

Research Article

Analysis of the $D\bar{D}^* K$ System with QCD Sum Rules

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In this article, we construct the color singlet-singlet-singlet interpolating current with $I(J^P) = (3/2)(1^-)$ to study the $D\bar{D}^* K$ system through QCD sum rules approach. In calculations, we consider the contributions of the vacuum condensates up to dimension-16 and employ the formula $\mu = \sqrt{M_{XY/Z}^2 - (2M_c)^2}$ to choose the optimal energy scale of the QCD spectral density. The numerical result $M_Z = 4.71_{-0.11}^{+0.19}$ GeV indicates that there exists a resonance state Z lying above the $D\bar{D}^* K$ threshold to saturate the QCD sum rules. This resonance state Z may be found by focusing on the channel $J/\psi\pi K$ of the decay $B \rightarrow J/\psi\pi K$ in the future.

1. Introduction

Since the observation of the $X(3872)$ by the Belle collaboration in 2003 [1], more and more exotic hadrons have been observed and confirmed experimentally, such as the charmonium-like X, Y, Z states, hidden-charm pentaquarks, etc. [2–4]. Those exotic hadron states, which cannot be interpreted as the quark-antiquark mesons or three-quark baryons in the naive quark model [5], are good candidates of the multi-quark states [6, 7]. The multi-quark states are color-neutral objects because of the color confinement and provide an important platform to explore the low energy behaviors of QCD, as no free particles carrying net color charges have ever been experimentally observed. Compared to the conventional hadrons, the dynamics of the multi-quark states is poorly understood and calls for more works.

Some exotic hadrons can be understood as hadronic molecular states [8], which are analogous to the deuteron as a loosely bound state of the proton and neutron. The most impressive example is the original exotic state, the $X(3872)$, which has been studied as the $D\bar{D}^*$ molecular state by many theoretical groups [9–17]. Another impressive example is the $P_c(4380)$ and $P_c(4450)$ pentaquark states observed by the LHCb collaboration in 2015, which are good candidates for the $\bar{D}\Sigma_c^*, \bar{D}^*\Sigma_c, \bar{D}^*\Sigma_c^*$ molecular states [8]. In

addition to the meson-meson type and meson-baryon type molecular state, there may also exist meson-meson-meson type molecular states; in other words, there may exist three-meson hadronic molecules.

In [18, 19], the authors explore the possible existence of three-meson system $D\bar{D}^* K$ molecule according to the attractive interactions of the two-body subsystems $DK, \bar{D}K, D^*K, \bar{D}^*K$, and $D\bar{D}^*$ with the Born-Oppenheimer approximation and the fixed center approximation, respectively. In this article, we study the $D\bar{D}^* K$ system with QCD sum rules.

The QCD sum rules method is a powerful tool in studying the exotic hadrons [20–25] and has given many successful descriptions; for example, the mass and width of the $Z_c(3900)$ have been successfully reproduced as an axial vector tetraquark state [26, 27]. In QCD sum rules, we expand the time-ordered currents into a series of quark and gluon condensates via the operator product expansion method. These quark and gluon condensates parameterize the nonperturbative properties of the QCD vacuum. According to the quark-hadron duality, the copious information about the hadronic parameters can be obtained on the phenomenological side [28, 29].

In this article, the color singlet-singlet-singlet interpolating current with $I(J^P) = (3/2)(1^-)$ is constructed to study

the $D\bar{D}^*K$ system. In calculations, the contributions of the vacuum condensates are considered up to dimension-16 in the operator product expansion and the energy-scale formula $\mu = \sqrt{M_{X/Y/Z}^2 - (2M_c)^2}$ is used to seek the ideal energy scale of the QCD spectral density.

The rest of this article is arranged as follows: in Section 2, we derive the QCD sum rules for the mass and pole residue of the $D\bar{D}^*K$ state; in Section 3, we present the numerical results and discussions; Section 4 is reserved for our conclusion.

2. QCD Sum Rules for the $D\bar{D}^*K$ State

In QCD sum rules, we consider the two-point correlation function,

$$\Pi_{\mu\nu}(p) = i \int d^4x e^{ip \cdot x} \langle 0 | T \{ J_\mu(x) J_\nu^\dagger(0) \} | 0 \rangle, \quad (1)$$

where

$$J_\mu(x) = \bar{u}^m(x) i\gamma_5 c^m(x) \bar{c}^n(x) \gamma_\mu d^n(x) \bar{u}^k(x) i\gamma_5 s^k(x), \quad (2)$$

and the m , n , and k are color indexes. The color singlet-singlet-current operator $J_\mu(x)$ has the same quantum numbers $I(J^P) = (3/2)(1^-)$ as the $D\bar{D}^*K$ system.

