Role of Faddeevian anomaly in the s-wave scattering of chiral fermion off dilaton black holes towards preservation of information

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Abstract

It was found that s-wave scattering of chiral fermion off dilaton black-hole when studied with a model generated from the chiral Schwinger model with standard Jackiw-Rajaraman type of anomaly provided information non preserving result. However this scattering problem when studied with a model generated from chiral Schwinger model with generalized Faddeevian type of anomaly rendered information preserving result and it had well agreement with Hawking's revised proposal related to information loss. A minute and equitable investigation in detail has been carried out here to show how Faddeevian type of anomaly scores over the Jackiw-Rajaraman type of anomaly in connection with the s-wave scattering of chiral fermion of dilaton black-hole related to preservation of information.

1. Introduction

Free chiral boson can be thought of in terms of chiral fermion in $(1 + 1)$ dimension which is a basic ingredient of heterotic string theory. So the theories involving chiral boson is interesting in its own right. There are quite a good mumbler of interesting field theoretical models involving free as well as interacting chiral boson [1–5]. So scattering of chiral fermion off dilaton black-

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hole would certainly be of interest and the model presented in the article [6] is so beautifully formulated that s-wave scattering of chiral fermion too can be studied in a significant manner. In the article [7], the authors first made an attempt to study the s-wave scattering of chiral fermion off dilaton black-hole, but they obtained a result which went against preservation of information. The interacting (gauged) chiral boson (fermion) part in that case was taken from the interacting model invented by Harada in [5]. This model was generated from the usual chiral Schwinger model with the standard one para meter class of anomaly introduced by Jackiw-Rajaraman [8] under the imposition of a chiral constraint by hand in the phase space of the theory. Recently, it is shown in the article [9], that it can be obtained directly from the gauged action of Siegel type chiral boson [1,3] too.

Although in the article [7], the s-wave scattering of chiral fermion off dilaton black-hole rendered a result with nonpreservation of information, in our article [10], we have shown that s-wave scattering of chiral fermion with generalized Faddeevian anomaly [11] leads to an information preserving result. This information preserving result related to information loss scenario of course is a gift of the Faddeevian anomaly and it indeed agrees with Hawking's revised suggestion [12]. In this article, an attempt has, therefore, been made to carry out a minute and detailed investigation how Faddeevian anomaly changes the scenario related to information loss in the scattering of chiral fermion off dilaton black-hole.

In this context, and for the sake of the readers benefit, we should mention a bit history related to information loss paradox. It is universally accepted that the issue that received special attention related to black-hole physics is the possible information loss when a black-hole evaporates through the form of thermal Hawking radiation. However from that very time of the discovery of this thermal radiation by Hawking [13], a controversy was initiated concerning his proposal related to information loss [13], since it carried an indication of a new level of unpredictability within the quantum mechanics by the gravity. After four decades, Hawking himself moved away from his own initial proposal and suggested that quantum gravity interaction did not lead to any loss of information and quantum coherence did maintain [12]. His revised suggestion on this issue although gave back a moderately pleasant stratus related to information loss puzzle, it would be fair to admit that the information loss related enigma would not yet been well settled down from all corners. Some investigations are still standing with information non-preserving result without having any plausible suggestion towards solvation of the puzzle.

The paper is organized as follows. Sec. 2 contains a brief discussion of the model used for studying the s-wave scattering of fermion off dilaton black-hole. Sec 3 is devoted to describe the s-wave scattering of chiral fermion with the Faddeevian anomaly with the specialized form developed by Mitra in the article [14] with the plausible explanation: why does it fail to preserve information? In Sec. 4, it is shown how the important and useful technique, e.g. imposition of chiral constraint developed by Harada [5] can miraculously solve the information loss problem. An over all discussion and conclusion is contained in Sec 5.

2. A brief review of the model used for studying s-wave scattering of fermion off dilaton black-hole

The backscattering of s-wave fermion with magnetically charged-Q dilaton black-hole is described by the action [6,15–18]

$$S_{AF} = \int d^4x \sqrt{g}[e^{-2\Phi}[R + 4(\nabla\Phi)^2 - \frac{1}{4}F^2] + i\bar{\Psi}\slashed{D}\Psi]. \tag{1}$$

The fields Ψ and Φ represent the charged fermion and the dilaton field respectively. The factor $\sqrt{g}$ is associated with the metric tensor $g_{\mu\nu}$, and $F_{\mu\nu}$ is the electromagnetic field strength. The geometry of this $(3+1)$ dimensional black-hole is constituted with three important regions: a symmetrical throat region far from the black-hole. As long as it proceeds near the black-hole the curvature begins to rise and finally enters into the mouth region. The metric is approximated to the flat $(1+1)$ dimensional Minkowsky space *times* the round metric on the two sphere with radius $|Q|$ far along the throat region. At large scale relative to the radius $|Q|$ the following action therefore results in

$$S_{AF} = \int d^2x \sqrt{g}[e^{-2\Phi}[R + 4(\nabla\Phi)^2 + \frac{1}{Q^2} - \frac{1}{4}F^2] + i\bar{\Psi}D\!\!\!\!/\Psi]. \tag{2}$$

