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Letter

Charge-screened nontopological solitons in a spontaneously broken U(1) gauge theory

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We numerically construct stationary and spherically symmetric nontopological soliton solutions in a system composed of a complex scalar field, a U(1) gauge field and a complex Higgs scalar field that causes spontaneous symmetry breaking. It is shown that the charge of the soliton is screened by counter charge everywhere.

Subject Index B71, E70, E73

1. *Introduction* Nontopological solitons, which are energy-minimum solutions under the condition of fixed conserved U(1) charge in classical field theories, appear in various theories: a coupled system of a complex scalar field and a real scalar field [1], a complex scalar field with nonlinear self-interactions [2], and so on.[1] In systems with gauge symmetry, nontopological soliton solutions have also been studied [5–7].

Gauge theory with spontaneous symmetry breaking is the most fundamental framework in modern physics. We present, in this article, nontopological soliton solutions in a system composed of a complex scalar field coupled to a U(1) gauge field, and a complex Higgs scalar field that causes spontaneous symmetry breaking. This is a generalization of Friedberg–Lee–Sirlin's model [1,6]. While a complicated potential form is assumed in a single scalar field model for the existence of nontopological solitons [5,7], we consider a model that has natural interaction terms. Thus, the present work could be useful in understanding how nontopological solitons can appear in gauge theories with large symmetries that include U(1) as a symmetry subgroup.

We show that the charges of nontopological solitons are perfectly screened [8]; namely, the charge density carried by the complex scalar field is canceled out by the counter-charge cloud carried by the other fields everywhere. In contrast to the known fact that the mass of a gauged nontopological soliton whose charge is not screened is bounded above [5–7], we show that charge-screened nontopological solitons can have any large amount of mass.

2. *Basic system* We consider the system described by the action

$$S = \int d^4x \left(-(D_\mu\psi)^*(D^\mu\psi) - (D_\mu\phi)^*(D^\mu\phi) - V(\phi) - \mu\psi^*\psi\phi^*\phi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} \right), \tag{1}$$

[1] Potentials inspired by supersymmetric theories also allowed nontopological soliton solutions [3,4].

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where ψ is a complex scalar field, ϕ is a complex Higgs scalar field with the potential

$$V(\phi) = \frac{\lambda}{4}(\phi^*\phi - \eta^2)^2, \tag{2}$$

where λ and η are positive constants, and $F_{\mu\nu} := \partial_\mu A_\nu - \partial_\nu A_\mu$ is the field strength of a U(1) gauge field A_μ. Here, D_μ in Eq. (1) is the covariant derivative defined by

$$D_\mu\psi := \partial_\mu\psi - ieA_\mu\psi, \quad D_\mu\phi := \partial_\mu\phi - ieA_\mu\phi, \tag{3}$$

where e is a coupling constant.

The action (1) is invariant under the local U(1) transformation and the global U(1) transformation:

$$\begin{aligned}
\psi(x) &\to \psi'(x) = e^{i(\chi(x)-\gamma)}\psi(x),\\
\phi(x) &\to \phi'(x) = e^{i(\chi(x)+\gamma)}\phi(x),\\
A_\mu(x) &\to A'_\mu(x) = A_\mu(x) + e^{-1}\partial_\mu\chi(x),
\end{aligned} \tag{4}$$

where $\chi(x)$ is an arbitrary function and γ is a constant. Owing to the invariance, the system has conserved currents

$$j^\nu_\psi := ie\left(\psi^*(D^\nu\psi) - \psi(D^\nu\psi)^*\right), \tag{5}$$

$$j^\nu_\phi := ie\left(\phi^*(D^\nu\phi) - \phi(D^\nu\phi)^*\right); \tag{6}$$

hence, the total charges of ψ and ϕ defined by

$$Q_\psi := \int \rho_\psi\, d^3x, \quad Q_\phi := \int \rho_\phi\, d^3x \tag{7}$$

are conserved, where $\rho_\psi := j^t_\psi$ and $\rho_\phi := j^t_\phi$.

