



$\Omega(2012)$ through the looking glass of flavour $SU(3)$

Maxim V. Polyakov^{a,b}, Hyeon-Dong Son^{a,*}, Bao-Dong Sun^{c,d}, Asli Tandogan^a

^a Ruhr-Universität Bochum, Fakultät für Physik und Astronomie, Institut für Theoretische Physik II, D-44780 Bochum, Germany

^b Petersburg Nuclear Physics Institute, Gatchina, 188300, St. Petersburg, Russia

^c Institute of High Energy Physics, Chinese Academy of Sciences, Beijing 100049, PR China

^d School of Physics, University of Chinese Academy of Sciences, Beijing 100049, PR China

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ABSTRACT

We perform the flavour $SU(3)$ analysis of the recently discovered $\Omega(2012)$ hyperon. We find that well known (four star) $\Delta(1700)$ resonance with quantum numbers of $J^P = 3/2^-$ is a good candidate for the decuplet partner of $\Omega(2012)$ if the branching for the three-body decays of the latter is not too large $\leq 70\%$. That implies that the quantum numbers of $\Omega(2012)$ are $I(J^P) = 0(3/2^-)$. The predictions for the properties of still missing Σ and Ξ decuplet members are made. We also discuss the implications of the $\bar{K}\Xi(1530)$ molecular picture of $\Omega(2012)$. In the molecular picture, $\Omega(2012)$ state can have quantum numbers of either $I(J^P) = 0(3/2^-)$ or $I(J^P) = 1(3/2^-)$. Crucial experimental tests to distinguish various pictures of $\Omega(2012)$ are suggested.

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1. Introduction

An Ω hyperon is a baryon with strangeness $S = -3$. It is a good marker of the flavour $SU(3)$ decuplet, if its isospin is zero, or of the exotic 27-plet, if its isospin is one. The Δ partner is another unique marker for the decuplet. If the Ω is in the decuplet then it must be accompanied by the Ξ , Σ and Δ flavour partners, see Fig. 1.

Moreover, if we know the masses of Δ and Ω from the decuplet, we can predict masses of the Σ and Ξ partner using the approximate $SU(3)$ symmetry. Additionally, the $SU(3)$ symmetry allows to predict the partial decay widths of all partners if we know one of decuplet partners, e.g. decays of Ω .

The recently discovered $\Omega(2012)$ hyperon by the BELLE collaboration has a mass and width [1]:

$$M = 2012.4 \pm 0.7(\text{stat}) \pm 0.6(\text{syst}) \text{ MeV},$$

$$\Gamma_{\text{tot}} = 6.4_{-2.0}^{+2.5}(\text{stat}) \pm 1.6(\text{syst}) \text{ MeV}, \quad (1)$$

neither the spin-parity nor the isospin are measured.

* Corresponding author.

E-mail addresses: maxim.polyakov@tp2.ruhr-uni-bochum.de (M.V. Polyakov), hyeon-dong.son@tp2.ruhr-uni-bochum.de (H.-D. Son), sunbd@ihep.ac.cn (B.-D. Sun), Asli.TandoganKunkel@ruhr-uni-bochum.de (A. Tandogan).

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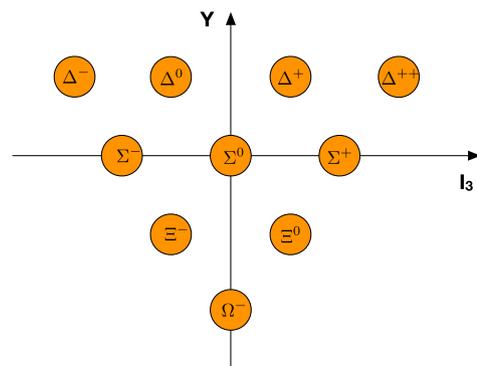


Fig. 1. $SU(3)$ decuplet.

Our aim here is to identify the Δ partner for the recently discovered $\Omega(2012)$. The information about the properties of the Δ resonance, e.g. mass, partial widths and quantum numbers, are obtained from well studied πN scattering. Therefore, using the approximate flavour $SU(3)$ symmetry we can check which one of the known Δ resonances belongs to the same decuplet as $\Omega(2012)$. Finding the Δ partner, we can establish the quantum numbers of $\Omega(2012)$ and make predictions for the properties of the other, Σ and Ξ , partners of the corresponding decuplet. Additionally, as

the properties of the new Ω hyperon are well measured, we can narrow down the uncertainties in the properties of its Δ partner.