On the phenomenological side, a complete set of intermediate hadronic states, which has the same quantum numbers as the current operator $J_\mu(x)$, is inserted into the correlation function $\Pi_{\mu\nu}(p)$ to obtain the hadronic representation [28, 29]. We isolate the ground state contribution Z from the pole term, and get the result:

$$\begin{aligned} \Pi_{\mu\nu}(p) &= \frac{\lambda_Z^2}{M_Z^2 - p^2} \left(-g_{\mu\nu} + \frac{p_\mu p_\nu}{p^2} \right) + \dots \\ &= \Pi(p^2) \left(-g_{\mu\nu} + \frac{p_\mu p_\nu}{p^2} \right) + \dots, \end{aligned} \quad (3)$$

where the pole residue λ_Z is defined by $\langle 0 | J_\mu(0) | Z(p) \rangle = \lambda_Z \varepsilon_\mu$, the ε_μ is the polarization vector of the vector hexaquark state Z .

At the quark level, we calculate the correlation function $\Pi_{\mu\nu}(p)$ via the operator product expansion method in perturbative QCD. The u , d , s , and c quark fields are contracted with the Wick theorem, and the following result is obtained:

$$\begin{aligned} \Pi_{\mu\nu}(p) &= -i \int d^4x e^{ip \cdot x} \left\{ \text{Tr} \left[\gamma_\mu D^{nm'}(x) \gamma_\nu C^{n'n}(-x) \right] \right. \\ &\quad \cdot \text{Tr} \left[i\gamma_5 C^{mm'}(x) i\gamma_5 U^{m'm}(-x) \right] \text{Tr} \left[i\gamma_5 S^{kk'}(x) \right. \\ &\quad \cdot i\gamma_5 U^{k'k}(-x) \left. \right] - \text{Tr} \left[\gamma_\mu D^{nm'}(x) \gamma_\nu C^{n'n}(-x) \right] \\ &\quad \cdot \text{Tr} \left[i\gamma_5 C^{mm'}(x) i\gamma_5 U^{m'k}(-x) i\gamma_5 S^{kk'}(x) \right. \\ &\quad \left. \cdot i\gamma_5 U^{k'm}(-x) \right] \left. \right\}, \end{aligned} \quad (4)$$

where the $U_{ij}(x)$, $D_{ij}(x)$, $S_{ij}(x)$, and $C_{ij}(x)$ are the full u , d , s , and c quark propagators, respectively. We give the full quark propagators explicitly in the following, (the $P_{ij}(x)$ denotes the $U_{ij}(x)$ or $D_{ij}(x)$),

$$\begin{aligned} P_{ij}(x) &= \frac{i\delta_{ij}\not{x}}{2\pi^2 x^4} - \frac{\delta_{ij}\langle \bar{q}q \rangle}{12} - \frac{\delta_{ij}x^2 \langle \bar{q}g_s\sigma Gq \rangle}{192} \\ &\quad - \frac{ig_s G_{\alpha\beta}^n t_{ij}^n (\not{x}\sigma^{\alpha\beta} + \sigma^{\alpha\beta}\not{x})}{32\pi^2 x^2} - \frac{1}{8} \langle \bar{q}_j \sigma^{\alpha\beta} q_i \rangle \sigma_{\alpha\beta} \\ &\quad + \dots, \end{aligned} \quad (5)$$

$$\begin{aligned} S_{ij}(x) &= \frac{i\delta_{ij}\not{x}}{2\pi^2 x^4} - \frac{\delta_{ij}m_s}{4\pi^2 x^2} - \frac{\delta_{ij}\langle \bar{s}s \rangle}{12} + \frac{i\delta_{ij}\not{x}m_s \langle \bar{s}s \rangle}{48} \\ &\quad - \frac{\delta_{ij}x^2 \langle \bar{s}g_s\sigma Gs \rangle}{192} + \frac{i\delta_{ij}x^2 \not{x}m_s \langle \bar{s}g_s\sigma Gs \rangle}{1152} \\ &\quad - \frac{ig_s G_{\alpha\beta}^n t_{ij}^n (\not{x}\sigma^{\alpha\beta} + \sigma^{\alpha\beta}\not{x})}{32\pi^2 x^2} - \frac{1}{8} \langle \bar{s}_j \sigma^{\alpha\beta} s_i \rangle \sigma_{\alpha\beta} + \dots, \end{aligned} \quad (6)$$

$$\begin{aligned} C_{ij}(x) &= \frac{i}{(2\pi)^4} \int d^4k e^{-ik \cdot x} \left\{ \frac{\not{k} + m_c}{k^2 - m_c^2} \delta_{ij} \right. \\ &\quad - g_s t_{ij}^n G_{\alpha\beta}^n \frac{(\not{k} + m_c) \sigma^{\alpha\beta} + \sigma^{\alpha\beta} (\not{k} + m_c)}{4(k^2 - m_c^2)^2} \\ &\quad - \frac{g_s^2 (t^n t^m)_{ij} G_{\alpha\beta}^n G_{\mu\nu}^n (f^{\alpha\beta\mu\nu} + f^{\alpha\mu\beta\nu} + f^{\alpha\mu\nu\beta})}{4(k^2 - m_c^2)^5} \\ &\quad \left. + \dots \right\}, \end{aligned} \quad (7)$$

