldsymbol{r})$:

$$\left[-\frac{\hbar^2}{\tilde{\mu}}\nabla^2 + V_{Q\bar{q}}(\boldsymbol{r}) - E\right]\Psi_{JM}(\boldsymbol{r}) = 0, \tag{18}$$

where $\tilde{\mu}$ denotes the reduced mass of the heavy meson system and $\Psi_{JM}$ stands for the wavefunction of the state with the total angular momentum $J$ and its third component $M$. To solve the Schrödinger equation numerically, we employ the GEM, which has been successfully applied to describe few-body systems such as light nuclei (see Ref. [36] and references therein).

In the GEM the wavefunction is expanded in terms of a set of $L^2$-integrable basis functions $\{\Phi^{LS}_{JM,k};\, k = 1 - k_{\rm max}\}$:

$$\Psi_{JM}(\boldsymbol{r}) = \sum_{k=1}^{k_{\rm max}} C^{(J)}_{k,LS}\Phi^{LS}_{JM,k}(\boldsymbol{r}), \tag{19}$$

and the Rayleigh–Ritz variational method is used. So, we are able to formulate a generalized eigenvalue problem given as

$$\sum_{m=1}^{k_{\rm max}} \left\langle \Phi^{LS}_{JM,k} \left| -\frac{\hbar^2}{\tilde{\mu}}\nabla^2 + V_{Q\bar{q}}(\boldsymbol{r}) - E \right| \Phi^{LS}_{JM,m} \right\rangle C^{(J)}_{m,LS} = 0. \tag{20}$$

The angular part of the basis function $\Phi^{LS}_{JM,k}$ is expressed in terms of standard spherical harmonics and the normalized radial part $\phi^L_k(r)$ is written in terms of the Gaussian basis functions

$$\phi^L_k(r) = \left(\frac{2^{2L+\frac{7}{2}} r_k^{-2L-3}}{\sqrt{\pi}(2L+1)!!}\right)^{1/2} r^L e^{-(r/r_k)^2}, \tag{21}$$

where $r_k$, $k = 1, 2, ..., k_{\rm max}$ designate variational parameters. When it comes to the case of a two-body problem, the total number of variational parameters is reduced by using geometric progression in the form of $r_k = r_1 a^{k-1}$, which provides a good convergence of the results. Thus, in the two-body problem, we need only three variational parameters, i.e., $r_1$, $a$, and $k_{\rm max}$.[1] Once the Schrödinger equation is solved, the energy eigenvalue $E_N$ is found and the mass of the heavy meson is determined by

$$M = M_Q + M_q + E_N + \Delta E_q, \tag{22}$$

[1] For more details, see Refs. [35–39].

**Table 1.** Free parameters of the model: $m_s$ denote the dynamical mass of the strange quark, $\varkappa$ stands for the string tension, $\sigma$ and $\sigma_I$ designate the smearing parameters corresponding to point-like interactions in Eqs. (7) and (12), $\Delta E_{u,d}$ and $\Delta E_s$ are the constant overall energy shifts of mesons corresponding to the up (down) and strange quark constituents, and $n$ is the density of the instanton medium.

| Model | $m_s$ [GeV] | $\varkappa$ [GeV$^2$] | $\sigma$ [GeV] | $\sigma_I$ [GeV] | $\Delta E_{u,d}$ [GeV] | $\Delta E_s$ [GeV] | $n$ [fm$^{-4}$] |
|---|---|---|---|---|---|---|---|
| I′ | 0.450 | 0.169 | 1.43 | – | −0.365 | −0.299 | – |
| I | 0.450 | 0.169 | 1.43 | 1.18 | −0.365 | −0.299 | 1.0 |
| II | 0.490 | 0.165 | 0.95 | 1.19 | −0.347 | −0.287 | 1.0 |
| III | 0.470 | 0.163 | 0.93 | 1.17 | −0.339 | −0.274 | 0.9 |

**Table 2.** The results of the charmed $D$-meson masses in units of MeV. The second column lists the results without the instanton-induced quark–quark interactions and is called Model I′. The third, fourth, and fifth columns list those of Models I, II, and III. The last column shows the corresponding experimental data taken from PDG [40].

