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Search for Abelian dominance in the effective $SO(N)$ theory

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ABSTRACT

Recently a new quadratic gauge has been introduced in the $SU(N)$ QCD in which interesting non perturbative characteristics of QCD has been brought to light. One of the characteristics is Abelian dominance, which is one of the signatures of color confinement in a dual superconductor picture. The $SO(N)$ QCD has been used quite often for various studies. In this paper, we extend the application of the quadratic gauge to $SO(N)$ QCD and probe different vacua of the effective theory in which ghosts condense to search existence of Abelian dominance. We begin with the vacuum whose $SU(N)$ group version leads us to find the evidence of Abelian dominance in $SU(N)$ QCD. Contrary to expectation, we do not observe Abelian dominance in such vacuum here. Taking hint from this consequence, another vacuum is then examined in which existence of Abelian dominance is strongly indicated. The geometric explanation for the results in both the vacua has been provided.

1. Introduction

The physical mechanism by which quarks and gluons are confined is unknown and this is one of the most important unanswered questions in the non perturbative QCD. A classical model for confinement is a linear potential between static quarks. In the mid 1970s, Nambu [1], 't Hooft [2], and Mandelstam [3] proposed a dual version of the type-II superconductor as quark confinement mechanism. The magnetic field is trapped in the form of one-dimensional Abrikosov vortex tubes inside the type-II superconductor, i.e., inside the medium of condensed electric charges [4]. In the same way but dually the idea of these proposals for the quark confinement mechanism is that the electric field due to quarks is trapped in the form of vortex tubes in a phase in which magnetic monopoles are condensed. This gives rise to forces which are parameterized by a constant string tension and a linear potential between static quarks. Here the crucial factor is that the dual picture is based on the Abelian gauge theory whereas QCD is non-Abelian, therefore one needs to demonstrate that QCD reduces to an effective Abelian theory at an infra-red scale. Secondly, it requires a new concept of condensed magnetic monopoles. Need for such an effective theory led to the concept of "Abelian dominance" [5].

According to the proposal of Abelian dominance, at a low energy scale, QCD can be effectively expressed in terms of Abelian gauge degrees of freedom [5]. It is usually discussed in terms of off-diagonal gluons i.e., gluons not associated with the Cartan subalgebra of a Lie group. In the $SU(N)$ gauge theory, there are $N(N-1)$ off-diagonal gluons. In $SO(N)$ theory, the number of off-diagonal gluons is classified in two sets as follows: for the odd dimension say, $2N+1$, there are $2N^2$ off-diagonal gluons and for the even dimension say, $2N$, there are $2N(N-1)$ of them. Exactly this number of gluons attaining large dynamical masses is presumed to provide the required evidence of existence of Abelian dominance. In the infra-red limit, the massive off-diagonal gluons decouple, leaving behind the massless diagonal gluons as the only dominant degrees of freedom. Thus, one gets as many copies of Abelian gauge theory as the rank of the Lie group, one for each diagonal gluon. Hence, for $SU(N)$ we have $N-1$ copies of Abelian theory and for $SO(2N)$ and $SO(2N+1)$, we get N copies of the same.

The $SO(N)$ group does not preserve some nice properties such as the symplectic structure that the $SU(N)$ group keeps. Broadly this turns out to be the reason we do not observe Abelian dominance in the vacuum whose $SU(N)$ version contains evidence of Abelian dominance in $SU(N)$ QCD as we see later. Therefore, we explore further to another vacuum in which Abelian dominance is strongly indicated. The $SO(N)$ QCD however is similar to the $SU(N)$ theory by construction and therefore has been a point of the study [6–8]. Some of them has been devoted to the non perturbative aspects of the theory [9,10], which caters as the motivation to advance our previous non perturbative study involving the quadratic gauge in the $SU(N)$ QCD to the $SO(N)$ QCD.

Now we present the plan of this paper. In the next section, we review the newly introduced quadratic gauge with the effec-

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tive $SU(N)$ QCD in it and discuss, in section 3, the mechanism of ghost condensation which leads to Abelian dominance [11] to set the stage for the current matter. In section 4, we extend the application of the quadratic gauge to the $SO(N)$ QCD and deal with the effective $SO(N)$ theory to check the possible existence of Abelian dominance. We then would begin the search with the same form of ghost condensation which was followed in $SU(N)$ theory to eventually learn that Abelian dominance does not exist in the present vacuum as opposed to the $SU(N)$ theory. Then, the vacuum with another form of ghost condensation is explored to have a strong indication of existence of Abelian dominance. We also give the geometric reason behind the results in both vacua in this section. The last section is kept for the conclusion.