$$f^{\lambda\alpha\beta} = (\not{k} + m_c) \gamma^\lambda (\not{k} + m_c) \gamma^\alpha (\not{k} + m_c) \gamma^\beta (\not{k} + m_c),$$

$$\begin{aligned} f^{\alpha\beta\mu\nu} &= (\not{k} + m_c) \gamma^\alpha (\not{k} + m_c) \gamma^\beta (\not{k} + m_c) \gamma^\mu (\not{k} + m_c) \\ &\quad \cdot \gamma^\nu (\not{k} + m_c), \end{aligned} \quad (8)$$

and $t^n = \lambda^n/2$; the λ^n is the Gell-Mann matrix [29]. We compute the integrals in the coordinate space for the light quark propagators and in the momentum space for the charm quark propagators and obtain the QCD spectral density $\rho(s)$ via taking the imaginary part of the correlation function: $\rho(s) = \lim_{\varepsilon \rightarrow 0} (\text{Im} \Pi(s+i\varepsilon)/\pi)$ [26]. In the operator product expansion, we take into account the contributions of vacuum condensates up to dimension-16 and keep the terms which are linear in the strange quark mass m_s . We take the truncation $k \leq 1$ for the operators of the order $\mathcal{O}(\alpha_s^k)$ in a consistent way and discard the perturbative corrections. Furthermore, the condensates $\langle \bar{q}q \rangle \langle \alpha_s GG/\pi \rangle$, $\langle \bar{q}q \rangle^2 \langle \alpha_s GG/\pi \rangle$, and $\langle \bar{q}q \rangle^3 \langle \alpha_s GG/\pi \rangle$ play a minor important role and are neglected.

According to the quark-hadron duality, we match the correlation function $\Pi(p^2)$ gotten on the hadron side and at the quark level below the continuum threshold s_0 and

perform Borel transform with respect to the variable $P^2 = -p^2$ to obtain the QCD sum rule:

$$\lambda_Z^2 \exp\left(-\frac{M_Z^2}{T^2}\right) = \int_{4m_c^2}^{s_0} ds \rho(s) \exp\left(-\frac{s}{T^2}\right), \quad (9)$$

where the QCD spectral density is

$$\begin{aligned} \rho(s) = & \rho_0(s) + \rho_3(s) + \rho_4(s) + \rho_5(s) + \rho_6(s) + \rho_8(s) \\ & + \rho_9(s) + \rho_{10}(s) + \rho_{11}(s) + \rho_{12}(s) + \rho_{13}(s) \\ & + \rho_{14}(s) + \rho_{16}(s), \end{aligned} \quad (10)$$

and the subscripts 0, 3, 4, 5, 6, 8, 9, 10, 11, 12, 13, 14, and 16 denote the dimensions of the vacuum condensates, the T^2 is the Borel parameter, and the lengthy and complicated expressions are neglected for simplicity. However, for the explicit expressions of the QCD special densities, the interested readers can obtain them through emailing us.

We derive (9) with respect to $1/T^2$ and eliminate the pole residue λ_Z to extract the QCD sum rule for the mass:

$$M_Z^2 = \frac{\int_{4m_c^2}^{s_0} ds (d/d(-1/T^2)) \rho(s) \exp(-s/T^2)}{\int_{4m_c^2}^{s_0} ds \rho(s) \exp(-s/T^2)}. \quad (11)$$

3. Numerical Results and Discussions

In this section, we perform the numerical analysis. To extract the numerical values of M_Z , we take the values of the vacuum condensates $\langle \bar{q}q \rangle = -(0.24 \pm 0.01 \text{ GeV})^3$, $\langle \bar{s}s \rangle = (0.8 \pm 0.1) \langle \bar{q}q \rangle$, $\langle \bar{q}g_s \sigma Gq \rangle = m_0^2 \langle \bar{q}q \rangle$, $\langle \bar{s}g_s \sigma Gs \rangle = m_0^2 \langle \bar{s}s \rangle$, $m_0^2 = (0.8 \pm 0.1) \text{ GeV}^2$, $\langle \alpha_s GG/\pi \rangle = (0.33 \text{ GeV})^4$ at the energy scale $\mu = 1 \text{ GeV}$ [28–30], choose the \overline{MS} masses $m_c(m_c) = (1.275 \pm 0.025) \text{ GeV}$, $m_s(\mu = 2 \text{ GeV}) = (0.095_{-0.003}^{+0.009}) \text{ GeV}$ from the Particle Data Group [2], and neglect the up and down quark masses, i.e., $m_u = m_d = 0$. Moreover, we consider the energy-scale dependence of the input parameters on the QCD side from the renormalization group equation,