The energy of the system is given by

$$\begin{aligned}
E = \int d^3x\Big(&|D_t\psi|^2 + (D_i\psi)^*(D^i\psi) + |D_t\phi|^2 + (D_i\phi)^*(D^i\phi)\\
&+ V(\phi) + \mu|\psi|^2|\phi|^2 + \frac{1}{2}\left(E_iE^i + B_iB^i\right)\Big),
\end{aligned} \tag{8}$$

where $E_i := F_{i0}$, $B^i := 1/2\epsilon^{ijk}F_{jk}$, and i denotes the spatial index. In the vacuum state, which minimizes the energy (8), ψ, ϕ, and A_μ should satisfy

$$\psi = 0,\ \phi^*\phi = \eta^2, \quad \text{and} \quad D_\mu\phi = 0. \tag{9}$$

Equivalently, the fields should take the form

$$\psi = 0,\ \phi = \eta e^{i\theta(x)}, \quad \text{and} \quad A_\mu = e^{-1}\partial_\mu\theta, \tag{10}$$

where θ is an arbitrary function. Since the gauge field is pure gauge, then $F_{\mu\nu} = 0$. By the vacuum expectation value of the Higgs scalar field η, the gauge field A_μ and the complex scalar field ψ acquire the masses $m_A = \sqrt{2}e\eta$ and $m_\psi = \sqrt{\mu}\eta$, respectively. The real scalar field that denotes a fluctuation of the amplitude of ϕ around η also acquires the mass $m_\phi = \sqrt{\lambda}\eta$. In the vacuum state (10), a global U(1) symmetry still exists.

By varying Eq. (1) with respect to ψ^*, ϕ^*, and A_μ, we obtain equations of motion:

$$D_\mu D^\mu \psi - \mu\phi^*\phi\psi = 0,$$
$$D_\mu D^\mu \phi - \frac{\lambda}{2}\phi(\phi^*\phi - \eta^2) - \mu\phi\psi^*\psi = 0,$$
$$\partial_\mu F^{\mu\nu} - j^\nu_\phi - j^\nu_\psi = 0. \tag{11}$$

We assume that the fields are stationary and spherically symmetric in the form

$$\psi = e^{i\omega t}u(r), \quad \phi = e^{i\omega' t}f(r), \quad A_t = A_t(r), \quad \text{and} \quad A_i = 0, \tag{12}$$

where ω and ω' are constants and $u(r)$ and $f(r)$ are real functions of r. Using the gauge transformation (4) we fix the variables as

$$\phi(r) \to f(r),$$
$$\psi(t,r) \to e^{i\Omega t}u(r) := e^{i(\omega-\omega')t}u(r),$$
$$A_t(r) \to \alpha(r) := A_t(r) - e^{-1}\omega'. \tag{13}$$

Substituting Eq. (13) into the field equations (11), we obtain

$$\frac{d^2u}{dr^2} + \frac{2}{r}\frac{du}{dr} + (e\alpha - \Omega)^2 u - \mu f^2 u = 0,$$
$$\frac{d^2f}{dr^2} + \frac{2}{r}\frac{df}{dr} - \frac{\lambda}{2}f(f^2 - \eta^2) + e^2\alpha^2 f - \mu u^2 f = 0,$$
$$\frac{d^2\alpha}{dr^2} + \frac{2}{r}\frac{d\alpha}{dr} + \rho_\psi + \rho_\phi = 0. \tag{14}$$

where the charge densities ρ_ψ, ρ_ϕ are given by

$$\rho_\psi = -2e(e\alpha - \Omega)u^2, \quad \rho_\phi = -2e^2\alpha f^2. \tag{15}$$

In Eqs. (14), the parameter Ω characterizes the solutions. We seek configurations of the fields with a nonvanishing value of Ω.

The set of Eqs. (14) is derived from the effective action in the form

$$S_{\text{eff}} = \int r^2 dr \left(\left(\frac{du}{dr}\right)^2 + \left(\frac{df}{dr}\right)^2 - \frac{1}{2}\left(\frac{d\alpha}{dr}\right)^2 - U_{\text{eff}} \right), \tag{16}$$

$$U_{\text{eff}} := -\frac{\lambda}{4}(f^2 - \eta^2)^2 - \mu f^2 u^2 + (e\alpha - \Omega)^2 u^2 + e^2 f^2 \alpha^2. \tag{17}$$

If we regard the coordinate r as resembling "time", the effective action (16) describes a mechanical system of three degrees of freedom, u, f, and α, where the "kinetic" term of α has the wrong sign.

