



# Virtual axion-like particle Complement to Euler-Heisenberg-Schwinger action

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## ABSTRACT

We modify action in an external electromagnetic field to include effects of virtual axion-like particle (ALP) excitations. A measurable addition to QED-Euler-Heisenberg-Schwinger (EHS) action is obtained and incorporated into experimental constraints placed on ALP mass and coupling to two photons. The regime of these constraints in which the ALP vacuum effect surpasses the EHS effect is characterized. We show that probing of the virtual vacuum effect offers an alternative method in search for physics related to ALPs.

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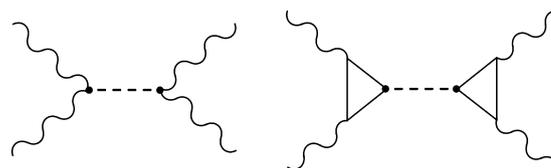
## 1. Introduction

We show that a pseudoscalar coupling between two photons and axion-like particles (ALP) produces a nonlinear in EM field addition to the QED vacuum effect akin to a specific term in the Euler-Heisenberg-Schwinger (EHS) action [1,2]. We report on this additional effect and the conditions required for it to be of or above in magnitude to the QED vacuum fluctuation EHS result.

The Adler-Bell-Jackiw anomaly is often used in study of pseudoscalar decay process  $\pi^0 \rightarrow \gamma\gamma$  [3,4]. This was followed by proposed Axion-Like-Particle (ALP)  $\leftrightarrow \gamma\gamma$  processes. Searches for these processes focus on possible flux of on-mass shell ALP particles from astronomical sources [5,6]. Another consequence of the ALP  $\leftrightarrow \gamma\gamma$  interaction is a possible modification of the vacuum by virtual fluctuations which we describe here.

The virtual EHS effect of the electron loop on electromagnetic processes has motivated measurement of vacuum birefringence in strong magnetic fields [7,8]. We complement this effective QED-EHS action including effects on vacuum structure by two other effects, the virtual ALP excitations and pions – the ALP couples to two photons with point particle form factor while the pion via quark triangle loop, see Fig. 1.

The ALP supplement to EHS action is obtained by rescaling ALP fields to include photon-loop corrections. This produces a fourth-order in EM field addition to EHS action, a consequence of ALP



**Fig. 1.** EHS complements: vacuum fluctuation contributions fourth order in the photon field. Left: four photon interactions via virtual ALP; and Right via PCAC pion anomaly.

fluctuations introducing an extra diagram contributing to vacuum polarization shown in the left-hand-side of Fig. 1.

The strength of the ALP fluctuation supplement to EHS action is characterized by ALP- $\gamma\gamma$  coupling involving the ratio of coupling and mass  $G_A/m_A$ . Observations of astrophysical ALP sources have provided constraints on  $G_A$ ,  $m_A$ : see [5,6] and references therein. Unlike in the case of axions, for ALPs  $G_A/m_A$  is not fixed. While the fixed value of pseudoscalar- $\gamma\gamma$  coupling to mass ratio (or product of mass and decay constant) excludes significant additions to EHS action in the case of axions and pions, the ALP contribution does not have this restriction. We obtain an ALP action supplement to the EHS result valid for external field photon frequency  $\omega < m_A$ , that is for external fields varying slowly over the ALP Compton wavelength. Ongoing search for ALP effects in vacuum birefringence in quasi-constant magnetic fields is carried out by PVLAS [9–13] and BMV [14,15]. While these experiments use laser wavelengths outside of domain  $\omega < m_A$ , they probe  $m_A$  and  $G_A$  in a regime where the here obtained ALP supplement surpasses EHS action. Existence of an ALP with parameters in this regime would significantly modify nonlinear vacuum effects, manifest as

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light-light scattering by long wavelength lasers: see [16–18] and references therein.

In the following we evaluate the size of the ALP contribution complement to the EHS effective action in section 2. We describe the experimental constraints in section 3, where the domain of  $G_A/m_A$  is shown in which the ALP contribution surpasses the corresponding contribution from the EHS action.