2. $SU(N)$ QCD in the quadratic gauge

Here we discuss a model which we intend to extend to the $SO(N)$ QCD. The model relies on the new quadratic gauge fixing of Yang–Mills action. In its simplest form, the gauge is also free of Gribov ambiguity as it is purely an algebraic gauge. However this gauge is not suitable for usual perturbation theory. The new quadratic gauge has been introduced as follows [11],

$$H^a[A^\mu(x)] = A^a_\mu(x)A^{\mu a}(x) = f^a(x); \quad \text{for each } a \tag{1}$$

where $f^a(x)$ is an arbitrary function of x. We note that this is not an Abelian projection [11] as the gauge condition does not take values in the Lie algebra. Furthermore, the condition of an Abelian projection is stipulated to be covariant in order to ensure survival of an Abelian component. The above gauge condition does not meet this requirement either. The consequences of such a condition to lifting the Gribov ambiguity were studied explicitly in [12]. The Faddeev–Popov determinant in this gauge is given by

$$\det\left(\frac{\delta(A^{a\epsilon}_\mu A^{\mu a\epsilon})}{\delta\epsilon^b}\right) = \det\left(2A^a_\mu(\partial^\mu\delta^{ab} - gf^{acb}A^{\mu c})\right). \tag{2}$$

Therefore, the resulting effective Lagrangian density contains gauge fixing and ghost terms as follows,

$$\mathcal{L}_{\rm GF} + \mathcal{L}_{\rm ghost} = -\frac{1}{2\zeta}\sum_a (A^a_\mu A^{\mu a})^2 - 2\sum_a \overline{c^a}A^{\mu a}(D_\mu c)^a, \tag{3}$$

where ζ is an arbitrary gauge fixing parameter and $(D_\mu c)^a = \partial_\mu c^a - gf^{abc}A^b_\mu c^c$. Now onwards, we shall drop the summation symbol, but the summation over an index a will be understood when it appears repeatedly, including when repeated *thrice* as in the ghost terms above. In particular,

$$-\overline{c^a}A^{\mu a}(D_\mu c)^a = -\overline{c^a}A^{\mu a}\partial_\mu c^a + gf^{abc}\overline{c^a}c^c A^{\mu a}A^b_\mu \tag{4}$$

where the summation over indices a, b and c each runs independently over 1 to N^2-1. With this understanding, we write the full effective Lagrangian density in this quadratic gauge as

$$\begin{aligned}\mathcal{L}_Q &= -\frac{1}{4}F^a_{\mu\nu}F^{\mu\nu a} - \frac{1}{2\zeta}(A^a_\mu A^{\mu a})^2 - 2\overline{c^a}A^{\mu a}(D_\mu c)^a \\ &= -\frac{1}{4}F^a_{\mu\nu}F^{\mu\nu a} + \frac{\zeta}{2}F^{a2} + F^a A^a_\mu A^{\mu a} - 2\overline{c^a}A^{\mu a}(D_\mu c)^a, \end{aligned} \tag{5}$$

where the field strength $F^a_{\mu\nu} = \partial_\mu A^a_\nu(x) - \partial_\nu A^a_\mu(x) - gf^{abc}A^b_\mu(x)A^c_\nu(x)$ and in the second version the F^a are a set of auxiliary fields called Nakanishi–Lautrup fields. As shown in [12], the Lagrangian is BRST invariant [13,14] which is essential for the ghost independence of green functions and unitarity of the S-matrix. Due to quark confinement signatures and absence of Gribov ambiguity, we find the gauge to be suitable for studying non perturbative aspects of $SU(N)$ QCD. Recently an interesting way of connecting two different regimes, namely non perturbative and perturbative, of QCD by further extending FFBRST method has been demonstrated [15].