This action was obtained treating the throat region of $(3+1)$ dimensional black-hole as a compactified form of $(3+1)$ dimension to $(1+1)$ dimension. The black-hole solution of which was provided for in the article [19]. It is known that in the extremal limit the geometry is non singular having no horizon, but when a low energy particle is thrown into that; it acquires a singularity along with an event horizon [6,15]. We should mention at this stage that following the review article [15] the diatonic equation of motion computed from the action (2) reads

$$e^{-2\Phi}[R + 4\lambda^2 + 4\nabla^2\Phi - 4(\nabla\Phi)^2 - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}] = 0, \tag{3}$$

where $\lambda^2 = \frac{1}{4Q^2}$. However the linear diatonic vacuum which is characterized by

$$R = \nabla^2\Phi = 0, \quad (\nabla\Phi)^2 = \lambda^2. \tag{4}$$

So we can introduce a coordinate system $(\tilde{x}, \tau)$ such that $g_{\mu\nu} = \eta_{\mu\nu}$ and $\Phi = -\lambda\tilde{x}$ in the vacuum. So the square of the line element gets simplified to $ds^2 = \eta_{\mu\nu}d\tilde{x}^\mu d\tilde{x}^\nu$. Certainly, τ stands for $\tilde{x}^0$. The natural dilatonic coupling for this theory is $e^{\Phi(\tilde{x})}$ indeed. Therefore, for sufficiently low energy and incoming fermion and negligible gravitational effect equation (2) reduces to

$$\mathcal{S}_f = \int d^2\tilde{x}[i\bar{\Psi}[\partial\!\!\!/ + ieA\!\!\!/]\Psi - \frac{1}{4}e^{-2\Phi(\tilde{x})}F_{\mu\nu}F^{\mu\nu}]. \tag{5}$$

Here $\gamma^\mu\partial_\mu = \partial\!\!\!/$ and $\gamma^\mu A_\mu = A\!\!\!/$ and e indicates the coupling constant having the dimension of mass. The Lorentz indices μ and ν takes the values 0 and 1 in $(1+1)$ dimensional Minkowsky space time. The non dynamical dilatonic background represented by the field Φ which is associated here with the electro magnetic background $F_{\mu\nu}$ has an incredible role in this model towards making the coupling constant position dependent. For later convenience let us now define $\eta^2(x) = e^{2\Phi(x)}$. The construction of the model is so powerful that it itself provides a room to take the effect of anomaly into account which enters from the one loop correction during the course of bosonization [7,10,20,21]. This very anomaly plays a crucial role to dictate whether there would be disastrous information loss or it would be free from that disaster. Our present work does explore that the crucial role of anomaly lies in the central position in obtaining the information preserving result when Faddeevian anomaly is considered during the course of investigation of s-wave scattering of chiral fermion with the intriguing supportive feature of imposition of chiral constraint developed by Harada in his seminal work [5].

   In this article we are intended to present a minute and equitable analysis of the scattering of chiral fermion in detail in presence of Faddeevian anomaly proposed by Mitra [14]. So in the present situation chiral interaction will take the place of vector interaction in the same manner as it was done in [7,21]. Thus the action in this situation in terms of fermion reads

$$\mathcal{S}_{CF} = \int d^2\tilde{x}[i\bar{\Psi}[\slashed{\partial} + ie\slashed{A}(1 + \gamma_5)]\Psi - \frac{1}{4}e^{-2\Phi(x)}F_{\mu\nu}F^{\mu\nu}]. \tag{6}$$

In fact, it is the chiral quantum electrodynamics with a dilatonic background along with the standard electromagnetic background. Our investigation begins with the bosonized version of the above action (6) to which we now turn. For the shake of notational convenience we will call $\tilde{x}$ as x in the rest part of the article.

3. Study of s-wave scattering of fermion off dilaton black-holes with Faddeevian anomaly

The generating functional of the above theory defined by the action (6) can be written down as

$$Z[A] = \int d\phi e^{i\mathcal{S}_{CB}}, \tag{7}$$

where

$$\mathcal{S}_{CB} = \int d^2x \mathcal{L}_{CH}, \tag{8}$$

with

$$L = \int dx \mathcal{L}_{CH} = \int dx[\frac{1}{2}\partial_\mu\phi\partial^\mu\phi + e(\eta^{\mu\nu} - \epsilon^{\mu\nu})\partial_\mu\phi A_\nu + \frac{1}{2}e^2[A_0^2 - 3A_1^2 - 2A_1A_0]$$
$$- \frac{1}{4}e^{-2\Phi(x)}F_{\mu\nu}F^{\mu\nu}]. \tag{9}$$