## 2. Pseudoscalar coupling input to effective EHS action

### 2.1. ALP complement to EHS

We consider the effective QED action using EM field invariants

$$S = \frac{1}{2}(\mathcal{E}^2 - \mathcal{B}^2), \quad P = \mathcal{E} \cdot \mathcal{B}, \quad (1)$$

in the respective terms

$$\mathcal{L} = S + \mathcal{L}_{EHS} + \mathcal{L}_\varphi + \mathcal{L}_{int}. \quad (2)$$

We see first the Maxwell action, complemented by renormalized effective EHS action, since in the interaction term bare  $(e_0\mathcal{E}_0)(e_0\mathcal{B}_0)$  is equivalent to renormalized  $(e\mathcal{E})(e\mathcal{B})$ :

$$\mathcal{L}_{EHS} = m_e^4 f\left(\frac{S}{m_e^4}, \frac{P^2}{m_e^8}\right), \quad (3)$$

where  $m_e$  is electron mass and the function  $f$  is well known [1,2]. Note that for reason of parity conservation by the QED vacuum, a  $P$ -term must have even powers in Eq. (3). The low field expansion generates the well known term

$$\mathcal{L}_{EHS}^{(1)} = \frac{e^4}{(4\pi)^2} \frac{2}{45m_e^4} (4S^2 + 7P^2), \quad (4)$$

where superscript (1) denotes action up to the leading nonlinear EM contribution.

The two supplemental and here relevant ALP terms in Eq. (2) include the pseudoscalar mass contribution

$$\mathcal{L}_\varphi = -\frac{m_A^2}{2}\varphi^2, \quad (5)$$

where  $\varphi$  is the ALP field. We neglect the kinetic energy term, as we are interested in the infrared limit of vacuum fluctuation contribution. For the effective ALP with two photon interaction term [5,6]

$$\mathcal{L}_{int} = \pm G_A \varphi P, \quad (6)$$

where ALP to two photon coupling  $G_A[\text{GeV}^{-1}]$  is constrained by experimental observation. Note that the sign in the effective action Eq. (6) requires further consideration: its value will not appear in our computations which are quadratic in this term, thus we choose in further considerations the positive sign.

The total action we thus consider comprises three contributions: EHS Eq. (4) and the two ALP terms

$$\mathcal{L}^{(1)} = S - \frac{m_A^2}{2}\varphi^2 + G_A \varphi P + \frac{e^4}{(4\pi)^2} \frac{2}{45m_e^4} (4S^2 + 7P^2). \quad (7)$$

The ALP degrees of freedom can be ‘rotated’ to diagonalize the ALP- $\gamma\gamma$  interaction. Completing the square,

$$\begin{aligned} -\frac{m_A^2}{2}\varphi^2 + G_A \varphi P &= -\frac{m_A^2}{2}\left(\varphi - G_A \frac{P}{m_A^2}\right)^2 + \frac{G_A^2}{2m_A^2} P^2 \\ &= -\frac{m_A^2}{2}\tilde{\varphi}^2 + \frac{G_A^2}{2m_A^2} P^2, \end{aligned} \quad (8)$$

where the ALP-field is configuration mixed with electromagnetic  $\gamma\gamma$ -contribution

$$\tilde{\varphi} = \varphi - G_A \frac{P}{m_A^2}. \quad (9)$$

After configuration mixing there is an additional contribution, the last term in Eq. (8), which like the considered term of EHS action (second term in Eq. (4)) is proportional to  $P^2$  but only depends on ALP properties and electron charge, and is thus independent of electron mass.

The total action up to 4th order in EM fields is

$$\mathcal{L} = S - \frac{m_A^2}{2}\tilde{\varphi}^2 + \tilde{\mathcal{L}}_{EHS}, \quad (10)$$

where the first term is the QED Maxwell field action, followed by ALP mass contribution and

$$\tilde{\mathcal{L}}_{EHS} = \mathcal{L}_{EHS} + \frac{G_A^2}{2m_A^2} P^2, \quad (11)$$

independent of the sign of the ALP interaction term in Eq. (6), as mentioned earlier. The ALP supplement to EHS action is valid for EM fields characterized by external photon frequency  $\omega < m_A$ . EM fields must be quasi-constant over the ALP Compton wavelength, a distance greater than the electron-positron Compton wavelength for the ALP masses we consider in the section 3.