$$\begin{aligned} \langle \bar{q}q \rangle(\mu) &= \langle \bar{q}q \rangle(1 \text{ GeV}) \left[\frac{\alpha_s(1 \text{ GeV})}{\alpha_s(\mu)} \right]^{4/9}, \\ \langle \bar{s}s \rangle(\mu) &= \langle \bar{s}s \rangle(1 \text{ GeV}) \left[\frac{\alpha_s(1 \text{ GeV})}{\alpha_s(\mu)} \right]^{4/9}, \\ \langle \bar{q}g_s \sigma Gq \rangle(\mu) &= \langle \bar{q}g_s \sigma Gq \rangle(1 \text{ GeV}) \left[\frac{\alpha_s(1 \text{ GeV})}{\alpha_s(\mu)} \right]^{2/27}, \\ \langle \bar{s}g_s \sigma Gs \rangle(\mu) &= \langle \bar{s}g_s \sigma Gs \rangle(1 \text{ GeV}) \left[\frac{\alpha_s(1 \text{ GeV})}{\alpha_s(\mu)} \right]^{2/27}, \\ m_s(\mu) &= m_s(2 \text{ GeV}) \left[\frac{\alpha_s(\mu)}{\alpha_s(2 \text{ GeV})} \right]^{4/9}, \end{aligned}$$

$$m_c(\mu) = m_c(m_c) \left[\frac{\alpha_s(\mu)}{\alpha_s(m_c)} \right]^{12/25},$$

$$\begin{aligned} \alpha_s(\mu) &= \frac{1}{b_0 t} \left[1 - \frac{b_1 \log t}{b_0^2 t} + \frac{b_1^2 (\log^2 t - \log t - 1) + b_0 b_2}{b_0^4 t^2} \right], \end{aligned} \quad (12)$$

where $t = \log(\mu^2/\Lambda^2)$, $b_0 = (33 - 2n_f)/12\pi$, $b_1 = (153 - 19n_f)/24\pi^2$, $b_2 = (2857 - (5033/9)n_f + (325/27)n_f^2)/128\pi^3$, $\Lambda = 213 \text{ MeV}$, 296 MeV and 339 MeV for the flavors $n_f = 5$, 4 and 3 , respectively [2].

For the hadron mass, it is independent of the energy scale because of its observability. However, in calculations, the perturbative corrections are neglected, the operators of the orders $\mathcal{O}_n(\alpha_s^k)$ with $k > 1$ or the dimensions $n > 16$ are discarded, and some higher dimensional vacuum condensates are factorized into lower dimensional ones; therefore, the corresponding energy-scale dependence is modified. We have to take into account the energy-scale dependence of the QCD sum rules.

In [26, 31–34], the energy-scale dependence of the QCD sum rules is studied in detail for the hidden-charm tetraquark states and molecular states, and an energy-scale formula $\mu = \sqrt{M_{X/Y/Z}^2 - (2\mathbb{M}_c)^2}$ is come up with to determine the optimal energy scale. This energy-scale formula enhances the pole contribution remarkably, improves the convergent behaviors in the operator product expansion, and works well for the exotic hadron states. In this article, we explore the DD^*K state Z through constructing the color singlet-singlet-singlet type current based on the color-singlet $q\bar{q}$ substructure. For the two-meson molecular states, the basic constituent is also the color-singlet $q\bar{q}$ substructure [33, 34]. Hence, the previous works can be extended to study the DD^*K state. We employ the energy-scale formula $\mu = \sqrt{M_{X/Y/Z}^2 - (2\mathbb{M}_c)^2}$ with the updated value of the effective c -quark mass $\mathbb{M}_c = 1.85 \text{ GeV}$ to take the ideal energy scale of the QCD spectral density.

At the present time, no candidate is observed experimentally for the hexaquark state Z with the symbolic quark constituent $c\bar{c}d\bar{u}s\bar{u}$. However, in the scenario of four-quark states, the $Z_c(3900)$ and $Z(4430)$ can be tentatively assigned to be the ground state and the first radial excited state of the axial vector four-quark states, respectively [35], while the $X(3915)$ and $X(4500)$ can be tentatively assigned to be the ground state and the first radial excited state of the scalar four-quark states, respectively [36, 37]. By comparison, the energy gap is about 0.6 GeV between the ground state and the first radial excited state of the hidden-charm four-quark states. Here, we suppose the energy gap is also about 0.6 GeV between the ground state and the first radial excited state of the hidden-charm six-quark states and take the relation $\sqrt{s_0} = M_Z + (0.4 - 0.6) \text{ GeV}$ as a constraint to obey.