Another possible interpretation of the $\Omega(2012)$ is the $\bar{K}\Xi(1530)$ bound state. In this molecular picture the isospin of $\Omega(2012)$ can be either zero (decuplet) or one (27-plet). We discuss below the predictions for such picture.

2. Identification of the Δ decuplet partner for $\Omega(2012)$

To make an identification of the Δ partner of $\Omega(2012)$ we use the flavour $SU(3)$ formalism described in details in reviews [2,3]. According to Gell-Mann–Okubo mass formula, the mass splitting in the decuplet is equidistant with typical mass splitting of order 100–150 MeV. Therefore, we can expect the Δ partner of $\Omega(2012)$ in the mass range of 1500–1700 MeV. In this mass range, Particle Data Group (PDG) [4] reports three Δ resonances; the $\Delta(1600)$ with $J^P = 3/2^+$, the $\Delta(1620)$ with $J^P = 1/2^-$, and the $\Delta(1700)$ with $J^P = 3/2^-$. In the most recent global $SU(3)$ analysis, [3], the Ξ and Ω decuplet partners of these Δ 's are missing.¹ For the $\Omega(2012)$, we can repeat the analysis to see which decuplet it may belong to.

There are only three open strong decay channels, $\Xi\bar{K}$, $\Xi\bar{K}\pi$ and $\Omega^-\pi\pi^2$ for the $\Omega(2012)$ from the decuplet, so we can use the sum of three-body decay branching ratios:

$$b = 1 - \frac{\Gamma(\Omega(2012) \rightarrow \Xi\bar{K})}{\Gamma_{\text{tot}}}, \quad 0 \leq b \leq 1 \quad (2)$$

as a single free parameter and calculate the two particle decay of $\Omega(2012)$ as

$$\Gamma(\Omega(2012) \rightarrow \Xi\bar{K}) = (1 - b) \cdot \Gamma_{\text{tot}}, \quad (3)$$

where the experimental total width is given by Eq. (1). By knowing the partial decay width of $\Omega(2012)$ we can now make predictions for the partial decay widths of Δ decuplet partner and compare them with the experimental data on the Δ resonances depending on the free parameter b .

For the partial decay widths of the decay $B_1 \rightarrow B_2 M$ (M is a pseudoscalar meson of the mass m) we shall use two different formulae which differ by $SU(3)$ symmetry-violating corrections of order $O(m_s)$. In this way we can estimate the systematic uncertainties of our $SU(3)$ analysis. The first formula used in the global $SU(3)$ analyses of Refs. [2,3] is:

$$\Gamma(B_1 \rightarrow B_2 M) = \frac{g_{B_1 B_2 M}^2}{8\pi} \left(\frac{k}{M_0}\right)^{2J+\mathcal{N}} \left(\frac{k}{M_1}\right) M_0. \quad (4)$$

The second formula used was obtained in Ref. [5]. It is in the framework of effective chiral Lagrangian for baryon resonances of any spin, and defined as:

$$\Gamma(B_1 \rightarrow B_2 M) = \frac{g_{B_1 B_2 M}^2}{8\pi} \left(\frac{k}{M_0}\right)^{2J-1} \left(\frac{k}{M_1}\right) \frac{[(M_1 - \mathcal{N}M_2)^2 - m^2]}{M_1}. \quad (5)$$

In both formulae J is the spin of decaying baryon, $\mathcal{N} = P(-1)^{J-1/2}$ is its normality [5,6], P is the parity of the resonance. M_0 is the

¹ The Σ partner was clearly identified in Ref. [3] only for $\Delta(1620)$, it is $\Sigma(1750)$ with $J^P = 1/2^-$.

² For the isovector $\Omega(2012)$ from the 27-plet an additional strong decay $\pi\Omega^-$ channel opens.

Table 1
 $SU(3)$ relations for coupling constants $g_{B_1 B_2 M}$ for $\mathbf{10} \rightarrow \mathbf{8} + \mathbf{8}$ and $\mathbf{10} \rightarrow \mathbf{10} + \mathbf{8}$ transitions.