| Model | I′ | I | II | III | Exp. |
|---|---|---|---|---|---|
| $D^{\pm}(1^1S_0)$ | 1867.7 | 1787.0 | 1868.3 | 1868.0 | $1869.65 \pm 0.05$ |
| $D^{*\pm}(2^1S_0)$ | 2013.5 | 2006.4 | 2009.7 | 2010.2 | $2010.26 \pm 0.05$ |
| $D_1(1^1P_1)$ | 2461.2 | 2461.5 | 2458.7 | 2456.7 | $2423.2 \pm 2.4$ |
| $D_2^*(1^3P_2)$ | 2462.2 | 2461.2 | 2461.7 | 2460.1 | $2465.4 \pm 1.3$ |
| $D^*(1^3S_1)$ | 2639.0 | 2593.4 | 2634.1 | 2630.4 | $2637 \pm 2 \pm 6$ |
| $(2^3S_1)$ | 2737.0 | 2732.6 | 2724.0 | 2719.8 | |

where $\Delta E_q$ is the overall energy shift in the spectra depending on the light-quark content of the meson and plays the role of a simple tuning parameter. As mentioned already, $M_Q$ and $M_q$ are the dynamical masses of the heavy and light quarks, respectively. Note that $M_Q$ also contains the mass shift arising from the instanton vacuum. In this work we will slightly vary the total mass of the strange quark mass $M_s$ and try to analyze the corresponding effects.

Since some of the remaining parameters cannot be determined theoretically, we construct several sets of parameters and call them Model I′, Model I, Model II, and Model III, respectively. The numerical values of the model parameters are listed in Table 1 and we use them to calculate the spectra of the heavy mesons.[2]

The results of the charmed meson masses corresponding to the different models are listed in Table 2 in comparison with the experimental data taken from the Particle Data Group (PDG) [40]. In the second column, the results without instanton-induced quark–quark interactions are presented. This is called Model I′, which is obtained by including only the confining and Coulomb-like type interactions. One can assume that in this model the nonperturbative effects are only taken into account by means of dynamically generated masses of the corresponding light quarks. It is seen that the results are relatively in good agreement with the experimental data. This indicates that a nonrelativistic approach to the heavy–light quark system works even quantitatively, at least for the mass spectra of the conventional heavy mesons.

Model I has the same parameter set as Model I′ except for the instanton-induced potentials, which means that the parameters are not tuned but the instanton-induced heavy–light quark interactions are taken into account. By doing this, we can examine how the instanton-induced quark–quark

[2] A corresponding explanation of the model parameters will be given hereafter in the text.

**Table 3.** The results of the charmed strange $D_s$-meson masses in units of MeV. Other notations are the same as in Table 1.

| Model | I′ | I | II | III | Exp. |
|---|---|---|---|---|---|
| $D_s^{\pm}(1^1S_0)$ | 1969.1 | 1887.9 | 1969.0 | 1968.9 | $1968.34 \pm 0.07$ |
| $D_s^{*\pm}(2^1S_0)$ | 2108.3 | 2100.8 | 2113.2 | 2111.5 | $2112.2 \pm 0.4$ |
| $D_{s1}^{\pm}(1^1P_1)$ | 2538.3 | 2538.1 | 2543.1 | 2540.5 | $2535.10 \pm 0.06$ |
| $D_{s2}^{*}(1^3P_2)$ | 2546.2 | 2545.1 | 2555.2 | 2551.8 | $2569.1 \pm 0.8$ |
| $D_s^{*}(1^3S_1)$ | 2703.7 | 2661.6 | 2697.4 | 2696.0 | $2708.3^{+4.0}_{-3.4}$ |
| $(2^3S_1)$ | 2792.6 | 2788.2 | 2780.5 | 2778.5 | |

interactions affect the mass of each charmed meson. The effects of instanton-induced quark–quark interactions are clearly seen in the ground state $D^{\pm}$ meson, while they are rather tiny on other charmed mesons. In particular, the effects are almost negligible on the $P$-wave charmed meson spectra. One can conclude that in general instanton-induced interactions do not greatly affect the spectra of heavy mesons and only play a role in the fine-tuning level.

Thus, we present the results of Model II, in which the free parameters are fitted to the experimental data. One can see that the results slightly change in comparison with Model I′, showing that the instanton-induced quark–quark interactions seem to be important in the fine-tuning level. In Model III, we also change the density of the instanton medium slightly, considering it as an input parameter. This is allowed, as was already discussed in Ref. [33] in detail. All other parameters are fitted to the experimental data as in the case of Model II.

The results of Model III are slightly better than those of Model II. As expected from the comparison of Model I with Model I′, the prediction of Model III is not much different from that of Model I′. Thus the potential from the instanton vacuum in the present form slightly changes the mass spectrum of the charmed mesons and does not quantitatively affect the results from the calculation without instanton-induced quark–quark interactions.

Table 3 lists the results of the charmed strange meson masses. As in Table 2, we first compute the masses of the charmed strange mesons without the instanton contributions, which are listed in the second column of Table 2. Then we include the instanton-induced quark–quark interactions, the results of which are presented in the other columns. The effects of the instantons are similar to the case of the charmed mesons; i.e., the instanton effects are noticeable only on the ground state $D_s^{\pm}$ meson whereas they are negligibly small on the $P$-wave charmed strange mesons. Though the results of Model III seem slightly better than those of Model I′, for the quark–quark potential from the instanton vacuum, at least in the present form, the improvement is marginal in the charmed strange meson mass spectrum. Moreover, the effects of the instanton-induced potential on the charmed strange mesons are even smaller than on the charmed nonstrange ones.