Equation (9) describes the exact bosonized version of the fermionic model descried in (6) details of which is presented in the articles [14,22,23]. We will carry out investigation with the bosonized version of this model (9) because from the article [7] we have learned that the study of chiral fermion in presence of the obstacle posed by the gravitational anomaly would be possible with the bosonized version. However, this process of bosonization is crucially connected with the regularization. During the course of integrating out the fermions one gets forced to incorporate the important one loop correction term in order to get rid of the divergence appeared in the fermionic determinant. This term may include different contribution depending on the choice of regularization. The issue that has to be taken care of with prime importance during regularization is indeed the maintenance of physical Lorentz symmetry. In this regard, we should mention that apparent Lorentz non-covariance not always stands as a hindrance in order to maintain physical Lorentz invariance [14,22,23]. Here we choose the Faddeevian type [11] of anomaly developed and studied in the articles [14,22,23]. With this specialized form of Faddeevian anomaly the bosonized action reads

$$\mathcal{S}_{CB} = \int d^2x[\frac{1}{2}\partial_\mu\phi\partial^\mu\phi + e(\eta^{\mu\nu} - \epsilon^{\mu\nu})\partial_\mu\phi A_\nu + \frac{1}{2}e^2[A_0^2 - 3A_1^2 - 2A_1A_0]$$
$$- \frac{1}{4}e^{-2\Phi(x)}F_{\mu\nu}F^{\mu\nu}]. \tag{10}$$

Note that there is no Lorentz co-variance at the action level however in due course we will find that it will not in any way hamper the physical Lorentz invariance of the theory even in the presence of dilatonic background. Here $\epsilon^{01} = -\epsilon_{01} = 1$ and the Minkowski metric $g^{\mu\nu} = diag(1, -1)$. The indices μ and ν takes the values 0 and 1 as have already been mentioned in Sec. 2. The advantage of using the bosonized version is that the anomaly automatically

gets incorporated within it. So the tree level bosonized theory acquires the effect of one loop correction. Equation (9), apart from the dilatonic factor was initially found in [14] where the author termed it as chiral Schwinger model with Faddeevian anomaly.

It is now necessary to carry out the Hamiltonian analysis of the theory in order to observe the role of dilaton field on the theoretical spectra. What happened to the Lorentz symmetry in the present situation will also be explored from our investigation apart from our main perspective of studying information loss scenario. We need to find out the theoretical spectrum of this model which necessities to proceed through the constraint analysis of this theory. From the standard definition the canonical momenta corresponding to the chiral boson field ϕ, the gauge field components A_0 and A_1 are computed as follows.

$$\frac{\partial \mathcal{L}_{CH}}{\partial \dot{\phi}} = \pi_\phi = \dot{\phi} + e(A_0 - A_1), \tag{11}$$

$$\frac{\partial \mathcal{L}_{CH}}{\partial \dot{A}_1} = \pi_1 = e^{-2\Phi(x)}(\dot{A}_1 - A_0') = \frac{1}{\eta^2}(\dot{A}_1 - A_0), \tag{12}$$

$$\frac{\partial \mathcal{L}_{CH}}{\partial \dot{A}_0} = \pi_0 \approx 0. \tag{13}$$

Here π_ϕ represent the momentum corresponding to the chiral boson field ϕ, and π_1 and π_0 stand for the momenta corresponding to the gauge field components A_0 and A_1. Using equations (11), (12) and (13) the canonical Hamiltonian of the theory is obtained.

$$H_C = \int dx[\frac{1}{2}e^{2\Phi}\pi_1^2 + \frac{1}{2}(\pi_\phi^2 + \phi'^2) + \pi_1 A_0' - e(A_0 - A_1)(\pi_\phi + \phi') + 2e^2 A_1^2]. \tag{14}$$

Note that $\pi_0 \approx 0$ does not contain any time derivative of the any field. So it is the primary constraint of the theory. The preservation of this constraint leads to secondary constraint:

$$G = \pi_1' + e(\pi_\phi + \phi') \approx 0. \tag{15}$$

This is generally termed as the Gauss law of the theory. Therefore, the effective Hamiltonian is given by

$$H_{Ef} = H_C + u\pi_0 + v(\pi_1' + e(\pi_\phi + \phi')). \tag{16}$$

Out of these two lagrange multipliers (velocities) u and v the velocity v is determinable and that comes out to be

$$v = \frac{1}{2}(\pi_\phi + \phi') - e(A_0 + A_1), \tag{17}$$

but the velocity u remains undetermined at this stage. It just plays the role of a Lagrange multiplier. The preservation of the constant G after imposition of the velocity v in the effective Hamiltonian gives a new constraint.

$$T = (A_0 + A_1) \approx 0. \tag{18}$$

There is no other constraint in the phase space of the theory. So the full set of constraints at a glance is

$$\Omega_1 = \pi_0 \approx 0, \tag{19}$$

$$\Omega_2 = A_0 + A_1 \approx 0, \tag{20}$$

$$\Omega_3 = \pi_1' + e(\pi_\phi + \phi') \approx 0. \tag{21}$$

According to the Dirac terminology these constraints are all weak conditions at this stage. It is found that the Poisson bracket between $G(x)$ and $G(y)$ gives a non vanishing contribution

$$[G(x), G(y)] = 2\delta(x - y)', \tag{22}$$

which is in sharp contrast to the Poisson bracket in the case of usual chiral Schwinger model [8] where it was found to render vanishing contribution. Faddeev initially noticed that anomaly might make Poisson bracket between $G(x)$ and $G(y)$ nonzero [11] and the constraint became second class itself at that stage and gauge invariance got lost. He, however, showed that it would be possible to quantize the theory but in that situation system might posses extra degrees of freedom. A careful look revels that the presence of this *extra degrees of freedom* causes a problem in the preservation of information in the scattering problem and that problem gets eradicated by the imposition of the chiral constraint which we are going to explored in our present investigation.