Using the ansatz (13), we rewrite the energy (8) for the symmetric system as

$$E = 4\pi \int_0^\infty r^2 dr \left(\left(\frac{du}{dr}\right)^2 + \left(\frac{df}{dr}\right)^2 + \frac{1}{2}\left(\frac{d\alpha}{dr}\right)^2 + \frac{\lambda}{4}(f^2 - \eta)^2 + \mu f^2 u^2 + (e\alpha - \Omega)^2 u^2 + e^2 f^2 \alpha^2 \right). \tag{18}$$

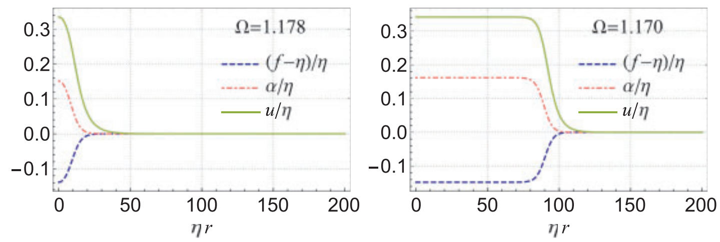

Fig. 1. Numerical solutions $u(r), f(r)$, and $\alpha(r)$ are shown for $\Omega = 1.178$ (left panel) and for $\Omega = 1.170$ (right panel).

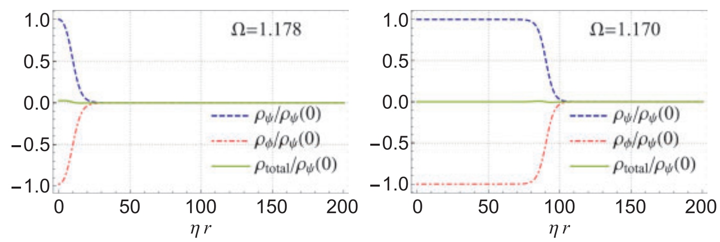

Fig. 2. The charge densities, ρ_ψ, ρ_ϕ, and $\rho_{\text{total}} := \rho_\psi + \rho_\phi$ normalized by the central value of ρ_ψ are shown as functions of r for $\Omega = 1.178$ (left panel) and for $\Omega = 1.170$ (right panel).

At the origin, we impose the regularity conditions for the spherically symmetric fields as

$$\frac{du}{dr} \to 0,\ \frac{df}{dr} \to 0,\ \frac{d\alpha}{dr} \to 0 \quad \text{as} \quad r \to 0. \tag{19}$$

On the other hand, we require that the fields should be in the vacuum state (9) at spatial infinity. Therefore, we impose the conditions

$$u \to 0,\ f \to \eta,\ \alpha \to 0 \quad \text{as} \quad r \to \infty. \tag{20}$$

3. *Numerical solutions* To obtain numerical solutions to the coupled ordinary differential equations (14), we use the relaxation method. In numerics, we set $\eta = 1$, and the dimensional quantities are scaled as $r \to \eta r, f \to \eta^{-1} f, u \to \eta^{-1} u, \alpha \to \eta^{-1}\alpha$, and $\Omega \to \eta^{-1}\Omega$, respectively. We set $\lambda = 1, e = 1$, and $\mu = 1.4$.

In Fig. 1, we show numerical solutions $u(r), f(r)$, and $\alpha(r)$ for $\Omega = 1.178$ and $\Omega = 1.170$ as examples. In both cases of Ω, the functions are nonvanishing in a finite region, and at large distances, Eq. (20) is achieved. Therefore, these solutions represent solitons. In the case of $\Omega = 1.178$, the functions are Gaussian-function-like, while in the case of $\Omega = 1.170$, the functions are step-function-like. The soliton in the latter case represents a homogeneous ball, all of the functions take constant values, $u(r) = u_0, f(r) = f_0$, and $\alpha(r) = \alpha_0$, within a radius of the ball, and at the surface, $r = r_s$, the functions decay quickly. This type of solution is discussed in the thin-wall approximation in the literature [1,2,5–7].

We show the charge densities ρ_ψ, ρ_ϕ in Fig. 2 as functions of r. In both cases of Ω, we find that ρ_ψ is canceled out by ρ_ϕ, and the total charge density $\rho_{\text{total}} := \rho_\psi + \rho_\phi$ almost vanishes everywhere, i.e., the charge of the field ψ is perfectly screened [8].

The total charge of the scalar field ψ, $Q_\psi (= -Q_\phi)$, depends on Ω as shown in Fig. 3. The solution exists for Ω in the range $\Omega_{\min} < \Omega < \Omega_{\max}$, where $\Omega_{\max}$ and $\Omega_{\min}$ are given later. At $\Omega = \Omega_{\min/\max}$, Q_ψ diverges, respectively.