In (11), there are two free parameters: the Borel parameter T^2 and the continuum threshold parameter s_0 . The extracted hadron mass is a function of the Borel parameter T^2 and the

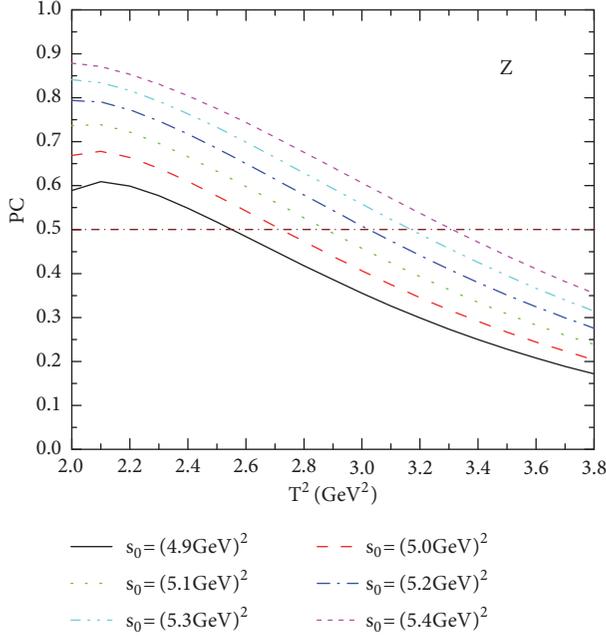


FIGURE 1: The pole contribution with variation of the Borel parameter T^2 .

continuum threshold parameter s_0 . To obtain a reliable mass sum rule analysis, we obey two criteria to choose suitable working ranges for the two free parameters. One criterion is the pole dominance on the phenomenological side, which requires the pole contribution (PC) to be about (40 – 60)%. The PC is defined as

$$\text{PC} = \frac{\int_{4m_c^2}^{s_0} ds \rho(s) \exp(-s/T^2)}{\int_{4m_c^2}^{\infty} ds \rho(s) \exp(-s/T^2)}. \quad (13)$$

The other criterion is the convergence of the operator product expansion. To judge the convergence, we compute the contributions of the vacuum condensates $D(n)$ in the operator product expansion with the formula:

$$D(n) = \frac{\int_{4m_c^2}^{s_0} ds \rho_n(s) \exp(-s/T^2)}{\int_{4m_c^2}^{s_0} ds \rho(s) \exp(-s/T^2)}, \quad (14)$$

where the n is the dimension of the vacuum condensates.

In Figure 1, we show the variation of the PC with respect to the Borel parameter T^2 for different values of the continuum threshold parameter s_0 at the energy scale $\mu = 2.9$ GeV. From the figure, we can see that the value $\sqrt{s_0} \leq 5.0$ GeV is too tiny to obey the pole dominance criterion and result in sound Borel window for the state Z . To warrant the Borel platform for the mass m_Z , we take the value $T^2 = (2.8 - 3.2)$ GeV². In the above Borel window, if we choose the value $\sqrt{s_0} = (5.1 - 5.3)$ GeV, the PC is about (39 – 63)%. The pole dominance condition is well satisfied.

In Figure 2, we draw the absolute contribution values of the vacuum condensates $|D(n)|$ at central values of the above input parameters. From the figure, we can observe

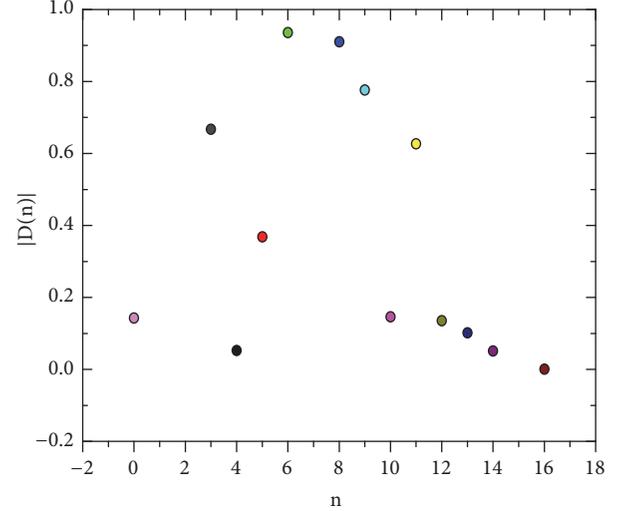


FIGURE 2: The absolute contributions of the vacuum condensates with dimension n in the operator product expansion.

that the contribution of the perturbative term $D(0)$ is not the dominant contribution; the contributions of the vacuum condensates with dimensions 3, 6, 8, 9, and 11 are very great. If we take the contribution of the vacuum condensate with dimension 11 as a milestone, the absolute contribution values of the vacuum condensates $|D(n)|$ decrease quickly with the increase of the dimensions n , and the operator product expansion converges nicely.