This is a system endowed with three constraints in the phase space. So ordinary Poisson bracket becomes inadequate and for the determination of the phase space structure of this system one needs the Dirac bracket [24] which is defined by

$$[A(x), B(y)]^* = [A(x), B(y)] - \int [A(x), \Omega_i(\eta)] C_{ij}^{-1}(\eta, z)[\Omega_j(z), B(y)] d\eta dz, \tag{23}$$

where $C_{ij}^{-1}(x, y)$ can be computed using the following equation

$$\int C_{ij}^{-1}(x, z)[\Omega_j(z), \Omega_k(y)] dz = \delta(x - y)\delta_{ik}. \tag{24}$$

The system under consideration described by the Lagrangian (9), has the following $C_{ij}(x, y)$:

$$C_{ij}(x, y) = \begin{pmatrix} 0 & -\delta(x - y) & 0 \\ \delta(x - y) & 0 & -\delta'(x - y) \\ 0 & -\delta'(x - y) & 2e^2\delta'(x - y) \end{pmatrix} \tag{25}$$

The non singular nature of the matrix C_{ij} indicates that it certainly has an inverse which is

$$C_{ij}^{-1}(x, y) = \begin{pmatrix} -\frac{1}{2e^2}\delta'(x - y) & \delta(x - y) & \frac{1}{2e^2}\delta(x - y) \\ -\delta(x - y) & 0 & 0 \\ \frac{1}{2e^2}\delta(x - y) & 0 & \frac{1}{4e^2}\epsilon(x - y) \end{pmatrix}. \tag{26}$$

The reduced Hamiltonian is obtained after the imposition of the constraints (19), (20) and (21) in equation (14) treating these constraints as strong conditions.

$$\mathcal{H}_R = \frac{1}{2}(e^{\Phi(x)}\pi_1^2 + \phi'^2) + \frac{1}{2e^2}\pi_1'^2 + \pi_\phi\phi' - \pi_1 A_1' + 2e^2 A_1^2 \tag{27}$$

Although the field Φ within the Hamiltonian has space dependence, it has no explicit time dependence. Hence the Hamiltonian (27) preserves in time. From the definition (23), the Dirac brackets of the fields constituting the reduced Hamiltonian (27) are computed:

$$[A_1(x), A_1(y)]^* = -\frac{1}{2e^2}\delta'(x - y), \tag{28}$$

$$[A_1(x), \pi_1(y)]^* = \delta(x - y), \tag{29}$$

$$[A_1(x), \phi(y)]^* = -\frac{1}{2e}\delta(x - y), \tag{30}$$

$$[\pi_1(x), \pi_1(y)]^* = 0, \tag{31}$$

$$[\phi(x), \phi(y)]^* = \frac{1}{4}\epsilon(x - y). \tag{32}$$

Here the superscript $(*)$ indicates the Dirac Bracket. Using the above Dirac brackets the following first order differential equations of motion are obtained from the Hamiltonian (27).

$$\dot{\pi}_1 = \pi_1' - 4e^2 A_1', \tag{33}$$

$$\dot{A}_1 = e^{2\Phi(x)}\pi_1 - A_1', \tag{34}$$

$$\dot{\phi} = -\phi' - \frac{1}{e}\pi_1' + 2eA_1. \tag{35}$$

After a little algebra the above three equations give the following theoretical spectra (two second order and one first order differential equation):

$$(\Box + 4e^2 e^{2\Phi(x)})A_1 = 0, \tag{36}$$

$$(\Box + 4e^2 e^{2\Phi(x)})\pi_1 = 0, \tag{37}$$

$$\partial_+[\phi + \frac{1}{2e}(\dot{A}_1 + A_1')] = 0. \tag{38}$$

Here $\partial_+ = \partial_0 + \partial_1$. Note that the equations of motion are all Lorentz invariant. So Physical Lorentz invariance is maintained. The two second order equations (36) and (37) are describing a free massive boson and its momentum respectively and equation (38) is representing a self dual (chiral) boson. The theoretical spectra therefore contain a massive boson with mass $2ee^{\Phi(x)}$ and an massless chiral boson [3,4]. This very chiral boson is the very *extra degrees of freedom* as predicted by Faddeev in [11]. In $(1 + 1)$ dimension, it can be thought of as chiral fermion. What follows next is the description of the disaster that was been brought in by this very chiral fermion towards preservation of information.

