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Strong cosmic censorship in charged black-hole spacetimes: As strong as ever

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Abstract

It is proved that dynamically formed Reissner-Nordström-de Sitter (RNdS) black holes, which have recently been claimed to provide counter-examples to the Penrose strong cosmic censorship conjecture, are characterized by unstable (singular) inner Cauchy horizons. The proof is based on analytical techniques which explicitly reveal the fact that *charged* massive scalar fields in the charged RNdS black-hole spacetime are characterized, in the large-coupling regime, by quasinormal resonant frequencies with $\Im\omega^{\text{min}} < \frac{1}{2}\kappa_+$, where κ_+ is the surface gravity of the black-hole event horizon. This result implies that the corresponding relaxation rate $\psi \sim e^{-\Im\omega^{\text{min}}t}$ of the collapsed charged fields is slow enough to guarantee, through the mass-inflation mechanism, the instability of the dynamically formed inner Cauchy horizons. Our results reveal the physically important fact that, taking into account the unavoidable presence of *charged* matter fields in dynamically formed *charged* spacetimes, non-asymptotically flat RNdS black holes are globally hyperbolic and therefore respect the fundamental strong cosmic censorship conjecture.

1. Introduction

The physically important and mathematically elegant singularity theorems of Hawking and Penrose [1,2], published almost five decades ago, have forever changed our understanding of

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the physical properties of curved spacetimes. In particular, these theorems have revealed the intriguing fact that spacetimes singularities, regions in which classical theories of gravity lose their predictive power, may naturally be formed from the dynamical gravitational collapse of compact enough matter configurations.

In order to maintain the utility of general relativity in successfully describing gravitational phenomena in our universe, Penrose [2] has put forward a physically intriguing conjecture, known as the cosmic censorship hypothesis. In its strong version, the conjecture asserts that, starting with physically reasonable generic initial data, the dynamics of self-gravitating physical systems, which are governed by the non-linearly coupled Einstein-matter field equations, will always produce globally hyperbolic spacetimes. The strong cosmic censorship (SCC) conjecture therefore implies that, in curved spacetimes that can be formed dynamically from physically acceptable initial conditions, singularities only appear on spacelike or null hypersurfaces and thus, until the very moment of encounter, a spacetime singularity has no diverging influence on physical observers that move along timelike trajectories.

It is well known that eternal charged and spinning black holes are characterized by timelike singularities and inner Cauchy horizons [3] which, according to the weak version of the cosmic censorship conjecture [1,2], are covered by larger event horizons. Interestingly, the presence of Cauchy horizons inside the canonical black-hole solutions of the Einstein field equations signals a potential breakdown of determinism within classical general relativity. In particular, observers crossing a Cauchy horizon on a timelike trajectory enter into a spacetime region inside the black hole in which past directed null geodesics may terminate on the inner singularity [4,5]. The future history of these observers cannot be determined uniquely from the initial data and the classical Einstein field equations. This physically intriguing fact implies that eternal charged and rotating black-hole spacetimes with inner Cauchy horizons are not globally hyperbolic [3–5].

Nevertheless, it has been proved that the mass-inflation mechanism [4–11], associated with the exponential blue-shift amplification of matter and radiation fields inside black holes, turns the pathological Cauchy horizons of asymptotically flat black holes into spacetime singularities, inner boundaries of the spacetimes beyond which the future evolution governed by the classical field equations ceases to make sense. The mass-inflation scenario [4–11] is based on the fact that physically realistic black-hole spacetimes, which are formed dynamically from the collapse of self-gravitating matter configurations with generic initial conditions, are characterized by the unavoidable presence of time-dependent remnant fields which, in the exterior spacetime regions, decay asymptotically as an inverse power of time [12–14]:

$$\psi(t \to \infty) \sim t^{-p} \ . \tag{1}$$

The associated energy flux of the infalling fields along the black-hole event horizon decays accordingly as an inverse power of the standard advanced null coordinate v [4,15]: $\mathcal{F}(v \to \infty) \sim v^{-2(p+1)}$.