3. Abelian dominance via ghost condensation

The significance of ghosts in the IR limit comes in terms of their condensates. It is imperative to study the mechanism of the ghost condensation thoroughly citing its value for the present purpose. The inspection of degrees of freedom in the second term of the ghost Lagrangian in Eq. (4) tells us that if certain ghost bilinears are replaced by c-numbers, we would get the massive gluons as ghost bilinears multiply terms quadratic in gauge fields in this term. This is suggestive of the gluon mass matrix as follows

$$(M^2)^{ab}_{\rm dyn} = 2g\sum_{c=1}^{N^2-1} f^{abc}\langle\overline{c^a}c^c\rangle \tag{6}$$

whereas diagonal components of $M^2_{\rm dyn}$ are zero since $f^{aac} = 0$. To obtain a spectrum of the theory i.e., to obtain masses of gluons, we must diagonalize the matrix and find eigenvalues. Required demonstration is simple in an $SU(N)$ symmetric state, where all ghost–anti-ghost condensates are identical i.e.,

$$\begin{aligned}&\langle\overline{c^1}c^1\rangle = \ldots = \langle\overline{c^1}c^{N^2-1}\rangle = \ldots = \langle\overline{c^{N^2-1}}c^1\rangle = \ldots \\ &\quad = \langle\overline{c^{N^2-1}}c^{N^2-1}\rangle \equiv K.\end{aligned} \tag{7}$$

This was achieved by introducing a Lorenz gauge fixing term for one of the diagonal gluons, in addition to the purely quadratic terms of Eq. (1). This gauge fixing gives the propagator to the corresponding ghost field. Using this ghost propagator, one can give nontrivial vacuum values to bilinears $\overline{c^a}c^c$ within the framework Coleman–Weinberg mechanism as described in [11]. Thus, when all the condensates are identical the mass matrix becomes

$$(M^2)^{ab}_{\rm dyn} = 2g\sum_{c=1}^{N^2-1} f^{abc}K, \tag{8}$$

which is an antisymmetric matrix due to the antisymmetry of the structure constant f^{abc}. The resulting mass matrix for the gluons has $N(N-1)$ non-zero eigenvalues only and thus has nullity $N-1$. This implies $N(N-1)$ off-diagonal gluons have become massive and $N-1$ diagonal gluons remain massless.

Now we give the crucial geometric reason behind this peculiar property [11]. The matrix in Eq. (8) is $N^2-1\times N^2-1$ dimensional adjoint representation of $\mathfrak{g}$, the Lie algebra vector space. For simple Lie groups such as $SU(N)$ and $SO(N)$, the coadjoint action Ad^* which is dual of the adjoint is the same as the adjoint $Ad = g^{-1}Fg$, where F is Lie algebra valued and the element g belongs to for example, the $SU(N)$ group. Therefore, the orbit resulting from the coadjoint action by elements g of $SU(N)$ on $\mathfrak{g}^*$, *dual vector space* is given by

$$\mathcal{O}_F = \{Ad^*F = g^{-1}Fg, \quad F\in\mathfrak{g}^*, \ \forall g\in SU(N)\}.$$

The orbit $\mathcal{O}_F$, passing through F, is known as the coadjoint orbit. Depending upon the starting point F the orbit may be generic or non generic. This action has a stabilizer, i.e., a set of elements in $SU(N)$ that leave the elements F invariant. Therefore, orbits are isomorphic to G/H with H being the stabilizer. For generic orbits, the maximal torus group acts as the stabilizer in a compact connected Lie group. For the $SU(N)$, the maximal torus group is $U(1)^{N-1}$. Thus, generic coadjoint orbits of the $SU(N)$ group are isomorphic to a manifold $SU(N)/U(1)^{N-1}$ i.e.,

$$\mathcal{O}_F \sim SU(N)/U(1)^{N-1} \sim \mathbb{C}P^{N-1} \otimes \mathbb{C}P^{N-2} \otimes \ldots \otimes \mathbb{C}P^1.$$

It is an $N(N-1)$ dimensional symplectic manifold with a symplectic form $Tr(F[X,Y])$, where $F \in \mathfrak{g}^*$ and $X, Y \in \mathfrak{g}$, defined on it [16]. In the present problem, the mass matrix $(M^2)^{ab}$ is the symplectic form of the generic orbit of $SU(N)$, $(M^2)^{ab} \sim Tr(F[T^a, T^b])$ with $F = \sum_{c=1}^{N^2-1} T^c$ and T^a, T^b, T^c being basis generators. Contrary to this as we see in the next section, in the $SO(N)$ effective theory the similar gluon mass matrix is the symplectic form of a non generic orbit. The rank of a symplectic form is always equal to a dimension of the coadjoint orbit. Since $(M^2)^{ab}$ is also a normal matrix, its rank and number of non-zero eigenvalues are equal. So, the rank and therefore the number of non-zero eigenvalues are $N(N-1)$ and thus nullity is $N-1$.