Thus, we obtain the values $T^2 = (2.8 - 3.2)$ GeV², $\sqrt{s_0} = (5.1 - 5.3)$ GeV and $\mu = 2.9$ GeV for the state Z . Considering all uncertainties of the input parameters, we get the values of the mass and pole residue of the state Z :

$$\begin{aligned} M_Z &= 4.71_{-0.11}^{+0.19} \text{ GeV}, \\ \lambda_Z &= (4.60_{-0.69}^{+1.15}) \times 10^{-4} \text{ GeV}^8, \end{aligned} \quad (15)$$

which are shown explicitly in Figures 3 and 4. Obviously, the energy-scale formula $\mu = \sqrt{M_{X/Y/Z}^2 - (2M_c)^2}$ and the relation $\sqrt{s_0} = M_Z + (0.4 - 0.6)$ GeV are also well satisfied. The central value $M_Z = 4.71$ GeV is about 337 MeV above the threshold $M_{K+D+\bar{D}^*} = 497.6 + 1865 + 2010 = 4372.6$ MeV, which indicates that the Z is probably a resonance state. For some exotic resonances, the authors have combined the effective range expansion, unitarity, analyticity, and compositeness coefficient to probe their inner structure in [38, 39]. Their studies indicated that the underlying two-particle component (in the present case, corresponding to three-particle component) plays an important or minor role; in other words, there are the other hadronic degrees of freedom inside the corresponding resonance. Hence, a resonance state embodies the net effect. Considering the conservation of the angular momentum, parity and isospin, we list out the possible hadronic decay patterns of the hexaquark state Z :

$$Z \longrightarrow J/\psi \pi K, \eta_c \rho(770) K, \bar{D}\bar{D}^* K. \quad (16)$$

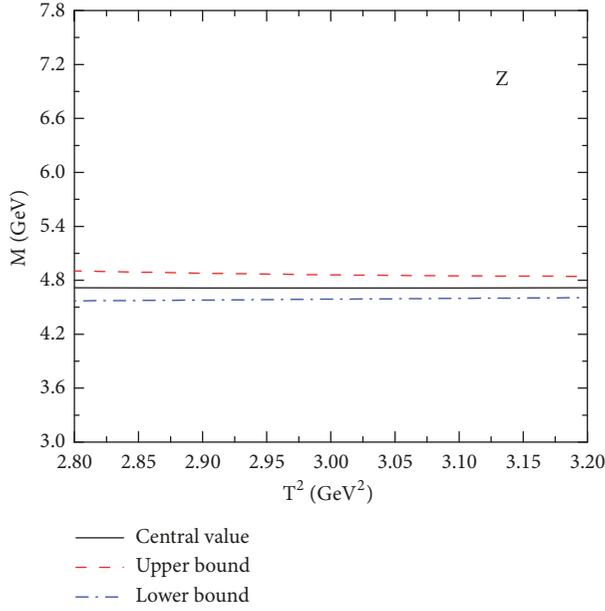


FIGURE 3: The mass with variation of the Borel parameter T^2 .

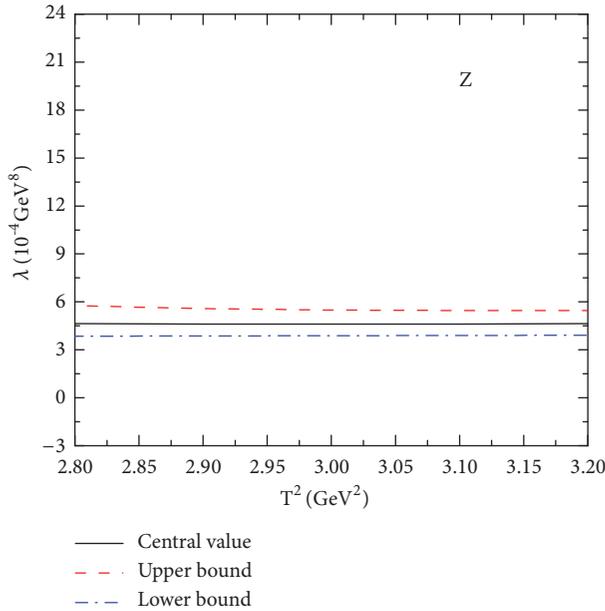


FIGURE 4: The pole residue with variation of the Borel parameters T^2 .

To search for the X(3872), Belle, BaBar, and LHCb have collected numerous data in the decay $B \rightarrow J/\psi\pi\pi K$. Thus, the hexaquark state Z may be found by focusing on the easiest channel $J/\psi\pi K$ in the experiment.