Note that because of the presence of the position dependent factor η^2 mass of this massive boson varies with the position which is although astonishing, this position dependence of mass cries lots of interesting surprises. The mass in this situation increases limitlessly in the negative x^1 direction because being inspired by the by the linear dilatonic vacuum of $(1 + 1)$ dimensional gravity $\Phi(x) = -x^1$ is chosen in the similar fashion as it was found in the in the earlier articles [6,7,10,16,20,21,25–28]. So in the exterior space, where the space like coordinate $x^1 \to \infty$, the coupling $\eta^2(x)$ vanishes exponentially [6]. However, when $x^1 \to -\infty$ the coupling constant will diverge that can be considered as an infinite throat in the interior of certain magnetically charged black-hole. Therefore, the massive particle will not be able to travel an arbitrary distance so it will be return back and any finite energy contribution must therefore be totally come back with unit probability. So an observer at $x^1 \to \infty$ will be able to retrieve all the information of the massive field. More rigorously, we can say that because of the position dependent factor η within the mass term it will start journey with the vanishing value near the mouth but as it goes into the throat region mass will increases limitlessly. Since in $(1 + 1)$ a massless scalar can be thought of as a massless fermion so a fermion which will proceed into the black-hole will not be able to advance an arbitrary long distance and will certainly be reflected back with a unit probability. Therefore, the massive boson will not create any problem to maintain preservation of information. However the mass less chiral fermion field will be able to travel smoothly within the black-hole without facing any obstruction. So the observer at $x^1 \to \infty$ will not find this massless chiral field to return back. Hence the total information corresponding to the mass lees chiral fermion will be lost. Even though it is a puzzling outcome with this setting and it goes against the Hawking's revised

proposal related to information preserving scenario [12] it is unavoidable at this stage. Although the anomaly considered here is Faddeevian type, the result has gone against the preservation of information. So a natural question may be initiated: how does the information preserving result for s-wave scattering of chiral fermion off dilaton black-hole was obtained in the article [10]? In the following Sec. 4, we will proceed to take an attempt to uncover where the crux of the fact is lying hidden.

4. A remedy to the disaster by the imposition of chiral constraint

From the discussion of the earlier Sec. 3, it has been confirmed that the presence of the very chiral fermion (38) is in the source of all disaster towards non preservation of information. So this Sec. of the article will be devoted to eliminate this chiral fermion from the spectra with a suitable standard physical technique keeping our intense focus on the fact to recover the preservation of information.

To search for a remedy let us see how does the remarkable technique of imposition of a chiral constraint presented by Harada in the article [5] can be brought into the service? It is known from quite a long ago that Harada in his seminal work [5] introduced this powerful technique to develop the gauged version of Flourinini-Jackiw type of chiral boson [4] from the usual Chiral Scwinger model with Jackiw-Rajaraman type of anomaly. We are now going to exploit the functionally active power of that fascinating technique to protect this model from this disastrous information loss problem.

At this stage, we will therefore proceed to impose a chiral constrain in the model described in equation (9). During the course of imposition of chiral constraint, we will not take into account the kinetic part of electro magnetic background accompanied with the dilaton field, because it remains unaffected during the course of imposition of chiral constraint. We indeed take that part into account in due course when we will proceed to determine the theoretical spectrum of this interacting model. Apart from the kinetic part of electromagnetic field entangled with non dynamical dilatonic background the model reads

$$\mathcal{L}_{CHB} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi + e(\eta^{\mu\nu} - \epsilon^{\mu\nu})\partial_\mu\phi A_\nu + \frac{1}{2}e^2[A_0^2 - 3A_1^2 - 2A_1A_0]. \tag{39}$$

In order to impose the chiral constraint we need to calculate the momentum corresponding to the field ϕ. From the standard definition the momentum corresponding to the field ϕ comes out as

$$\frac{\partial\mathcal{L}_{CHB}}{\partial\dot{\phi}} = \pi_\phi = \dot{\phi} + e(A_0 - A_1). \tag{40}$$

The Hamiltonian of this system as usual is obtained through the Legendre transformation:

$$H_B = \int d^2x[\pi_\phi\dot{\phi} - \mathcal{L}_{CHB}]. \tag{41}$$

Explicitly, the Hamiltonian density of the system as obtained from the Legendre transformation (41), is

$$\mathcal{H}_B = \frac{1}{2}[\pi_\phi - e(A_0 - A_1)]^2 + \frac{1}{2}\phi'^2 - 2e\phi'(A_0 - A_1) - \frac{1}{2}e^2(A_0^2 - 2A_0A_1 - 3A_1^2). \tag{42}$$