$\mathbf{10}$	Decay mode	$\rightarrow \mathbf{8} + \mathbf{8}$	$\rightarrow \mathbf{10} + \mathbf{8}$
Δ	$\rightarrow N\pi$	$-\sqrt{2}/2 A_{10}$	
	$\rightarrow \Sigma K$	$\sqrt{2}/2 A_{10}$	
	$\rightarrow \Delta \pi$		$\sqrt{10}/4 A'_{10}$
	$\rightarrow \Delta \eta$		$-\sqrt{2}/4 A'_{10}$
	$\rightarrow \Sigma^* K$		$1/2 A'_{10}$
Σ	$\rightarrow \Lambda \pi$	$-1/2 A_{10}$	
	$\rightarrow \Sigma \pi, \Xi K$	$\sqrt{6}/6 A_{10}$	
	$\rightarrow N \bar{K}$	$-\sqrt{6}/6 A_{10}$	
	$\rightarrow \Sigma \eta$	$1/2 A_{10}$	
	$\rightarrow \Sigma^* \pi, \Xi^* K, \Delta \bar{K}$		$\sqrt{3}/6 A'_{10}$
	$\rightarrow \Sigma^* \eta$		0
Ξ	$\rightarrow \Xi \pi, \Xi \eta, \Sigma \bar{K}$	$1/2 A_{10}$	
	$\rightarrow \Lambda \bar{K}$	$-1/2 A_{10}$	
	$\rightarrow \Xi^* \pi, \Xi^* \eta$		$\sqrt{2}/4 A'_{10}$
	$\rightarrow \Sigma^* \bar{K}$		$\sqrt{2}/2 A'_{10}$
	$\rightarrow \Omega K$		$1/2 A'_{10}$
Ω	$\rightarrow \Xi \bar{K}$	A_{10}	
	$\rightarrow \Xi^* \bar{K}, \Omega \eta$		$\sqrt{2}/2 A'_{10}$

mass constant parameter characterising details of the baryon dynamics,³ its precise value is irrelevant for our analysis and we, following [2], choose $M_0 = 1$ GeV. Eventually k is the c.m.s. momentum

$$k = \frac{\sqrt{(M_1^2 - (M_2 + m)^2)(M_1^2 - (M_2 - m)^2)}}{2M_1}. \quad (6)$$

The coupling constants $g_{B_1 B_2 M}$ for the various decays $\mathbf{10} \rightarrow \mathbf{8} + \mathbf{8}$ are related to each other by $SU(3)$ Clebsch-Gordan coefficients listed in Table 1.

From Table 1 we see that the partial decay $\mathbf{10} \rightarrow \mathbf{8} + \mathbf{8}$ widths for all members of the decuplet are fixed in terms of one constant A_{10} . This constant can be constrained by the experimental values for mass and the width of $\Omega(2012)$ by assuming various quantum numbers of $\Omega(2012)$. By doing this we predict that the $\Delta(1600)$ with $J^P = 3/2^+$ and the $\Delta(1620)$ with $J^P = 1/2^-$ are excluded as decuplet partners of the $\Omega(2012)$ because, from $SU(3)$ relations, the resulting partial decay widths of Δ 's are much smaller than the ones listed in PDG [4], even for the value of parameter $b = 0$.

For $\Delta(1700)$ with $J^P = 3/2^-$ we obtain that its πN partial decay width is in good agreement with the width from the well measured width of $\Omega(2012)$ if the parameter b (see Eq. (2)) is not too large. This is illustrated on Fig. 2 where we compare our $SU(3)$ prediction, for the πN partial decay width of $\Delta(1700)$, with results of various PWA [7–11] used by PDG in their estimations of the $\Delta(1700)$ properties.

In Fig. 2, we plot the predicted πN partial width of $\Delta(1700)$ for two values of the parameter $b = 0$ (left) and $b = 0.7$ (right) by the $SU(3)$ analysis. The bands are originated from the experimental error in Eq. (1) and two bands in different colours in each plot show our systematic uncertainties arising from the use of two different formulae for the partial decay width, Eq. (4) and Eq. (5).

Additionally our analysis allows to narrow down the πN partial width and mass of the $\Delta(1700)$ giving a slight preference to analyses of Cutkosky et al. [7] and Hoehler et al. [9], while disfavouring slightly the PWA of Sokhoyan et al. [11] and Arndt et al. [10].

From Fig. 2 we see that if the parameter $b \geq 0.7$, then the data on $\Delta(1700)$ decays are not compatible with its interpretation as

³ For example, $M_0 = f_\pi$ in the context of the effective chiral Lagrangian approach of Ref. [5].