4. Conclusion

In this article, we construct the color singlet-singlet-singlet interpolating current operator with $I(J^P) = (3/2)(1^-)$ to study the $D\bar{D}^*K$ system through QCD sum rules approach by taking into account the contributions of the vacuum

condensates up to dimension-16 in the operator product expansion. In numerical calculations, we saturate the hadron side of the QCD sum rule with a hexaquark molecular state, employ the energy-scale formula $\mu = \sqrt{M_{X/Y/Z}^2 - (2M_c)^2}$ to take the optimal energy scale of the QCD spectral density, and seek the ideal Borel parameter T^2 and continuum threshold s_0 by obeying two criteria of QCD sum rules for multi-quark states. Finally, we obtain the mass and pole residue of the corresponding hexaquark molecular state Z. The predicted mass, $M_Z = 4.71_{-0.11}^{+0.19}$ GeV, which lies above the $D\bar{D}^*K$ threshold, indicates that the Z is probably a resonance state. This resonance state Z may be found by focusing on the channel $J/\psi\pi K$ of the decay $B \rightarrow J/\psi\pi\pi K$ in the future.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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References

- [1] S. K. Choi, "Observation of a narrow charmoniumlike state in exclusive $B^\pm \rightarrow K^\pm \pi^+ \pi^- J/\psi$ decays," *Physical Review Letters*, vol. 91, Article ID 262001, 2003.
- [2] K. A. Olive, "Review of particle physics," *Chinese Physics C*, vol. 38, Article ID 090001, 2014.
- [3] S. L. Olsen, T. Skwarnicki, and D. Zieminska, "Nonstandard heavy mesons and baryons: experimental evidence," *Reviews of Modern Physics*, vol. 90, no. 1, Article ID 15003, 2018.
- [4] N. Brambilla, "Heavy quarkonium: progress, puzzles, and opportunities," *European Physical Journal C*, vol. 71, Article ID 1534, 2011.
- [5] S. Godfrey and N. Isgur, "Mesons in a relativized quark model with chromodynamics," *Physical Review D: Particles, Fields, Gravitation and Cosmology*, vol. 32, no. 1, pp. 189–231, 1985.
- [6] H.-X. Chen, W. Chen, X. Liu, and S.-L. Zhu, "The hidden-charm pentaquark and tetraquark states," *Physics Reports*, vol. 639, p. 1, 2016.
- [7] J. He, " $\bar{D}\Sigma_c^*$ and $\bar{D}^*\Sigma_c$ interactions and the LHCb hidden-charmed pentaquarks," *Physics Letters B*, vol. 753, pp. 547–551, 2016.
- [8] F. K. Guo, C. Hanhart, U. G. Meißner, Q. Wang, Q. Zhao, and B. S. Zou, "Hadronic molecules," *Reviews of Modern Physics*, vol. 90, no. 1, Article ID 015004, 2018.
- [9] C. Y. Wong, "Molecular states of heavy quark mesons," *Physical Review C*, vol. 69, Article ID 055202, 2004.
- [10] E. S. Swanson, "Short range structure in the X(3872)," *Physics Letters B*, vol. 588, pp. 189–195, 2004.
- [11] M. Suzuki, "X(3872) boson: Molecule or charmonium," *Physical Review D*, vol. 72, Article ID 114013, 2005.