The nonzero eigenvalues thus identified, being eigenvalues of an antisymmetric matrix, are purely imaginary and occur in conjugate pairs, viz., $M^2_{gluon} = \pm i m^2$, (m^2 positive real). This implies that masses of these gluons $M_{gluon} = \frac{1}{\sqrt{2}}(1 \pm i)m$ or $\frac{1}{\sqrt{2}}(-1 \mp i)m$. We ignore the latter choice since it gives $Re(M_{gluon})$ negative, which is not physical. Thus in this description, the off-diagonal gluons acquire masses with the positive real parts i.e., $Re(M_{gluon}) > 0$, which makes them short-ranged. Only the diagonal gluons mediate interactions in the IR limit which are long range, strongly suggesting Abelian dominance. This in turn hints at quark confinement. Having reviewed prerequisites, we are now in a position to discuss the consequences of this gauge in the $SO(N)$ QCD.

4. Exploration of Abelian dominance in $SO(N)$ QCD

Previous non perturbative studies in the $SO(N)$ QCD has prompted us with the idea to extend the application of the quadratic gauge, $A^a_\mu(x) A^{\mu a}(x) = f^a(x)$ (1) to the $SO(N)$ QCD in order to investigate non perturbative characteristic of the effective theory. Thus, there are $\frac{N(N-1)}{2}$ quadratic gauge conditions, one for each $SO(N)$ gluon (now onwards, will be called only gluon). The structure of the effective theory of course remains same as,

$$\mathcal{L}_Q = -\frac{1}{4} F^a_{\mu\nu} F^{\mu\nu a} - \frac{1}{2\zeta}(A^a_\mu A^{\mu a})^2 - 2\overline{c^a} A^{\mu a} (D_\mu c)^a, \tag{9}$$

where a description of the field strength and a ghost term is given as in Sec. 2. Here, the only difference is that a sum over the index a, including when repeated *thrice*, in the Lagrangian now rather runs from 1 to $\frac{N(N-1)}{2}$. With this in mind, we study what happens in the present occasion when ghosts condense. The second term of the ghost Lagrangian is rewritten as

$$g f^{abc} \overline{c^a} c^c A^{\mu a} A^b_\mu \tag{10}$$

with sum over all the indices a, b, c runs from 1 to $\frac{N(N-1)}{2}$ independently, and f^{abc} throughout this section is taken as the structure constant of $SO(N)$. Thus, in a state where the ghosts have condensed we get the following gluon mass matrix for $SO(N)$ group

$$(M^2)^{ab}_{\text{dyn}} = 2g \sum_{c=1}^{\frac{N(N-1)}{2}} f^{abc} \langle \overline{c^a} c^c \rangle. \tag{11}$$

Now, we start the hunt with the $SO(N)$ symmetric vacuum akin to $SU(N)$ symmetric vacuum in the last section, where all the condensates are equal. Such vacuum can be achieved by following the Coleman Weinberg method of Ref. [11] here as well. The vacuum is given by

$$\begin{aligned}\langle \overline{c^1} c^1 \rangle &= \ldots = \langle \overline{c^1} c^{\frac{N(N-1)}{2}} \rangle = \ldots = \langle \overline{c^{\frac{N(N-1)}{2}}} c^1 \rangle = \ldots \\ &= \langle \overline{c^{\frac{N(N-1)}{2}}} c^{\frac{N(N-1)}{2}} \rangle \equiv J.\end{aligned} \tag{12}$$

In this vacuum, the mass matrix above for $SO(2N)$ and $SO(2N+1)$ is given respectively by

$$(M^2)^{ab}_{\text{dyn}} = 2gJ \sum_{c=1}^{N(2N-1)} f^{abc} \text{ for } SO(2N)$$

or

$$= 2gJ \sum_{c=1}^{N(2N+1)} f^{abc} \text{ for } SO(2N+1). \tag{13}$$

We have segregated the analysis into $SO(2N)$ and $SO(2N+1)$ groups for a better clarity of further discussion as the rank of both the groups is same, N. Diagonalizing this matrix, we will get masses of gluons. The eigenvalues will be purely imaginary and occur in conjugate pairs due to an antisymmetric nature of the structure constants f^{abc}. As in $SU(N)$ theory, the square root of an eigenvalue with a positive real part gives mass of a gluon.