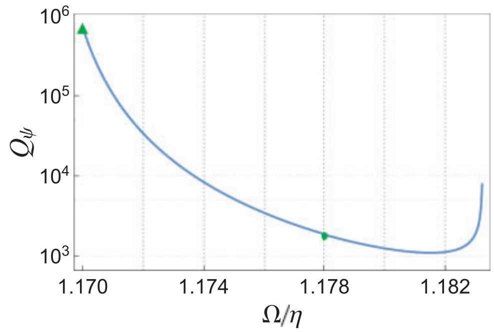

Fig. 3. The total charge of ψ, Q_ψ, is plotted as a function of Ω. Q_ψ diverges at $\Omega = \Omega_{\min}$ and $\Omega = \Omega_{\max}$. The circle in the figure, $\Omega = 1.178$, corresponds to the left panel in Fig. 1, while the triangle, $\Omega = 1.170$, corresponds to the right panel.

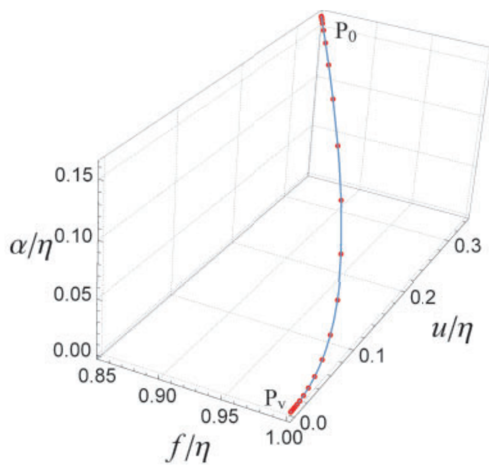

Fig. 4. Trajectory of the numerical solution for $\Omega = 1.170$ in the (u, f, α) space. It starts from P_0 and ends at P_v. Dots on the trajectory denote laps of r.

First, we determine $\Omega_{\max}$. Since $u, f - \eta$, and α are small at large distances, solving the linearized equations of Eq. (14), we have

$$u(r) \propto \frac{1}{r} \exp\left(-\sqrt{m_\psi^2 - \Omega^2}\; r\right). \tag{21}$$

If we require the solutions to be localized in a finite region, the parameter Ω should satisfy

$$\Omega^2 < \Omega_{\max}^2 = m_\psi^2. \tag{22}$$

Next, we determine $\Omega_{\min}$. If Ω takes a value near $\Omega_{\min}$, we have a homogeneous ball solution as shown in the right panel of Fig. 1 as an example. As Ω approaches $\Omega_{\min}$, Q_ψ increases, and the radius of the ball increases greatly. The homogeneous ball solution with a large radius is described by a bounce solution of Eq. (14): a point in the 3D space (u, f, α) sits on a stationary point of the potential $U_{\rm eff}$, say P_0, for a long time; it then moves to another stationary point, P_v, i.e., the true vacuum, for a short period, and finally stays there. A trajectory of the solution in (u, f, α) space for $\Omega = 1.170$ is shown in Fig. 4. The stationary point P_v exists at $(u, f, \alpha) = (0, \eta, 0)$, and P_0 at

$(u, f, \alpha) = (u_0, f_0, \alpha_0)$, where

$$\alpha_0 = \frac{1}{e(4\mu - \lambda)}\left((\mu - \lambda)\Omega + \sqrt{\mu(2\lambda + \mu)\Omega^2 - \mu\lambda(4\mu - \lambda)\eta^2}\right),$$
$$f_0 = \frac{1}{\sqrt{\mu}}(\Omega - e\alpha_0), \quad u_0 = \frac{1}{\sqrt{\mu}}\sqrt{e\alpha_0(\Omega - e\alpha_0)}. \tag{23}$$

Here, $0 < e\alpha_0 < \Omega$ should hold for real values of u_0. This condition with Eq. (22) requires $\lambda < \mu$. For the homogeneous ball solution with a large radius, the damping terms, first derivative terms that are in proportion to $1/r$, in Eq. (14) are negligible around $r = r_s$. In this case,

$$E_{\text{eff}} := \left(\frac{df}{dr}\right)^2 + \left(\frac{du}{dr}\right)^2 - \frac{1}{2}\left(\frac{d\alpha}{dr}\right)^2 + U_{\text{eff}}(u, f, \alpha) \tag{24}$$

is conserved during the "evolution" in r, and then the bounce solution appears if the potential heights at the two stationary points are same, i.e.,

$$U_{\text{eff}}(\text{P}_\text{v}) = U_{\text{eff}}(\text{P}_0). \tag{25}$$

From the numerical calculations, we see that this occurs for Ω_{min}. Solving Eq. (25), we have

$$\Omega_{\text{min}} = \sqrt{m_\phi(2m_\psi - m_\phi)}. \tag{26}$$

Therefore, the allowed range of Ω, $\Omega_{\text{min}} < \Omega < \Omega_{\text{max}}$, is rewritten as

$$2m_\psi m_\phi - m_\phi^2 < \Omega^2 < m_\psi^2, \tag{27}$$

or, equivalently,

$$2\sqrt{\lambda\mu} - \lambda < (\Omega/\eta)^2 < \mu. \tag{28}$$

Then, the nontopological soliton solution exists for the model parameters satisfying $\lambda < \mu$.