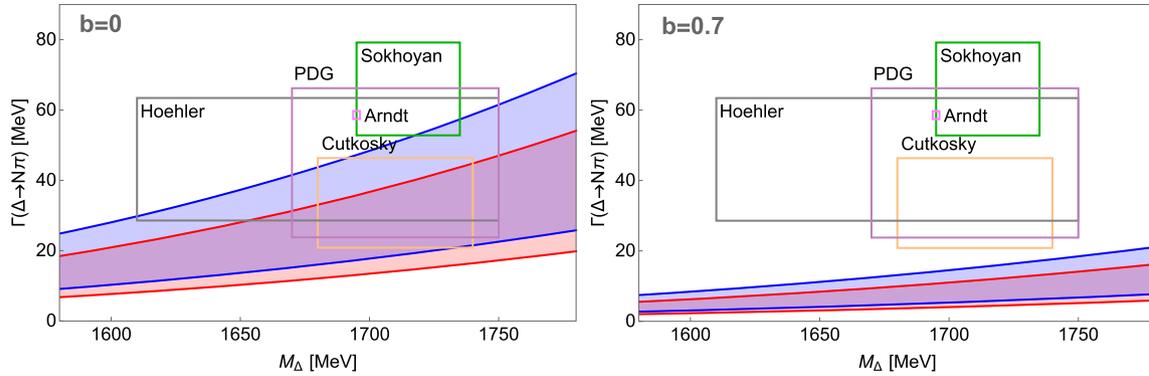


Fig. 2. $\Gamma(\Delta \rightarrow \pi N)$ versus M_Δ . Red and blue bands are our prediction of the decay width $\Gamma(\Delta \rightarrow \pi N)$ based on Eq. (4) and Eq. (5), respectively. Left plot corresponds to the parameter $b=0$, Eq. (2) and right one to $b=0.7$. The rectangles show the PWA analyses [7–11] results and PDG estimations [4].

the decuplet partner of $\Omega(2012)$. In this case, one of the interpretations of the $\Omega(2012)$ hyperon can be a $\bar{K}\Xi(1530)$ bound state with quantum numbers $J^P = 3/2^-$ (S -wave bound state). In these two interpretations, we obtain that the quantum numbers of the recently discovered $\Omega(2012)$ are $J^P = 3/2^-$.

The excited Ω hyperon with the mass around 2000 MeV and quantum numbers $I(J^P) = 0(3/2^-)$ was obtained in different models before the discovery of $\Omega(2012)$; in the quark model [12], in the lattice simulations [13] and in the Skyrme model [14]. We note, however, that in the quark model calculations of Ref. [12], the equidistant mass splitting between the decuplet partners is strongly violated. The lattice calculations of Ref. [13] give rather large mass splitting in the discussed decuplet of $\delta_{10} = 155 \pm 50$ MeV, although compatible with the result of our analysis $\delta_{10} = 104 \pm 15$ MeV within the error bars.

After its discovery the $\Omega(2012)$ hyperon was considered in the framework of chiral quark model Ref. [15] and in the framework of QCD sum rules in Ref. [16]. In both studies, the authors obtained $J^P = 3/2^-$ quantum numbers as well. The decay of $\Omega(2012)$ in the chiral quark model is dominated by $\Xi\bar{K}$ mode, so the $\bar{K}\Xi(1530)$ molecular scenario is excluded in chiral quark model. Therefore, the chiral quark model of Ref. [15] can be tested, i.e. it should predict $J^P = 3/2^-$ Δ with the decay properties as shown in Fig. 2. Unfortunately, $SU(3)$ partners of the $\Omega(2012)$ hyperon were not discussed in Refs. [15,16].

3. Properties of new $J^P = 3/2^-$ decuplet of baryons

We obtain that if the parameter b of Eq. (2) is smaller than 0.7, the $\Omega(2012)$ hyperon and $\Delta(1700)$ are good candidates for the members of the same decuplet. The mass splitting δ_{10} in the decuplet is equidistant and is $\delta_{10} = 104 \pm 15$ MeV, where the error bar is dominated by the uncertainty of the $\Delta(1700)$ mass. Using this value we can predict properties of other members of the decuplet, i.e. Σ and Ξ hyperons. Their quantum numbers are $J^P = 3/2^-$ and masses $M_\Sigma = 1805 \pm 40$ MeV and $M_\Xi = 1910 \pm 40$ MeV.