The ALP contribution is written as an addition to the lowest order term in EHS action (Eq. (4)),

$$\tilde{\mathcal{L}}_{EHS}^{(1)} = \frac{e^4}{(4\pi)^2} \frac{2}{45m_e^4} (4S^2 + 7P^2) + \frac{G_A^2}{2m_A^2} P^2. \quad (12)$$

This is equivalent to modification of the  $P^2$  coefficient in Eq. (4) by

$$P^2 \rightarrow P^2 \left(1 + \frac{45m_e^4 G_A^2}{28\alpha^2 m_A^2}\right), \quad (13)$$

where  $\alpha = e^2/4\pi$ . In order for the ALP effect to be equal or larger than the QED effect, the ratio of ALP- $\gamma\gamma$  coupling to mass  $m_A$  amounts to

$$\frac{G_A^2}{m_A^2} \geq \frac{28\alpha^2}{45m_e^4}. \quad (14)$$

This condition will be used to characterize presence of a virtual pseudoscalar effect on electromagnetism comparable to electron loop effects, in context of constraints from astronomical observation and vacuum birefringence experiment. We first discuss the differences between ALPs, axions and pions in context of Eq. (14).

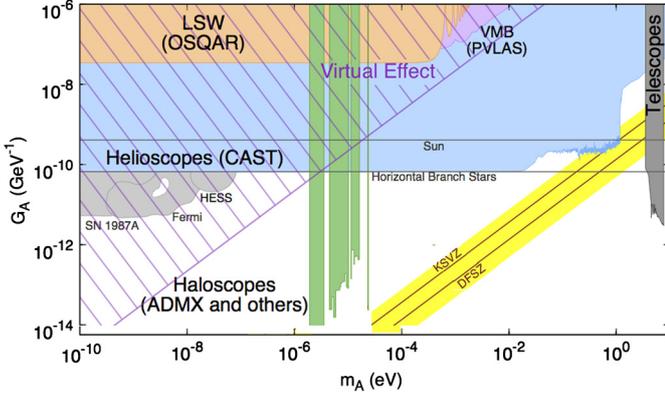
### 2.2. Pion and axion

Evaluation of the  $P^2$  rescaling in Eq. (13) is repeated for the pion, a result of supplementary diagram to QED vacuum polarization shown in the right-hand-side of Fig. 1. Eq. (6) is replaced with

$$\mathcal{L}_{int} = \frac{\alpha}{2\pi f_\pi} \varphi P, \quad (15)$$

where pion decay constant  $f_\pi = 93 \text{ MeV}$  [19]. This is reminiscent of Schwinger’s result characterizing pion decay, which was missing a phase factor enforcing gauge invariance, summarized and reconciled with PCAC in [20–22].

Repeating steps in section 2.1, with  $\varphi$  now denoting a pion field and  $m_A \rightarrow m_\pi \sim 135 \text{ MeV}$ ,



**Fig. 2.** We adapt Figure 111.1 from Ref. [6], adding purple lines to denote a new region labeled ‘virtual effect’, where according to Eq. (20) the ALP supplement to effective action surpasses the first nonlinear EHS effect.

$$P^2 \rightarrow P^2 \left( 1 + \frac{45m_e^4}{112\pi^2} \frac{1}{f_\pi^2 m_\pi^2} \right) = P^2 (1 + \mathcal{O}(10^{-10})), \quad (16)$$

producing a negligible addition to EHS action. Using instead an axion-field, the result is still characterized by a product  $f_a m_a$  on the order of  $f_\pi m_\pi$  [5,6]: the virtual axion effect does not produce a significant contribution to  $\mathcal{L}_{EHS}$  either.

Even though the result for pions is very small, we note that the decay constant  $f_\pi$  is measured in timelike kinematic domain with pion momentum  $Q_\pi^2 \rightarrow m_\pi^2$ . Whether application of  $f_\pi = 93$  MeV to the infrared domain  $Q_\pi^2 \rightarrow 0$  of external EM fields is valid remains an open question. While this issue of kinematic domains awaits resolution, we only consider vacuum fluctuations of ALPs and not pions nor axions. We proceed to experimental constraints for ALPs.