The nontopological soliton obtained in this paper can be regarded as a condensate of particles of the scalar field ψ, where the Higgs field plays the role of glue against repulsive force by the U(1) gauge field. We compare the energy of the soliton, E_{NTS}, given by Eq. (18) with the mass energy of the free particles of ψ. The number of particles is defined by

$$N_\psi := |Q_\psi|/e, \tag{29}$$

so that the free particles, as a whole, have the same amount of charge as the soliton. Then, the mass energy of the free particles of ψ is given by $E_{\text{free}} = m_\psi N_\psi$.

In Fig. 5, we plot the ratio $E_{\text{NTS}}/E_{\text{free}}$ as a function of Ω. We see that there exists Ω_{cr} such that $E_{\text{NTS}}/E_{\text{free}} < 1$ in the range

$$\Omega_{\text{min}} < \Omega < \Omega_{\text{cr}}. \tag{30}$$

The nontopological soliton in the range (30) is preferable energetically: then the soliton does not decay into free particles. In Fig. 6, $E_{\text{NTS}}/E_{\text{free}}$ is plotted as a function of N_ψ. We see that there exists a lower limit on the numbers of condensed particles for stable solitons, N_{cr}, but no upper limit. A stable nontopological soliton with any large amount of mass possibly exists. This is a significant difference from the case of a gauged nontopological soliton whose charge is not screened.[2]

[2] Large gauged Q-balls are proposed by using models with complicated self-interactions [9,10].

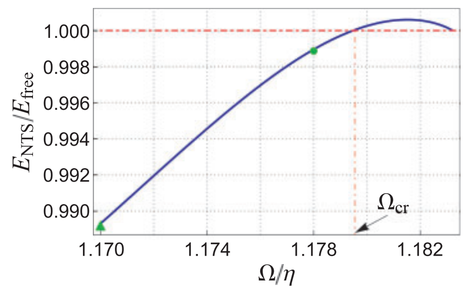

Fig. 5. The ratio $E_{\rm NTS}/E_{\rm free}$ as a function of Ω. For $\Omega_{\rm min} < \Omega < \Omega_{\rm cr}$, $E_{\rm NTS}/E_{\rm free} < 1$. The circle in the figure denotes $\Omega = 1.178$, and the triangle $\Omega = 1.170$.

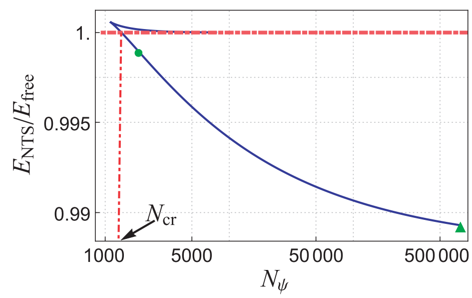

Fig. 6. The ratio $E_{\rm NTS}/E_{\rm free}$ as a function of N_ψ. The branch in the region $E_{\rm NTS}/E_{\rm free} < 1$ corresponds to $\Omega_{\rm min} < \Omega < \Omega_{\rm cr}$. For $N_{\rm cr} < N_\psi$ on the branch, $E_{\rm NTS}/E_{\rm free} < 1$. The circle in the figure denotes $\Omega = 1.178$, and the triangle $\Omega = 1.170$.

4. *Summary and discussions* In this article, we have shown that nontopological solitons exist stably in a system consisting of a complex scalar field coupled to a U(1) gauge field and a complex Higgs scalar field that causes spontaneous symmetry breaking. The characteristic property of the solitons in this system is the perfect charge screening [8], namely, cancellation of the charge density of the complex scalar field by the counter-charge cloud of the other fields everywhere. This is a desirable property for nontopological solitons as dark matter [11–13]. It is interesting to consider how many charge-screened nontopological solitons were produced during the evolution of the early stage of the universe [14–17].

Owing to the perfect charge screening, infinitely heavy solitons are allowed in this system. Then, two solitons would merge by collision and form a larger soliton [11], and solitons on an astrophysical scale would finally appear. It is important to investigate the gravitational effects of large solitons: soliton stars, seeds of supermassive black holes, and so on [18–21].

The system considered here should be embedded in more realistic field theories. Generalization of the model is an interesting issue. Furthermore, the stability of the solutions should be clarified from various points of view [22–26].

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