As discussed in the introduction, to prove an existence of Abelian dominance we need exactly $2N^2$ gluons for $SO(2N+1)$ and $2N(N-1)$ for $SO(2N)$ to become massive as the rest of the massless diagonal gluons signify dominant Abelian degrees. We find in general that the mass matrix does not have the required number of non zero eigenvalues. The matrix has less non zero eigenvalues by even number than required or in other words nullity is higher by even number than the rank of $SO(N)$. In an instance of the $SO(4)$, the matrix has only two non zero eigenvalues whereas the required number for Abelian dominance is 4. This means in addition to the diagonal gluons, even number of off-diagonal gluons whose corresponding generators do not commute (not associated Cartan sub algebra of the group) remain massless. Therefore, along with diagonal gluons, these off-diagonal gluons also carry the interaction at infrared scale destroying the dominance of only Abelian components viz., Abelian dominance. Thus, counter intuitively we do not observe Abelian dominance in the vacuum whose $SU(N)$ version contains signs of Abelian dominance in $SU(N)$ QCD. We here mention that in a special instance such as $SO(3)$ group, the matrix in Eq. (13) may belong to generic orbit and therefore Abelian dominance can still exist in this case in a given vacuum. However, the conclusion is neither general nor interesting. We shall relook at this point in the upcoming matter.

The failure in the given occasion however hints at the possible direction of further exploration in which we continue our search to examine other vacua of ghost condensates where we can likely find the required signature. Before that we give the geometric explanation of the result obtained above.

4.1. A geometric explanation

The coadjoint orbit of the $SO(N)$ group, $\mathcal{O}_F = \{Ad^*F = g^{-1}Fg,\ \ F \in \mathfrak{g}^*,\ \forall g \in SO(N)\}$ is also a symplectic manifold with a well defined symplectic form, $Tr(F[X,Y])$, where $F \in \mathfrak{g}^*$ and $X, Y \in \mathfrak{g}$. For a normal symplectic form, there are as many non zero eigenvalues as the dimension of a corresponding orbit. The gluon mass matrix in Eq. (11) is normal and is the symplectic form,

$$\sim Tr\left(\sum_{c=1}^{N(2N\mp1)} T^c[T^a, T^b]\right) \text{ where } T^a \text{ is a basis,}$$

of a non generic orbit of $SO(2N)$ (or $SO(2N+1)$) rather of a generic orbit contrary to the case of the $SU(N)$. The dimension of a non generic orbit is less by even number than that of a generic orbit. Hence, for a normal symplectic form of a non generic orbit, nullity is higher by even number than the rank of the group as for generic orbits nullity equals the rank. Therefore, it suggests that the number of massive gluons is always less than the required for Abelian dominance in the state at hand. In an example of the $SO(4)$ group, the matrix in Eq. (11) is the symplectic form of the orbit $\sim \mathbb{S}^2$, which is a non generic orbit and therefore has only two non zero eigenvalues whereas the generic orbit is $\sim \mathbb{S}^2 \times \mathbb{S}^2$ [17].

4.2. Further exploration

We dealt with the specific mass matrix of Eq. (13) which corresponds to non generic orbit. Now, the geometric consideration suggests that if we deal with a more general matrix then we might get the goal as more general matrix is likely to correspond to the higher dimensional orbit. With this in sight, we analyze the following form of ghost condensates

$$\langle \overline{c^1} c^1 \rangle = \ldots = \langle \overline{c^{\frac{N(N-1)}{2}}} c^1 \rangle = K_1 \neq \langle \overline{c^1} c^2 \rangle = \ldots = \langle \overline{c^{\frac{N(N-1)}{2}}} c^2 \rangle = K_2$$
$$\neq \ldots \neq \langle \overline{c^1} c^{\frac{N(N-1)}{2}} \rangle = \ldots = \langle \overline{c^{\frac{N(N-1)}{2}}} c^{\frac{N(N-1)}{2}} \rangle = K_{\frac{N(N-1)}{2}}. \qquad (14)$$