### 3. Evaluated ALP vacuum fluctuation effect

#### 3.1. Constraints on $G_A$ and $m_A$

We wish to add our result Eq. (14) into constraints on  $G_A$  and  $m_A$ , based on results provided in Figure 111.1 from [6]. All these constraints are based on real axions, with the exception of virtual effects probed by the vacuum birefringence measurements carried out by the PVLAS [10–13] collaboration. Our addition is a virtual effect, a supplement to EHS effective action.

For clarity we describe in detail how the new domain arises: Taking the square root of Eq. (14) and writing units explicitly,

$$\frac{G_A[\text{GeV}^{-1}]}{m_A[\text{eV}]} \geq \sqrt{\frac{28}{45}} \alpha \frac{1}{(0.511 \text{ MeV})^2}. \quad (17)$$

We write

$$G_A[\text{GeV}^{-1}] = \frac{10^y}{\text{GeV}}, \quad m_A[\text{eV}] = 10^x \text{ eV}, \quad (18)$$

where the coefficients  $y, x$  serve as variables  $\log_{10} G_A[\text{GeV}^{-1}]$ ,  $\log_{10} m_A[\text{eV}]$  in Fig. 2. Eq. (17) becomes

$$\frac{10^{y-x-9}}{(\text{eV})^2} \geq \sqrt{\frac{28}{45}} \alpha \frac{10^{-12}}{(0.511)^2 (\text{eV})^2} = \frac{2.21 \cdot 10^{-14}}{(\text{eV})^2}. \quad (19)$$

Taking  $\log_{10}$  and keeping only  $y$  on the left hand side,

$$y \geq x + \log_{10} 2.21 \cdot 10^{-5} = x - 4.66. \quad (20)$$

With this we define boundary at which ALP and QED effects equal:

$$y_{QED} = x_{QED} - 4.66 \rightarrow \frac{G_{A(QED)}}{m_{A(QED)}} = \frac{10^{-4.66}}{(\text{GeV})(\text{eV})}. \quad (21)$$

In Fig. 2, the region of Figure 111.1 of [6] where  $y, x$  satisfy Eq. (20) is shaded. The boundary of the shaded region, at which the ALP and QED effects are equal is given by Eq. (21). This boundary is parallel to the KSVZ and DFSZ models for the axion shown in the figure, which have fixed coupling to mass ratios (denoted axion):

$$y_{\text{axion}} \sim (x_{\text{axion}} - 4.66) - 5 \\ \rightarrow \frac{G_{A(\text{axion})}^2}{m_{A(\text{axion})}^2} = 10^{-10} \frac{G_{A(QED)}^2}{m_{A(QED)}^2}. \quad (22)$$

This is in agreement with the  $\sim 10^{-10}$  suppression of virtual pion vs QED effect in section 2.2, recalling the similar (differences are model-dependent) products of decay constants and masses of axions and pions.

#### 3.2. PVLAS result

The summary of observational data presented in Figure 111.1 in Ref. [6] and copied into our Fig. 2 includes an update from PVLAS, measuring vacuum birefringence and dichroism in external magnetic fields. PVLAS 1992 work excludes massless ALPs:  $m_A < 10^{-3}$  eV at coupling  $G_A > 3.6 \cdot 10^{-7}$   $\text{GeV}^{-1}$  [10]. PVLAS 2006 results suggested a coupling stronger by factor  $\sim 10$ , subsequently revised by more recent results [10–13]. Even without factor  $\sim 10$  enhancement of coupling, ALP constraints lie within the ‘virtual effect’ region in Fig. 2, where an ALP virtual contribution surpasses the EHS effect.

We find that the updated PVLAS range plotted, see [13], is included within the shaded region denoting prominent virtual effects:

$$\frac{G_{A(\text{PVLAS})}^2}{m_{A(\text{PVLAS})}^2} \sim \left( \frac{10^{-7} (\text{GeV}^{-1})}{10^{-3} (\text{eV})} \right)^2 = 10^{1.32} \frac{G_{A(QED)}^2}{m_{A(QED)}^2}, \quad (23)$$

using Eq. (21) in the last line. Thus the PVLAS constraint suggests a possible virtual pseudoscalar effect as much as 20 times the strength of the QED-EHS effect.