Finally, we would like to note that, although we have changed the density of the instanton medium $n$ in Model III in comparison with Model II, the mass contribution $\Delta M_Q$ is unchanged and kept in both cases equal to 0.086 GeV. However, $\Delta M_Q$ is proportional to $n$ and therefore it must also be modified if the value of $n$ changes. As a result, the eigenfunctions and eigenvalues of the Hamiltonian should also be altered. Consequently, a better fine-fitting of the whole mass spectra can be achieved by means of changes of instanton parameters in a self-consistent manner. Though these self-consistent changes of parameters are expected to improve the present results further, we have not done this because in the present work we aimed at examining the effects of the existing nonperturbative heavy–light quark potentials from the instanton vacuum on the conventional heavy mesons.

**Table 4.** The results of the instaton effects on the low-lying charmed heavy mesons in units of MeV. The values of the relevant parameters are taken from those for Model I.

| Heavy meson | Instanton contribution [MeV] | Exp. [MeV] |
|---|---|---|
| $D^{\pm}(1^1S_0)$ | 80.7 | $1869.65 \pm 0.05$ |
| $D^{*\pm}(1^3S_1)$ | 7.1 | $2010.26 \pm 0.05$ |
| $D_1(1^1P_1)$ | −0.3 | $2423.2 \pm 2.4$ |
| $D_2^*(1^3P_2)$ | 0.1 | $2465.4 \pm 1.3$ |
| $D^*(2^1S_0)$ | 45.6 | $2637 \pm 2 \pm 6$ |
| $(2^3S_1)$ | 4.4 | |
| $D_s^{\pm}(1^1S_0)$ | 81.2 | $1968.34 \pm 0.07$ |
| $D_s^{*\pm}(1^3S_1)$ | 7.5 | $2112.2 \pm 0.4$ |
| $D_{s1}^{\pm}(1^1P_1)$ | 0.2 | $2535.10 \pm 0.06$ |
| $D_{s2}^*(1^3P_2)$ | 1.1 | $2569.1 \pm 0.8$ |
| $D_s^*(2^1S_0)$ | 42.1 | $2708.3^{+4.0}_{-3.4}$ |
| $(2^3S_1)$ | 4.4 | |

In Table 4, we list the results of the contributions from the instanton-induced potentials. While they have visible effects on the masses of the $D^{\pm}$ and $D_s^{\pm}$ mesons, and marginal contributions to the radially excited $S$-wave $D^*(2^1S_0)$ and $D_s^*(2^1S_0)$ mesons, they have almost no impact on other excited $D$ and $D_s$ mesons. Thus, in conclusion, the present form of the instanton-induced potentials contributes to some of the $D$ and $D_s$ mesons as explicitly shown in Tables 2, 3, and 4, but its overall effects turn out to be marginal. Possible ways of improving the present results will be mentioned in the next section.

## 4. Summary and outlook

In the present work, we have investigated the effects of the heavy–light quark potential from the instanton vacuum on the mass spectra of the conventional charmed mesons. First, we considered the confining potential that is proportional to the relative distance between the heavy and light quarks. The Coulomb-like potential, which arises from one-gluon exchange, has been included. The spin-dependent potentials were generated from the central part. Then we computed the mass spectra of the charmed mesons, employing the Gaussian expansion method to solve the nonrelativistic Schrödinger equation. The results are in good agreement with the experimental data even without the inclusion of the potential from the instanton vacuum. Then, we introduced the central and spin-dependent potentials from the instanton vacuum. The additional spin part of the potential was obtained from the central part of the instanton-induced potential. While the instanton effects are noticeable on the $S$-wave charmed and charmed strange heavy mesons, the contribution to their masses from the instanton-induced potential is rather tiny.

Though the present form of the instanton-induced potential does not make any significant contribution to the heavy meson masses, there are some possible ways of elaborating the present analysis:

- The present work is based on the nonrelativistic Schrödinger equation, since we aimed mainly at investigating the effects of the instanton-induced potential. However, once the light quark is involved, the inclusion of certain relativistic effects is inevitable.
- The instanton-induced potentials used in the present work were derived from the random instanton gas model and are given as local ones. However, if one uses the instanton liquid model, the

interaction between the heavy and light quarks turns out to be nonlocal [44]. This nonlocality will have certain effects on the mass spectra of the heavy mesons.
- Recently, Ref. [45] showed that rescattering of gluons with instantons dynamically generates the effective momentum-dependent gluon mass that will cause the screened heavy-quark potential. This indicates that certain nonperturbative effects from the instanton vacuum will also contribute to the heavy–light quark system.

Thus, one needs to study systematically nonperturbative effects on both heavy mesons and heavy baryons, arising from the instanton vacuum. The corresponding investigations are underway.

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