These external remnant fields are then infinitely blue-shifted as they fall into the newly born charged and rotating black holes and propagate parallel to their inner Cauchy horizons. This blue-shift mechanism [4–11] turns the Cauchy horizons into singular hypersurfaces. In particular, the radiation flux of the infalling remnant fields, as measured by observers crossing the Cauchy horizon, is characterized by the exponentially divergent functional relation [4]

$$\mathcal{F} \sim v^{-2(p+1)} \times e^{2\kappa_- v} \to \infty \quad \text{for} \quad v \to \infty \ , \tag{2}$$

where κ_- is the surface gravity which characterizes the inner Cauchy horizon of the black-hole spacetime [16].

Interestingly, fully non-linear numerical simulations of the collapse of self-gravitating charged scalar fields in asymptotically flat spacetimes [9–11] have explicitly demonstrated that the inner Cauchy horizons of dynamically formed black holes are transformed into weak null singularities which are connected to the strong spacelike singularities of the central $r = 0$ hypersurfaces. Thus, as opposed to eternal black holes, dynamically formed black holes embedded in asymptotically flat spacetimes contain no pathological timelike singularities. These physically realistic black-hole spacetimes therefore respect the Penrose strong cosmic censorship conjecture [1,2].

Intriguingly, it has recently been claimed [17] that *non*-asymptotically flat charged Reissner-Nordström-de Sitter (RNdS) black holes may provide a genuine counter-example to the Penrose SCC conjecture. The physically interesting assertion made in [17] is based on the fact that, as opposed to asymptotically flat dynamical black-hole spacetimes which are characterized by external inverse *power-law* decaying tails [see Eq. (1)], non-asymptotically flat spacetimes are characterized by *exponentially* decaying remnant fields [4,17,18]. In particular, it has been demonstrated [4,17,18] that the relaxation phase of neutral perturbation fields in the exterior spacetime regions of charged RNdS black holes is dominated by exponentially decaying tails of the form

$$\psi(t \to \infty) \sim e^{-gt} \ , \tag{3}$$

where the spectral gap parameter $g = g(M, Q, \Lambda)$ depends on the physical parameters (mass, electric charge, cosmological constant) of the black-hole spacetime.

The effectiveness of the mass-inflation mechanism [4–11] in transforming the inner Cauchy horizons of non-asymptotically flat black-hole spacetimes, which are pathological from the point of view of the SCC conjecture, into singular non-extendable hypersurfaces depends on a delicate competition between two opposite physical mechanisms: (1) the decay of remnant perturbation fields in the exterior regions of the dynamically formed black-hole spacetimes, and (2) the exponential blue-shift amplification of the infalling fields inside the black holes. These two physical mechanisms are respectively controlled by the physical parameters g and κ_- [see Eqs. (2) and (3)].

Specifically, the fate of the inner Cauchy horizons inside dynamically formed non-asymptotically flat black-hole spacetimes depends on the dimensionless physical ratio [17,19–21]

$$\Gamma \equiv \frac{g}{\kappa_-} \ . \tag{4}$$

In particular, as explicitly shown in [17,19–21], if there exists a finite range of the black-hole physical parameters $\{M, Q, \Lambda\}$ for which

$$\Gamma(M, Q, \Lambda) > \frac{1}{2} \ , \tag{5}$$

then the corresponding black-hole spacetimes are physically extendable beyond their inner Cauchy horizons, a pathological fact which signals a breakdown of determinism (and therefore a violation of the fundamental Penrose SCC conjecture) within classical general relativity.

The physically interesting numerical study of Cardoso, Costa, Destounis, Hintz, and Jansen [17] has recently revealed the intriguing fact that the decay of *neutral* perturbation fields in near-extremal (highly charged) RNdS black-hole spacetimes is characterized by the pathological dimensionless relation

$$\frac{1}{2} < \Gamma(M, Q, \Lambda) \leq 1 \quad \text{for } \textit{neutral} \text{ massless fields} \ . \tag{6}$$

It was therefore asserted in [17,22] that non-asymptotically flat near-extremal charged RNdS black-hole spacetimes may provide physically interesting counter-examples to the fundamental Penrose SCC conjecture.