### 4. Conclusion

The ALP vacuum effect as shown in Fig. 2 covers a large domain provided by the astronomical observation constraints. These results rely on propagation of real ALPs, either over large distances from astronomical sources, or length scales probed by resonant cavities and light-through-wall experiments, see [5,6] and references therein.

In the virtual ALP experiments EM fields must be strong, preferably as near as possible to the critical field for the electron loop:  $E_c \equiv m_e^2 c^3 / e \hbar = 1.3 \cdot 10^{18}$  V/m, in order to probe the effects inherent to EHS action and virtual ALPs. We note that as EM fields exceed the domain in which low field expansion of action to order  $P^2$  is valid, higher order (photon number) ALP interactions must be evaluated: another paper is required to study this limit. The low field expansion studied here behaves differently for the ALP contribution than in EHS action. In the latter subcritical fields must satisfy  $\mathcal{E}^2/E_c^2 < 1$  and  $\mathcal{B}^2/E_c^2 < 1$ , while the ALP term requires  $PG_A^2/\alpha m_A^2 < 1$ . Thus a supercritical EM field in context of EHS action may be subcritical for ALP effects if  $P$  is small. Even for small mass  $m_A < 10^{-5}$  eV, subcritical requirement for low field expansion is satisfied for appropriate ratio  $G_A/m_A$ . However in

considering the possibly small ALP mass range in Fig. 2, we further need EM fields quasi-constant over the range of the Compton wavelength of the ALP:  $\omega < m_A < 10^{-5}$  eV. Study of light-light scattering ([16–18]) in this long wavelength limit offers a probe for significant nonlinear vacuum effects: the regime where virtual ALP modification of Maxwell equations surpasses that from EHS action.

The virtual effect region of Fig. 2 includes the domain of  $G_A$ ,  $m_A$  obtained by PVLAS, which we benchmark at  $m_A = 10^{-3}$  eV and  $G_A = 10^{-7}$  GeV $^{-1}$ . The PVLAS experiment probes virtual ALP and  $e^+e^-$  effects via vacuum birefringence in an external magnetic field [10–13]. The external field driving the 4-photon interaction in Fig. 1 consists of a magnetic field of 2.5 T constant over  $\lambda_c$ , and most recently a 1064 nm laser [13], similar to BMV parameters [14, 15]. While external fields strengths satisfy condition for low field expansion in  $P^2$  given above, the laser wavelength is smaller by a factor  $\sim 10^{-2}$  than Compton wavelength  $\lambda_c = \hbar/m_A c = 2 \times 10^{-4}$  m for  $m_A = 10^{-3}$  eV. A larger mass  $m_A \sim 10^{-1}$  eV is required in order for  $\omega < m_A$  to be satisfied. Constraints on this larger mass via study of vacuum effects on external EM fields await resolution [5]. However, we note that in context of EHS action virtual behavior of periodic fields may correspond to that of constant fields [23,24]. Whether this argument can be extended to ALP effects awaits resolution in future work: it is possible that the PVLAS experiment [10–13] using a standing laser wave and an external magnetic field probes virtual effects on the required length scale.

We note ongoing effort to study the very strong field environments with high intensity lasers at ELI [25,26], and the long lasting study of supercritical fields in relativistic heavy-ion collisions [27, 28]. Both methods offer encouraging prospects as probing methods for QED + ALP action though the length and mass scale is today quite different, with EM fields varying over a shorter range than the usually applied  $\lambda_c$  range.

We conclude that our consideration of virtual ALP effects adds a new method for detection of ALPs that does not rely on propagation of real ALPs. Should ALPs exist only virtually in the vacuum (like quarks and gluons), they will never be discovered as free-streaming particles, but could via vacuum fluctuations we evaluated in this work. This observation adds to the findings we presented in Fig. 2 an additional motivation to relevant experiments, such as PVLAS and BMV.

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