In principle, the masses of the Σ and Ξ decuplet members can be shifted from the equidistant rule due to the mixing with nearby members of $J^P = 3/2^-$ octet. However, according to the analyses of Refs. [2,3] there are no such nearby states, so we expect the Gell-Mann–Okubo mass formula should work well for the new decuplet.

There are no candidates for suggested Σ and Ξ decuplet states in PDG, so their existence is our prediction. Our predictions for the partial decay widths for Σ and Ξ members of the new decuplet are shown in Table 2. In this table we note that the Ξ of the discussed decuplet can be identified with known $\Xi(1950)$. However,

Table 2

Partial decay width predictions for the missing members of the new $J^P = 3/2^-$ decuplet assuming $b=0$ (see Eq. (2)). The predictions for other values of b can be obtained by rescaling above numbers by $(1-b)$. Masses of these states are predicted in the range $M_\Sigma = 1805 \pm 40$ MeV and $M_\Xi = 1910 \pm 40$ MeV. The larger value of decay widths correspond to the larger mass.

	Decay mode	$\Gamma_{\text{Eq. (4)}} [\text{MeV}]$	$\Gamma_{\text{Eq. (5)}} [\text{MeV}]$
Δ	$\rightarrow N\pi$	29–44	39–58
	$\rightarrow \Sigma K$	0–0.2	0–2
Σ	$\rightarrow \Lambda\pi$	10–16	11–18
	$\rightarrow \Sigma\pi$	4–7	4–7
	$\rightarrow N\bar{K}$	5–10	7–12
	$\rightarrow \Sigma\eta$	0.01–0.5	0.01–0.5
Ξ	$\rightarrow \Xi\pi$	5–9	5–9
	$\rightarrow \Sigma\bar{K}$	2–5	2–5
	$\rightarrow \Lambda\bar{K}$	5–9	5–10
	$\rightarrow \Xi\eta$	0–0.3	0–0.3
Ω	$\rightarrow \Xi\bar{K}$	Input	Input

in Ref. [3] it was shown that the $\Xi(1950)$ is fitted very well into the octet of baryons with $J^P = 5/2^-$.

In Table 2 we gave predictions for the partial decay width using two different formulae Eq. (4) and Eq. (5) to illustrate the systematic uncertainties inherent to our $SU(3)$ analysis.

We note that in Ref. [3], it was assumed that the hyperon masses have the equal-spacing $\Delta m = 150$ MeV for the multiplet including $\Delta(1700)$ since there were no experimentally found candidates at that moment. Hence, the assumption led rather larger masses of the decuplet members and thus larger decay widths.

4. On $\Omega(2012)$ as a $\bar{K}\Xi(1530)$ molecule

The mass of $\Omega(2012)$ is slightly below of the $\bar{K}\Xi(1530)$ threshold, so it could be the corresponding molecular state. If the $\bar{K}\Xi(1530)$ bound state is in S -wave the quantum numbers of the molecule are $J^P = 3/2^-$, its isospin can be either zero (decuplet) or one (27-plet).

We stress that the flavour $SU(3)$ formalism (Gell-Mann–Okubo mass formulae, relations between widths, etc.) does not work for the molecular states – the corresponding $SU(3)$ breaking corrections are of order $\sim m_s/(\text{binding energy})$ (not $\sim m_s/(\text{hadron mass})$) as for genuine resonances) and hence can be very large. For example, the deuteron nucleus belongs to the $SU(3)$ anti-decuplet of $B=2$ hypernuclei, but the corrections to the Gell-Mann–Okubo formula are large enough such that one cannot make precise predictions for the corresponding partner hypernuclei. The best one can do from the $SU(3)$ analysis is to identify the most attractive channels in hyperon-hyperon potential.

To obtain the $\Omega(2012)$ as the $\bar{K}\Xi(1530)$ bound state, one needs an attraction in the S -wave. Such an attraction can be provided by the Weinberg-Tomozawa (WT) term of the effective chiral Lagrangian. Detailed studies of that were performed in Refs. [17, 18]. The authors of Ref. [17] found an isospin zero (decuplet) $\bar{K}\Xi(1530)$ bound state with the mass of 1950 MeV. Probably, tuning the model parameters, one can adjust the mass of the state to ~ 2000 MeV. However, the channel with isospin one (27-plet) was not studied quantitatively in Ref. [17].