We are now in a position to impose the chiral constraint in the phase space of the system described by the Hamiltonian (42). The chiral constraint which has been brought here into the service is

$$\Omega(x) = \pi_\phi(x) - \phi'(x) \approx 0, \tag{43}$$

which has the following Poisson bracket with itself

$$[\Omega(x), \Omega(y)] = -2\delta'(x - y). \tag{44}$$

Therefore, this chiral constraint itself is second class in nature. After imposing the constraint $\Omega(x) \approx 0$, into the phase space the generating functional of the corresponding theory reads

$$Z_{CH} = \int d\phi d\pi_\phi \delta(\pi_\phi - \phi') \sqrt{det[\Omega, \Omega]} e^{i \int d^2 x (\pi_\phi \dot\phi - \mathcal{H}_B)},$$

$$= \int d\phi e^{i \int d^2 x \mathcal{L}_{CH}}, \tag{45}$$

with

$$\mathcal{L}_{CH} = \dot\phi \phi' - \phi'^2 + 2e(A_0 - A_1)\phi' - 2e^2 A_1^2. \tag{46}$$

This is the gauged Lagrangian density for Floreanini-Jackiw type chiral boson apart from the electromagnetic background and it is obtained from the bosonized version of chiral Schwinger with Faddeevian type anomaly [14] after implementing the chiral constraint in its phase space in the same fashion as it was found to be implemented by Harada in [5] in order to formulate a model of the gauged chiral boson from the usual chiral Schwinger model with one parameter class of anomaly introduced by Jackiw and Rajaraman [8]. The constraint analysis and the determination of the phase space structure corresponding to this model will almost alike to the work presented in [22]. The only difference that will appear is due to the presence of the non dynamical dilatonic factor entangled with the kinetic term of the electromagnetic background. Let us now take into account the electromagnetic background which is entangled with dilaton field crucially with the starting Lagrangian. The resulting model now takes the following form

$$\mathcal{L}_{CHC} = \dot\phi \phi' - \phi'^2 + 2e(A_0 - A_1)\phi' - 2e^2 A_1^2 + \frac{1}{2} e^{-2\Phi(x)} F_{01}^2. \tag{47}$$

Here ϕ represents a chiral boson field. The tree level bosonized theory has acquired the effect of anomaly in this situation too [22,23].

We will now proceed with the Hamiltonian analysis of the theory to observe the role of dilaton field on the theoretical spectrum in the present situation, i.e. after the imposition of the chiral constraint $\Omega(x) \approx 0$. To determine the theoretical spectrum of this interacting model in the present situation we have to use the constrained dynamics developed by Dirac [24]. So we need to calculate the momentum corresponding to the fields with which the theory is constituted. From the standard definition the canonical momenta corresponding to the chiral boson field ϕ, the gauge field A_0 and A_1 are obtained:

$$\frac{\partial \mathcal{L}_{CHC}}{\partial \dot\phi} = \pi_\phi = \phi', \tag{48}$$

$$\frac{\partial \mathcal{L}_{CHC}}{\partial \dot A_0} = \pi_0 \approx 0, \tag{49}$$

$$\frac{\partial \mathcal{L}_{CHC}}{\partial \dot A_1} = \pi_1 = e^{-2\phi(x)}(\dot A_1 - A_0') = \frac{1}{\eta^2}(\dot A_1 - A_0). \tag{50}$$

Here π_ϕ, π_0 and π_1 are the momenta corresponding to the field ϕ, A_0 and A_1. Note that there are two primary constraint in the phase space of the theory:

$$\tilde{\Omega}_1 = \pi_0 \approx 0, \tag{51}$$

$$\tilde{\Omega}_2 = \pi_1 + e\phi \approx 0. \tag{52}$$

Using the equations (48), (49) and (50) it is straightforward to obtain the canonical Hamiltonian through a Legendre transformation:

$$\mathcal{H}_C = \pi_\phi \dot{\phi} + \pi_1 \dot{A}_1 - \mathcal{L}. \tag{53}$$

The above transformation yields

$$H_C = \int dx [\frac{1}{2} e^{2\Phi} \pi_1^2 + \pi_1 A_0' + \phi'^2 - 2e(A_0 - A_1)\phi' + e^2 A_1^2]. \tag{54}$$

The Hamiltonian acquires an explicit space dependence through $\Phi(x)$, however it has no time dependence so it is preserved in time in this situation too. Equation (51) and (52) are the primary constraints of the theory. So it necessities to deal with the effective Hamiltonian:

$$H_{EFF} = H_C + \tilde{u}\pi_0 + \tilde{v}(\pi_\phi - \phi'). \tag{55}$$

Here $\tilde{u}$ and $\tilde{v}$ are two arbitrary Lagrange multipliers. The consistency of the theory requires the preservation of the constraint (51) and (52) in time. The preservation of the constraint (51) leads to a new constraint which is the Gauss law of the theory.