The main goal of the present paper is to reveal the physically important fact that, taking into account the unavoidable presence of *charged* fields in dynamically formed *charged* black-hole spacetimes, the seemingly pathological RNdS spacetimes actually respect the Penrose SCC conjecture [1,2]. To this end, we shall study below the linearized relaxation dynamics of newly born charged RNdS black holes. In particular, we shall use analytical techniques in order to calculate the characteristic quasinormal resonant frequencies (the characteristic damped oscillations) of linearized charged massive scalar fields in the charged RNdS black-hole spacetime. Interestingly, below we shall explicitly prove that, as opposed to the *neutral* perturbation fields considered in [17], the relaxation phase of *charged* fields in the large-coupling regime $qQ \gg 1$ [23] is characterized by the physically acceptable dimensionless relation $\Gamma(M, Q, \Lambda) < 1/2$ [see Eq. (31) below].

2. Description of the system

We analyze the linearized dynamics of charged massive scalar fields in the non-asymptotically flat charged Reissner-Nordström-de Sitter spacetime, whose spherically symmetric curved line element is given by [3,24,25]

$$ds^2 = -f(r)dt^2 + \frac{1}{f(r)}dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2) \quad \text{with} \quad f(r) = 1 - \frac{2M}{r} + \frac{Q^2}{r^2} - \frac{\Lambda r^2}{3} . \tag{7}$$

Here $\{M, Q\}$ are respectively the mass and electric charge of the central black hole [26], and $\Lambda > 0$ is the cosmological constant of the spacetime. The zeroes of the radial metric function,

$$f(r_*) = 0 \quad \text{with} \quad * \in \{-, +, \text{c}\} , \tag{8}$$

determine the radii $\{r_-, r_+, r_c\}$ of the inner (Cauchy), outer (event), and cosmological horizons which characterize the non-asymptotically flat charged black-hole spacetime [27].

The dynamics of charged massive scalar fields in the curved RNdS spacetime is determined by the Klein-Gordon differential equation [13,24,28]

$$[(\nabla^\nu - iqA^\nu)(\nabla_\nu - iqA_\nu) - \mu^2]\Psi = 0 , \tag{9}$$

where $\{\mu, q\}$ are respectively the proper mass and the charge coupling constant of the field [29], and $A_\nu = -\delta^0_\nu Q/r$ is the electromagnetic potential of the charged black-hole spacetime. Expanding the charged massive scalar field Ψ in the form [30]

$$\Psi(t, r, \theta, \phi) = \int \sum_{lm} \frac{\psi_{lm}(r;\omega)}{r} Y_{lm}(\theta) e^{im\phi} e^{-i\omega t} d\omega , \tag{10}$$

and using the differential relation $dy = dr/f(r)$ for the tortoise coordinate y, one finds from Eqs. (7) and (9) that the radial scalar function is determined by the Schrödinger-like ordinary differential equation [31]

$$\frac{d^2\psi}{dy^2} + V\psi = 0 . \tag{11}$$

The effective radial potential, $V = V(r; M, Q, \Lambda, \omega, q, \mu, l)$, which characterizes the composed charged-RNdS-black-hole-charged-massive-scalar-field system, is given by the compact functional expression [24]

$$V(r) = \Big(\omega - \frac{qQ}{r}\Big)^2 - \frac{f(r)G(r)}{r^2} \quad \text{with} \quad G(r) = \mu^2 r^2 + l(l+1) + \frac{2M}{r} - \frac{2Q^2}{r^2} - \frac{2\Lambda r^2}{3} \, . \tag{12}$$

The quasinormal resonant frequencies of the charged scalar fields in the non-asymptotically flat charged black-hole spacetime (7) are characterized by purely ingoing waves at the black-hole outer horizon and purely outgoing waves at the cosmological horizon [24]:

$$\psi \sim \begin{cases} e^{-i(\omega - qQ/r_+)y} & \text{for} \quad r \to r_+ \quad (y \to -\infty) \, ; \\ e^{i(\omega - qQ/r_c)y} & \text{for} \quad r \to r_c \quad (y \to \infty) \, . \end{cases} \tag{13}$$

These physically motivated boundary conditions for the eigenmodes of the charged scalar fields determine the discrete spectrum $\{\omega_n(M, Q, \Lambda, q, \mu, l)\}_{n=0}^{n=\infty}$ of complex quasinormal resonant frequencies which characterize the composed RNdS-black-hole-charged-massive-scalar-field system.