The occurrence of this form can be ascertained within physical mechanism by carrying out the procedure of the appendix A which lends strength to the physical relevance of the theory. In this new form of condensates, the mass matrix in Eq. (11) becomes

$$(M^2)^{ab}_{\text{dyn}} = 2g \sum_{c=1}^{\frac{N(N-1)}{2}} K_c f^{abc} = \sum_{c=1}^{\frac{N(N-1)}{2}} w_c [T^{ab}]^c, \qquad (15)$$

where T^c is a generator of $SO(N)$ in the adjoint representation and w_is are real parameters with a condition $w_1 \neq w_2 \neq \ldots \neq w_{\frac{N(N-1)}{2}}$. In case of several groups, we would achieve the same outcome with some of the w_is being equal. In a particular case like $SO(3)$, the outcome would not change when all the w_is are same. We are here considering the general situation. The matrix in Eq. (15) is a generic vector in the Lie algebra of $SO(N)$. Hence, the matrix is the symplectic form of the generic orbit of the $SO(N)$ unlike the matrix in Eq. (13) which corresponds to the non generic orbit of the $SO(N)$. The symplectic form of the generic orbit has nullity equal to the rank of the group which in this case is $SO(N)$ as explained earlier. Hence, the matrix in Eq. (15), for the $SO(2N)$, has exactly $2N(N-1)$ non zero eigenvalues and for the $SO(2N+1)$, has exactly $2N^2$ non zero eigenvalues. For example in $SO(4)$, the matrix in Eq. (15) has 4 non zero eigenvalues and it corresponds to the generic orbit $\sim \mathbb{S}^2 \times \mathbb{S}^2$. Thus, in the $SO(2N)$, $2N(N-1)$ and in the $SO(2N+1)$, $2N^2$ off-diagonal gluons obtain masses respectively strongly suggesting existence of Abelian dominance in the effective $SO(N)$ theory of the quadratic gauge albeit in a different vacuum than in $SU(N)$ QCD. Thus, in this new form of the ghost condensates, there is a signature of color confinement in the present theory.

5. Conclusion

Abelian dominance is one crucial signature of color confinement in the dual super conductor picture. In this paper, we extended our previous work employing the quadratic gauge to the $SO(N)$ QCD to investigate non perturbative sector of the theory. We followed the same ghost condensation whose $SU(N)$ version suggested existence of Abelian dominance in the $SU(N)$ QCD in the present effective theory. Interestingly despite the same structure of the vacua, we do not find Abelian dominance in the stated condensation. This is because the mass matrix in Eq. (13) is the symplectic form of a non generic orbit of $SO(N)$ group contrary to the mass matrix in $SU(N)$ QCD and hence the nullity is higher by even number than the rank of $SO(N)$. This implies in the framework of the present theory that there are generally at least two off-diagonal gluons along with diagonal gluons which carry an interaction at low energy scale as they remain massless. Therefore, only Abelian components no longer dominate the interaction in the IR limit in the given vacuum. Hence, taking hint from geometric consideration we explore further a new form of ghost condensates in which the mass matrix now belongs to the genetic orbit and thus nullity of the new matrix equals the rank of the $SO(N)$ which is the required number to signify presence of Abelian dominance in this new vacuum as only Abelian degrees will now carry the interaction. Therefore, according to a dual superconductor picture, quark confinement is suggested in the $SO(N)$ QCD with the quadratic gauge albeit in a different vacuum than in $SU(N)$ QCD.