$$\tilde{\Omega}_4(x) = (\pi_1' + 2e\phi')(x) \approx 0. \tag{56}$$

The preservation of the constraint (56) initially does not give rise to any new constraint since it fixes the velocity $\tilde{v}$ which comes out to be

$$\tilde{v} = \phi' - e(A_0 - A_1). \tag{57}$$

After substituting the velocity $\tilde{v}$ in (55) the consistency of the theory leads to a new constraint

$$\tilde{\Omega}_3 = (A_0 + A_1)' \approx 0. \tag{58}$$

We do not get any new constraints from the preservation of (58). So, we find that the phase space of the theory is endowed with the following four constraints:

$$\tilde{\Omega}_1 = \pi_0 \approx 0, \tag{59}$$

$$\tilde{\Omega}_2 = \pi_1 + e\phi \approx 0, \tag{60}$$

$$\tilde{\Omega}_3 = (A_0 + A_1) \approx 0, \tag{61}$$

$$\tilde{\Omega}_4 = \pi_\phi - \phi' \approx 0. \tag{62}$$

The four constraints (59), (60), (61) and (62) form a second class set and all are weak condition up to this stage. If we impose these constraints into the canonical Hamiltonian (54) treating these as strong conditions, the canonical Hamiltonian will be simplified into

$$H_{CR} = \int dx^1 [\frac{1}{2} e^{2\Phi(x)} \pi_1^2 + \frac{1}{4e_2} \pi_1'^2 - \pi_1' A_1 + 2e^2 A_1^2]. \tag{63}$$

The Hamiltonian H_{CR} lying in equation (63), is generally known as reduced Hamiltonian. According to Dirac's formalism of constrained dynamics, Poisson brackets become inadequate for this reduced Hamiltonian [24]. This reduced Hamiltonian however remains consistent with the Dirac brackets which has already been defined in equation (23)

For this constrained system the matrix constituted with the Poisson brackets among the constraints themselves is

$$
C_{ij} = \begin{pmatrix}
0 & 0 & -\delta(x-y) & 0 \\
0 & 0 & -\delta(x-y) & 2e\delta(x-y) \\
\delta(x-y) & e\delta(x-y) & 0 & 0 \\
0 & 2e\delta(x-y) & 0 & -2\delta'(x-y)
\end{pmatrix}
\tag{64}
$$

Since C_{ij} in this situation too remains nonsingular it is invertible. Using equation (24) the inverse matrix C_{ij}^{-1} is computed and it is given by

$$
C_{ij}^{-1} = \begin{pmatrix}
\frac{2}{e^2}\delta'(x-y) & \frac{1}{2e^2}\delta'(x-y) & \delta(x-y)) & -\frac{1}{2e}\delta(x-y) \\
-\frac{1}{2e}\delta'(x-y) & -\frac{1}{2e}\delta'(x-y) & 0 & \frac{1}{2e}\delta(x-y) \\
-\delta(x-y) & 0 & 0 & 0 \\
\frac{1}{2e}\delta(x-y) & -\frac{1}{2e}\delta(x-y) & 0 & 0
\end{pmatrix}.
\tag{65}
$$

From the definition of the Dirac brackets (23), we can compute the Dirac brackets between the fields A_1 and π_1 those which have been describing the reduced Hamiltonian H_{CR}. These are required to obtain the theoretical spectra in the present situation.

$$
[A_1(x), A_1(y)]^* = -\frac{1}{2e^2}\delta'(x-y),
\tag{66}
$$

$$
[A_1(x), \pi_1(y)]^* = \delta(x-y),
\tag{67}
$$

$$
[\pi_1(x), \pi_1(y)]^* = 0
\tag{68}
$$

The superscript (*) is representing the Dirac bracket as usual. Using the Dirac brackets (66), (67) and (68), we obtain the following two first order equations of motion from the reduced Hamiltonian (63):

$$
\dot{A}_1 = e^{2\Phi(x)}\pi_1 - A_1',
\tag{69}
$$

$$
\dot{\pi}_1 = \pi_1' - A_1'.
\tag{70}
$$

We find that both the field A_1 and π_1 and satisfy Klein Gordon type equation:

$$
(\Box + 4e^{2\Phi(x)}e^2)A_1 = 0,
\tag{71}
$$

$$
(\Box + 4e^{2\Phi(x)}e^2)\pi_1 = 0.
\tag{72}
$$

So the system contains only a massive boson with mass $2ee^{\Phi}(x)$. The equation (72) represents the equation of motion of momentum corresponding to the field A_1. Note that the fields A_1 and π_1 satisfy canonical Dirac bracket in the constrained sub space. It is amazing to note that there is no mass less chiral boson as was found in the previous situation according to the prediction made by Faddeev in the article [11]. The very presence of chiral boson in the preceding Sec. 3 are found to be in the root of the disastrous information loss because it was capable of travelling an arbitrary distance without getting any hindrance since the dilatonic background failed to pose any obstruction to the free chiral fermion and the observer $x^1 \to \infty$ did not find it with a backward journey. So information loss was inevitable there. But the present situation appears with a remarkable difference so far constraint structure is concerned and that in turn leads to a welcome change in the theoretical spectrum. The disturbing extra degrees of freedom in the form of chiral boson which can be thought of in terms of a chiral fermion in (1+1) dimension miraculously gets

disappeared from the theoretical spectrum and all credit goes to that important technique of imposition of chiral constraint due to Harada [5]. Since there is no chiral fermion in the theoretical spectrum, there will be no scope of information loss and a unitary s-matrix can be constructed for this situation. This results reminds us the s-wave scattering of Dirac fermion too as presented in the article [6]. Thus the scattering of chiral fermion with Faddeevian type of anomaly along with the imposition of the chiral constraint is found to be free from the dangerous information loss problem. This result is completely opposite to the observation of the authors of the article [7] where the authors considered the same problem with different type of anomaly.