3. The quasinormal resonance spectrum of the composed charged-RN*d*S-black-hole-charged-field system

In the present section we shall determine the discrete set of complex (damped) resonant frequencies which characterize the linearized dynamics of the charged massive scalar fields in the non-asymptotically flat charged RNdS black-hole spacetime. In particular, as we shall explicitly show below, the resonant spectrum can be determined *analytically* in the dimensionless large-coupling regime [23,32]

$$\alpha \equiv qQ \gg \max\{\mu r_+, l+1\} \tag{14}$$

of the composed charged-black-hole-charged-scalar-field system. In addition, as explicitly shown in [33–36], the Schwinger-type pair-production mechanism of charged particles in the charged black-hole spacetime yields the upper bound $Q/r_+^2 \ll \mu^2/q\hbar$ on the black-hole electric field, or equivalently

$$\alpha \ll \mu^2 r_+^2 \, . \tag{15}$$

In the dimensionless regime (14), the radial potential (12), which determines the dynamics of the charged massive scalar fields in the charged RNdS black-hole spacetime, has the form of an effective potential barrier whose fundamental quasinormal resonant modes can be determined analytically using standard WKB techniques [37–39]. In particular, as we shall now show, in the large-coupling regime (14), the peak $r = r_0$ of the effective potential barrier (12) is located in the near-horizon region

$$\frac{r_0 - r_+}{r_+ - r_-} \ll 1 \tag{16}$$

of the charged black-hole spacetime. Defining the dimensionless physical variables

$$x \equiv \frac{r - r_+}{r} \quad ; \quad \varpi \equiv \omega \cdot \frac{r_+}{\alpha} - 1 \, , \tag{17}$$

one can express the effective black-hole-charged-scalar-field potential (12) in the compact dimensionless form [40]

$$r_+^2 V(x;\varpi) = [\alpha(x+\varpi)]^2 - 2G_0\beta\cdot x[1+O(x/\beta)]\ , \tag{18}$$

where $G_0 \equiv G(x=0)$ [41], and the dimensionless parameter β is defined by the gradient relation [42,43]

$$\beta \equiv \frac{1}{2}\frac{df(x=0)}{dx} = \kappa_+ r_+\ . \tag{19}$$

From Eq. (18) one finds that the peak location, $x = x_0$, of the composed charged-black-hole-charged-scalar-field scattering potential is characterized by the simple dimensionless relation [44]

$$x_0 + \varpi = \frac{G_0\beta}{\alpha^2} \ll 1\ . \tag{20}$$

It has been explicitly proved in [37,38] that the characteristic quasinormal resonant frequencies of the Schrödinger-like radial differential equation (11) are determined by the WKB resonance equation

$$iK = n + \frac{1}{2} + \Lambda(n) + O[\Omega(n)] \quad ; \quad n = 0,1,2,... \tag{21}$$

where [38]

$$K = \frac{V_0}{\sqrt{2V_0^{(2)}}} \quad ; \quad \Lambda(n) = \frac{1}{\sqrt{2V_0^{(2)}}}\Big[\frac{1+(2n+1)^2}{32}\cdot\frac{V_0^{(4)}}{V_0^{(2)}} - \frac{28+60(2n+1)^2}{1152}\cdot\Big(\frac{V_0^{(3)}}{V_0^{(2)}}\Big)^2\Big]\ , \tag{22}$$

and the cumbersome mathematical expression of the sub-leading correction term $\Omega(n)$ is given by equation (1.5b) of [38]. Here the spatial derivatives $V_0^{(k)} \equiv d^kV/dy^k$ of the effective black-hole-field potential $V(y)$ are evaluated at the radial location $y = y_0$ of its scattering peak.

Substituting Eqs. (18) and (20) into Eq. (22), one finds the functional expressions [41,45]

$$K = \frac{G_0}{2\alpha}\cdot\frac{\varpi - \frac{G_0\beta}{2\alpha^2}}{\frac{G_0\beta}{\alpha^2} - \varpi} \quad ; \quad \Lambda(n) = -\frac{2\alpha}{G_0}\cdot(n+\frac{1}{2})^2 \quad ; \quad \Omega(n) = O[(\alpha/G_0)^2(n+1/2)^3] \tag{23}$$

for the various terms in the WKB resonance equation (21). From Eqs. (21) and (23), one obtains the (rather cumbersome) expressions

$$\varpi_R = \frac{G_0\beta}{2\alpha^2}\cdot\frac{G_0^2 + 2(2n+1)^2\alpha^2[1-(2n+1)\alpha/G_0]^2}{G_0^2 + (2n+1)^2\alpha^2[1-(2n+1)\alpha/G_0]^2}\cdot\big\{1+O[(\alpha/G_0)^2]\big\} \tag{24}$$