- [12] M. T. AlFiky, F. Gabbiani, and A. A. Petrov, “X(3872) Hadronic molecules in effective field theory,” *Physics Letters B*, vol. 640, p. 238, 2006.
- [13] S. Fleming, M. Kusunoki, T. Mehen, and U. van Kolck, “Pion interactions in the X(3872),” *Physical Review D*, vol. 76, Article ID 034006, 2007.
- [14] E. Braaten, M. Lu, and J. Lee, “Weakly-bound hadronic molecule near a 3-body threshold,” *Physical Review D*, vol. 76, Article ID 054010, 2007.
- [15] C. Hanhart, Y. S. Kalashnikova, A. E. Kudryavtsev, and A. V. Nefediev, “Reconciling the X(3872) with the near-threshold enhancement in the $D^0\bar{D}^{*0}$ final state,” *Physical Review D*, vol. 76, Article ID 034007, 2007.
- [16] M. B. Voloshin, “sospin properties of the X state near the $D\bar{D}^*$ threshold,” *Physical Review D*, vol. 76, Article ID 014007, 2007.
- [17] P. Colangelo, F. De Fazio, and S. Nicotri, “X(3872) $\rightarrow D\bar{D}^*\gamma$ decays and the structure of X(3872),” *Physics Letters B*, vol. 650, pp. 166–171, 2007.
- [18] L. Ma, Q. Wang, and U.-G. Meissner, “Double heavy tri-hadron bound state via delocalized π bond,” *Chinese Physics C*, vol. 43, Article ID 014012, 2019.
- [19] X. L. Ren, B. B. Malabarba, L. S. Geng, K. P. Khemchandani, and A. M. Torres, “ K^* mesons with hidden charm arising from $KX(3872)$ and $KZ_c(3900)$ dynamics,” *Physics Letters B*, vol. 785, pp. 112–117, 2018.
- [20] R. M. Albuquerque, K. P. Khemchandani, J. M. Dias et al., “QCD sum rules approach to the X, Y and Z states,” <https://arxiv.org/abs/1812.08207>.
- [21] W. Chen and S. L. Zhu, “The vector and axial-vector charmonium-like states,” *Physical Review D*, vol. 83, Article ID 034010, 2011.
- [22] W. Chen, T. G. Steele, H. X. Chen, and S. L. Zhu, “ $Z_c(4200)^+$ decay width as a charmonium-like tetraquark state,” *The European Physical Journal C*, vol. 75, p. 338, 2015.
- [23] J. R. Zhang and M. Q. Huang, “ $\{Q\bar{s}Q(\prime)s\}$ molecular states in QCD sum rules,” *Communications in Theoretical Physics*, vol. 54, pp. 1075–1090, 2010.
- [24] Z. Y. Di, Z. G. Wang, J. X. Zhang, and G. L. Yu, “Scalar hidden-charm tetraquark states with QCD sum rules,” *Communications in Theoretical Physics*, vol. 69, p. 191, 2018.
- [25] Z. G. Wang, “The lowest hidden charmed tetraquark state from QCD sum rules,” *Modern Physics Letters A*, vol. 29, Article ID 1450207, 2014.
- [26] Z. G. Wang and T. Huang, “Analysis of the X(3872), $Z_c(3900)$, and $Z_c(3885)$ as axial-vector tetraquark states with QCD sum rules,” *Physical Review D*, vol. 89, Article ID 054019, 2014.
- [27] Z. G. Wang and J. X. Zhang, “Possible pentaquark candidates: new excited Ω_c states,” *European Physical Journal C*, vol. 78, p. 14, 2018.
- [28] M. A. Shifman, A. I. Vainshtein, and V. I. Zakharov, “QCD and resonance physics. Applications,” *Nuclear Physics B*, vol. 147, no. 5, pp. 448–518, 1979.
- [29] L. J. Reinders, H. Rubinstein, and S. Yazaki, “Hadron properties from QCD sum rules,” *Physics Reports*, vol. 127, no. 1, p. 1, 1985.
- [30] P. Colangelo and A. Khodjamirian, *At the Frontier of Particle Physics: Handbook of QCD*, M. Shifman, Ed., vol. 3, World Scientific, Singapore, 2001.
- [31] Z. G. Wang, “Analysis of the $Z_c(4020)$, $Z_c(4025)$, Y(4360), and (Y4660) as vector tetraquark states with QCD sum rules,” *European Physical Journal C*, vol. 74, p. 2874, 2014.
- [32] Z. G. Wang and T. Huang, “The $Z_b(10610)$ and $Z_b(10650)$ as axial-vector tetraquark states in the QCD sum rules,” *Nuclear Physics A*, vol. 930, pp. 63–85, 2014.
- [33] Z. G. Wang and T. Huang, “Possible assignments of the X(3872), $Z_c(3900)$, and $Z_b(10610)$ as axial-vector molecular states,” *European Physical Journal C*, vol. 74, p. 2891, 2014.
- [34] Z. G. Wang, “Reanalysis of the Y(3940), Y(4140), $Z_c(4020)$, $Z_c(4025)$, and $Z_b(10650)$ as molecular states with QCD sum rules,” *European Physical Journal C*, vol. 74, p. 2963, 2014.
- [35] Z. G. Wang, “Analysis of the Z(4430) as the First Radial Excitation of the $Z_c(3900)$,” *Communications in Theoretical Physics*, vol. 63, p. 325, 2015.
- [36] Z. G. Wang, “Scalar tetraquark state candidates: X(3915), X(4500) and X(4700),” *European Physical Journal C*, vol. 77, p. 78, 2017.
- [37] Z. G. Wang, “Reanalysis of the X(3915), X(4500) and X(4700) with QCD sum rules,” *European Physical Journal A*, vol. 53, p. 19, 2017.
- [38] X. W. Kang, Z. H. Guo, and J. A. Oller, “General considerations on the nature of $Z_b(10610)$ and $Z_b(10650)$ from their pole positions,” *Physical Review D*, vol. 94, Article ID 014012, 2016.
- [39] R. Gao, Z. H. Guo, X. W. Kang, and J. A. Oller, “Effective-range-expansion study of near threshold heavy-flavor resonances,” <https://arxiv.org/abs/1812.07323>.