5. Discussion and conclusion

The analysis available in article [7] and the present analysis differs only in the choice of taking the anomaly into account which enters during the course of bosonization through a one loop correction, but that has brought a remarkable change in the information loss scenario. Note that the model used in the article [7] was also generated by the imposition of the same chiral constraint by Harada developed in the article [5], but the authors of the article [7] obtained a result which was not in agreement with the revised suggestion of Hawking [12]. This present result is consistent with the standard belief as well as it is in agreement with the Hawking's revised suggestion [12]. Therefore although the role of Fadeevian anomaly is crucial, the importance of the technique of imposition of chiral constraint is indeed instrumental along with that. On other hand, we would have got information preserving result in Sec. 3, where imposition of this very chiral constraint (43) was not implemented.

Apart from studying information related problem through the s-wave scattering of fermion off dilaton black-hole there are several approaches to study this information preservation related question [29–32]. The information preserving result as we have obtained hare also agrees with the recent result obtained in [29,30]. In order to get the information preserving result in the present situation we exploit the regularization ambiguity that crucially alters the anomaly structure. Along with that a very useful technique of imposition of chiral constraint [5] also has been employed. And these two are equally important for holding the information preserving result. These are very often found in quantum electro dynamics and chiral quantum electrodynamics. A remarkable instance in this context is the removal of the long suffering of chiral generation of Schwinger model [33,34] from non-unitarity problem by Jackiw and Rajaraman in their seminal work [8] inviting the anomaly into the model. We, therefore, conclude that with the support of the standard technique of imposition of chiral constraint due to Harada [5], the anomaly that entered into the theory as a one loop correction during bosonization has been found to play a crucial role in dictating whether the scattering of chiral fermion off magnetically charged dilaton black-hole will face the information loss problem or it will get averted from that danger. The main focus of this article is to establish that Faddeevian anomaly can save the model from dangerous information loss problem with the incredible support of the standard technique of imposition of chiral constraint due to Harada [5]. Surprisingly, it would not be possible for the standard anomaly considered by Jackiw-Rajaraman to protect the s-wave scattering of chiral fermion off dilaton black-holes from the disastrous information loss problem even if the support of the standard technique of imposition of chiral constraint would be taken there as has been found in the important study of Mitra [7]. Therefore, so far preservation of information is concerned through s-wave scattering of dilaton black-hole the Faddevian type of anomaly [11] scores over the Jackiw-Rajaraman type of anomaly [8].

We have seen various examples with the crucial dependence of information loss with anomaly via the exploitation of ambiguity in the articles presented earlier in different times [6,7,10,20,21]. The present one has enlarged this list to confirm more convincingly the crucial role of anomaly towards the information preserving scenario with the support of the standard and powerful technique of imposition of chiral constraint [5]. The crucial role of exploitation of ambiguity in the regularization has been found in different perspective also. It is very common in connection with the confinement scenario of fermion in (1+1) dimensional electrodynamics and chiral electrodynamics [10,20,22,35–38]. With the usual anomaly the fermion was found to be unconfined where as with the Faddeevian type of anomaly it was found to remain confined [22]. In that case also the supportive role of this powerful technique of imposition of chiral constraint developed by Harada [5] can be observed if a careful look is projected [22,36]. However, it is fair to admit that hitherto there is no known direct relation between the confinement of fermion with the anomaly.

We should mention that in the article [21], we considered the generalized Faddeevian anomaly, however in our present article we have considered the specialized Mitra type Faddeevian anomaly which was presented quite a long ago in the article [7]. The whole algorithm can be carried out with the generalized Fadeevian anomaly too which will provide a prototype description but that would render no new information.

Besides, we must mention here that the information loss problem as faced in the article [7] was much more severe than this one, since we have shown that although the use of the fascinating technique of imposition of chiral constraint developed by Harada [5] can save this model with Faddeevian anomaly from the danger of information loss, hitherto there is no known standard formalism to save the model studied in the article [7] from the disastrous information loss problem. Few literatures are available where attempts have been made to protect different models to save them from this disastrous information loss problem [20,21]. In the article [21], a very effective nonstandard technique has been used where the service of unparticle [39,40] was brought into action to make the scattering process information preserving. However, none of the techniques are found effective to save the model studied in the article [7] from this disastrous information loss. But one might be hopeful to save this model too from this problem. Intensive investigation is needed for that purpose indeed.

We must mention here that the general study of this problem [41] is accompanied with lots of technical complexities and computational difficulties. Therefore, most of the authors including us prefer to deal with this problem with this less complicated but well formulated interacting model presented in [6].

Acknowledgements

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