and

$$\varpi_I = -i\frac{G_0\beta}{2\alpha^2}\cdot\frac{(2n+1)\alpha G_0[1-(2n+1)\alpha/G_0]}{G_0^2 + (2n+1)^2\alpha^2[1-(2n+1)\alpha/G_0]^2}\cdot\big\{1+O[(\alpha/G_0)^2]\big\}\ , \tag{25}$$

for the dimensionless resonant frequencies which characterize the composed charged-black-hole-charged-scalar-field system in the large-coupling regime (14). Taking cognizance of the strong inequality [see Eq. (15) and [41]]

$$\frac{\alpha}{G_0} \ll 1 \; , \tag{26}$$

one obtains from (24) and (25) the approximated compact expressions

$$\varpi_R = \frac{G_0 \beta}{2\alpha^2} \cdot \{1 + O[(\alpha/G_0)^2]\} \tag{27}$$

and

$$\varpi_I = -i \frac{\beta}{\alpha}(n + 1/2)[1 - (2n+1)\alpha/G_0] \cdot \{1 + O[(\alpha/G_0)^2]\} \; . \tag{28}$$

Finally, substituting Eqs. (27) and (28) into the relation $\omega = (qQ/r_+) \cdot (\varpi + 1)$ [see Eqs. (14) and (17)], and using the identity (19), we obtain the remarkably compact functional expression

$$\omega_n = \frac{qQ}{r_+} + \frac{G_0 \kappa_+}{2qQ} - i\kappa_+ \cdot \Big[n + \frac{1}{2} - \frac{2qQ}{G_0} \cdot \Big(n + \frac{1}{2}\Big)^2\Big] \quad ; \quad n = 0, 1, 2, ... \tag{29}$$

for the quasinormal resonant spectrum of the composed charged-RNdS-black-hole-charged-massive-scalar-field system in the large-coupling regime (14).

In particular, the fundamental (least damped) resonant mode of the system, which dominates the relaxation phase of the dynamically formed non-asymptotically flat charged black-hole spacetimes, is characterized by the simple relation

$$\Im\omega_0 = \frac{1}{2}\kappa_+ \Big(1 - \frac{qQ}{G_0}\Big) \; . \tag{30}$$

Interestingly, the analytically derived expression (30) for the fundamental resonant mode reflects the fact that, to leading order in the dimensionless small quantity $qQ/\mu^2 r_+^2 \ll 1$ [see Eq. (15) and [41]], the characteristic relaxation time $\tau \equiv 1/\Im\omega_0 = 2/\kappa_+$ of the composed charged-RNdS-black-hole-charged-massive-scalar-field system is *universal*, that is, independent of the charged-field parameters q and μ.

4. Summary and discussion

The Penrose strong cosmic censorship conjecture [1,2] has attracted the attention of physicists and mathematicians during the last five decades. This physically intriguing conjecture asserts that, starting with generic initial conditions, the future evolution of the non-linearly coupled matter-curvature fields should always produce globally hyperbolic spacetimes.

It has long been known [4–11] that the presence of remnant power-law tails in the exterior spacetime regions of dynamically formed black holes in asymptotically flat spacetimes [12–14], together with the exponential blue-shift effect which characterizes the dynamics of the infalling fields in the interior spacetime regions of the black holes, guarantee, through the mass-inflation mechanism [4–11], the validity of the SCC conjecture.

As nicely shown in [17,19–21], the (in)stability properties of the inner Cauchy horizons of dynamically formed charged RNdS black holes, which ultimately determine the fate of the Penrose SCC conjecture in these non-asymptotically flat spacetimes, are controlled by the simple dimensionless ratio $\Gamma(M, Q, \Lambda) \equiv g/\kappa_-$ [see Eqs. (3) and (4)] between the decay rate g of remnant perturbation fields in the exterior regions of the black-hole spacetime and the blue-shift growth rate κ_- of the infalling fields inside the black holes.

In particular, as explicitly demonstrated in [17,19–21], non-asymptotically flat black-hole spacetimes which are characterized by the inequality $\Gamma > 1/2$ are physically extendable beyond their inner Cauchy horizons, a fact which signals the breakdown of determinism and the corresponding violation of the Penrose SCC conjecture in these classical black-hole spacetimes.