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Appendix A. Coleman Weinberg mechanism

In order to lend strength to the physical relevance of the theory here we lay down the physical mechanism and set up the feasible scenario within which the occurrence of condensates of the form given in Eq. (14) can be realized. The procedure is the same as given in Ref. [11] with some minor modifications. We consider a ghost bilinear, $\overline{c^m}c^n$, and introduce for them an auxiliary scalar field σ^{mn}, which is their putative condensate. We insert the following identity into the gauge fixed path integral

$$1 = \int \prod_{m=1,n=1}^{\frac{N(N-1)}{2},\frac{N(N-1)}{2}} \left((\mathcal{D}\sigma^{mn})\, e^{-i\int d^4x(\sigma^{mn} - \beta\,\overline{d^3}d^3 - \alpha_n\,\overline{c^m}c^n)^2} \right), \qquad (A.1)$$

where α_n, β are couplings and, couplings α_n satisfy

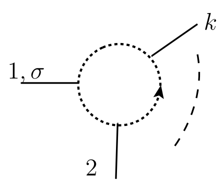
$$\alpha_1 \neq \alpha_2 \neq \ldots \neq \alpha_n. \qquad (A.2)$$

Ghost field d^3 arises as a result of additional Feynman gauge fixing on the gluon A_3 whose corresponding action would add to the effective $SO(N)$ action in the quadratic gauge. Without loss generality we take $\langle \overline{d^3}d^3 \rangle = 0$ as it in no ways affects dynamics of the theory in the vacuum. Due to Feynman fixing, the ghost d^3 acquires a usual propagator $\sim 1/p^2$ to be used in deriving the following effective potential for σ^{mn}. The identity amounts to the following effective Lagrangian which adds to Lagrangian in Eq. (9),

$$\mathcal{L}(\sigma) = \sum_{m=1,n=1}^{\frac{N(N-1)}{2},\frac{N(N-1)}{2}} \Big[(\sigma^{mn})^2 - \beta\sigma^{mn}\,\overline{d^3}d^3 - \alpha_n\sigma^{mn}\,\overline{c^m}c^n + \alpha_n\beta\,\overline{c^m}c^n\overline{d^3}d^3 + \beta^2\,(\overline{d^3}d^3)^2 + \alpha_n^2(\overline{c^m}c^n)^2 \Big] \tag{A.3}$$

Additional terms due to unity in the path integral and Feynman gauge fixing do not affect conclusions of the theory; therefore, we have not considered them in the main discussions of the paper and take into account the original effective Lagrangian (Eq. (9)) only. The effective potential for σ^{mn} may now be computed within the standard strategy of Coleman–Weinberg mechanism in which one-loop diagrams give the leading quantum correction. In the present case, this consists of all one-loop d^3 diagrams.

$$V^{1-loop}(\sigma^{mn}) = \sum_{k=1}^{\infty}$$



$$V^{1-loop}(\sigma^{mn}) = \int \frac{d^4p}{2i(2\pi)^4} \sum_{k=1}^{\infty} \frac{(-1)}{k}(\sigma^{mn})^k \frac{(i)^k}{(p^2)^k}\Big(-i\beta\Big)^k = \int \frac{d^4p}{2i(2\pi)^4} ln\Big(\frac{p^2-\beta\sigma^{mn}}{p^2}\Big) \tag{A.4}$$

Therefore, the effective potential is

$$V(\sigma^{mn}) = (\sigma^{mn})^2 + \int \frac{d^4p}{2i(2\pi)^4}\, ln\Big(\frac{p^2-\beta\sigma^{mn}}{p^2}\Big) \tag{A.5}$$

The extremum of the potential is given by the zero of the gap equation $V'(\sigma) = 0$, which in the minimal subtraction scheme of the dimensional regularization is given by

$$\sigma^{mn} + \frac{1}{32\pi^2}\beta^2\sigma^{mn}\Big(ln\big(\beta\sigma^{mn}4\pi\mu^2\big) + \gamma - 1\Big) = 0 \tag{A.6}$$

where γ is the Euler constant, 0.57721... and μ is an arbitrary mass scale.

Apart from a trivial solution, it has a non-trivial solution

$$\sigma_0^{mn} = \frac{1}{\beta}\,4\pi\mu^2 e^{(1-\gamma)}\,\exp\Big(\frac{-32\pi^2}{\beta^2}\Big) \tag{A.7}$$

which corresponds to the minimum of the potential because

$$V''(\sigma_0^{mn}) = \frac{1}{32\pi^2}\beta^2 > 0 \tag{A.8}$$

We also notice that all the putative condensates are equal. From the equation of motion for σ^{mn}, we can deduce that

$$\langle\overline{c^m}c^n\rangle \sim \frac{\sigma_0^{mn}}{\alpha_n} \tag{A.9}$$

Thus, we realize the desired vacuum in Eq. (14) in view of Eq. (A.2).

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