The studies, very similar to Ref. [17] in methods, but using different to Ref. [17] subtractions, were performed in Ref. [18]. The authors came to different results for the isospin zero channel with $S = -3$ (decuplet). In Ref. [18] it was obtained that the bound $\bar{K}\Xi(1530)$ state does not exist in this channel, instead the two coupled $\bar{K}\Xi(1530)$ and $\eta\Omega^-$ states with the pole position at $2141 - i38$ MeV were found. This pole is far away from $\Omega(2012)$. Again the studies of isospin one (27-plet) channel were not performed. The acute difference of the results for the $S = -3, I = 0$ channel of Ref. [17] and Ref. [18] calls for clarification. Also, potentially very interesting $S = -3, I = 1$ channel should be studied.

On general grounds we can estimate that, if a $\bar{K}\Xi(1530)$ bound state is formed in the isospin zero channel (decuplet), its main decay mode is $\Xi\bar{K}\pi$ with the corresponding partial width of ~ 10 MeV (order of the $\Xi(1530)$ width). Our qualitative conclusion, stating that the dominant decay mode of $\bar{K}\Xi(1530)$ bound state is $\Xi\bar{K}\pi$, is recently confirmed by the authors of Ref. [19,20] by model calculations in the effective field theory. However, they obtained for the corresponding partial decay width the value considerably smaller than our estimate of ~ 10 MeV. They attributed this difference to the binding energy of the molecule as well as to the kinetic energy of K^- inside the molecule. We note, however, that the naive phase space reduction of the width due to binding energy frequently cancels with the final state interactions, see examples of such cancellations for muon atoms in Refs. [21,22]. It could be that in the case of the $\bar{K}\Xi(1530)$ bound state similar cancellation happens, we shall study this elsewhere.

As we discussed above in the molecular scenario, the $\Delta(1700)$ can not be the partner of $\Omega(2012)$ and in this case one should expect one additional excited Ω hyperon with $J^P = 3/2^-$ in mass region of 2000–2150 MeV (see discussion in [3]).

The simplest experimental way to figure out the nature of $\Omega(2012)$ (genuine resonance or $\bar{K}\Xi(1530)$ bound state) is to measure its branching for the $\Xi\bar{K}\pi$ decay mode and to search for its possible charge partners.

5. Summary and outlook

We have found that the recently detected, by the BELLE collaboration, $\Omega(2012)$ hyperon and the well known $\Delta(1700)$ resonance can belong to the same $SU(3)$ decuplet of baryons with $J^P = 3/2^-$. Mass splitting in the new decuplet is determined with good accuracy $\delta_{10} = 104 \pm 15$ MeV, which can be used as a benchmark for various models of baryons. Properties such as masses and decay widths of the still missing Σ and Ξ members of the decuplet are predicted in details (see Table 2), these predictions can guide experiments on hyperon spectroscopy.

Our identification of the decuplet is correct only if the sum of branching ratios for the decays $\Omega(2012) \rightarrow \Xi\bar{K}\pi, \Omega^-\pi\pi$ is not too large ($\leq 70\%$).

For the large branching ratios of $\Omega(2012) \rightarrow \Xi\bar{K}\pi, \Omega^-\pi\pi$ decay modes, the most probable interpretation of the Ω hyperon is the S -wave $\bar{K}\Xi(1530)$ bound state with quantum numbers also $J^P = 3/2^-$. We note that the $\bar{K}\Xi(1530)$ bound state can have isospin one and hence it has an additional decay mode $\pi\Omega^-$. In a molecular picture, estimated width of $\Omega(2012)$ hyperon is

~ 10 MeV for isospin zero (decuplet), and considerably larger for the isospin one (27-plet). It seems that the latter option is incompatible with the small experimental width of $\Omega(2012)$. A measurement of the three-body decay branchings of new $\Omega(2012)$ and search for the charge partners of it are crucial tests of the nature of this hyperon.

We found also interesting that the studies of the strange hyperon properties can be a big help to constrain considerably the (frequently large) uncertainties of the classical PWA of πN processes. It is not surprising – mathematically PWA belongs to the class of so-called ill-posed (in Hadamard's sense [23]) problems and the $SU(3)$ relations can provide a regularisation of the ill-posed problem (see e.g. [24]).

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