Intriguingly, the recently published important numerical study of Cardoso, Costa, Destounis, Hintz, and Jansen [17] has revealed the fact that the decay rate of *neutral* perturbation fields in the exterior spacetime regions of highly charged (near-extremal) RNdS black holes is too fast to make their inner Cauchy horizons unstable. In particular, it has been explicitly proved that there is a finite volume in the phase space of the physical parameters $\{M, Q, \Lambda, l, m\}$, which characterize the composed charged-black-hole-neutral-scalar-field system, for which the dimensionless ratio g/κ_- is characterized by the pathological relation $1/2 < \Gamma(M, Q, \Lambda) \leq 1$ [17]. It was therefore concluded in [17] that these non-asymptotically flat black-hole spacetimes may provide physically interesting counter-examples to the fundamental SCC conjecture.

It should be realized that the existence of even one physically acceptable counter-example to the Penrose SCC conjecture would imply the undesirable breakdown of determinism in Einstein's classical general theory of relativity. It is therefore physically crucial to restore the predictive power of the Einstein field equations by proving that, due to the existence of some physical mechanism that has not been taken into account in previous analyzes, the dynamically formed RNdS black-hole spacetimes, which have been suspected to violate the Penrose SCC conjecture, are globally hyperbolic.

In the present paper we have pointed out that the presence of charged remnant fields in the exterior spacetime regions of dynamically formed RNdS black holes is an unavoidable feature of these non-asymptotically flat charged spacetimes. In particular, we have demonstrated that, as opposed to the *neutral* remnant fields considered in [17], the decay rates of *charged* remnant fields [46,47] in these dynamically formed non-asymptotically flat charged black-hole spacetimes are slow enough to guarantee, through the mass-inflation mechanism [4–11], the instability of the inner Cauchy horizons. Specifically, using analytical techniques, it has been explicitly proved that the relaxation phase of the composed charged-black-hole-*charged*-scalar-field system in the large-coupling regime $qQ \gg 1$ [23] is characterized by the physically acceptable dimensionless relation [see Eq. (30)] [48–50]

$$\Gamma(M, Q, \Lambda) < \frac{\kappa_+}{2\kappa_-} \leq \frac{1}{2} \quad \text{for } \textit{charged} \text{ massive fields} . \tag{31}$$

The analytically derived results presented in this paper therefore reveal the physically important fact that, taking into account the unavoidable presence of charged remnant fields in dynamically formed charged RNdS black holes, these non-asymptotically flat spacetimes respect the Penrose strong cosmic censorship conjecture [1,2].

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[41] Note that the strong inequalities (14) and (15) imply the dimensionless relation $\mu r_+ \gg 1$, and thus $G_0 \simeq \mu^2 r_+^2 [1 + O(1/\mu^2 r_+^2)]$.
[42] Note that $f(x = 0) \equiv 0$ at the radial location $r = r_+$ (or equivalently, $x = 0$) of the black-hole event horizon.
[43] Here κ_+ is the surface gravity which characterizes the event horizon of the charged black-hole spacetime.
[44] The last inequality in (20) characterizes the large-coupling regime (14) (see also [41]) of the composed charged-black-hole-charged-field system.
[45] It is worth noting that the strong inequality (15), which is a direct consequence of the Schwinger discharge mechanism [33–36] that characterizes the composed charged-black-hole-charged-massive-scalar-field system, implies that, in the regime $n \ll G_0/\alpha$ [41] of the fundamental (least damped) resonant modes, the various terms that appear on the r.h.s. of the WKB equation (21) are characterized by the dimensionless relations [41] $\Lambda(n)/(n + 1/2) = O[(n + 1/2)\alpha/G_0] \ll 1$ and $\Omega(n)/\Lambda(n) = O[(n + 1/2)\alpha/G_0] \ll 1$.
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[49] It is worth mentioning that there is a $\sim 11\%$ difference between the analytically derived formula (30) and the numerical result of [50]. This difference may stem from the fact that the physical parameters used in section 7.4 of [50] satisfy the assumed strong inequality [see Eq. (15)] $qQ \ll \mu^2 r_+^2$ only marginally (that is, $qQ/\mu^2 r_+^2 = 0.1$ in the numerical computation of [50]).
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