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Scattering of the Kalb-Ramond state from a dynamical Dp-brane with background fields

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Abstract

We apply the boundary state method and operator formalism to obtain tree-level scattering amplitude of the Kalb-Ramond state from a Dp-brane. The brane has a tangential dynamics, and it has been dressed by the antisymmetric tensor field, a $U(1)$ internal gauge potential and an open string tachyon field. By using the scattering amplitudes we acquire two DBI-like actions corresponding to the target branes. Our calculations are in the framework of the bosonic string theory.

1. Introduction

There are various properties of the strings and D-branes, such as the non-perturbative string theory, the D-brane thickness and extension of the D-brane effective action, that can be revealed via the following scattering processes: brane-brane, string-string and string-brane. In fact, the brane-brane scattering is drastically complicated, because it is entirely non-perturbative phenomenon [1]. However, development of string theory at a non-perturbative level has shown that scattering amplitude not only is related to the string-string collisions [2], but also it comprises the scattering of closed strings from D-branes [3–15]. This demonstrates that the string-brane scattering provides a remarkable insight on the non-perturbative string theory. Besides, one of

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the main tools to extract some terms of the D-brane effective action is scattering amplitude of a string from that D-brane [5–9]. In addition, the string-brane scattering conveniently gives the key for understanding some essential phenomena such as the elucidation of the sizes of the branes and strings.

On the other hand we have boundary states which represent the D-branes. A boundary state, which is a closed string state, accurately encodes all physical properties of the corresponding D-brane. This state elaborates that a D-brane appears as a source (sink) for emission (absorption) of any closed string state. Thus, this adequate formalism has been widely applied for various setups of the D-branes [16–33], and scattering of strings from the D-branes [1–15].

In this paper we shall investigate the elastic scattering of a specific massless closed string state, i.e. the Kalb-Ramond state, from a single Dp-brane. The brane has tangential rotation and linear motion, and has been furnished with a $U(1)$ gauge potential, an antisymmetric tensor field and an open string tachyon field. For this purpose we shall apply the string operator formalism and boundary state method in the framework of the bosonic string theory. The scattering amplitudes enable us to extract two DBI-like actions which are corresponding to the stable and unstable dynamical target branes.

In fact, each background field and the brane dynamics effectively impose a potential to the incoming and outgoing string states, and hence the scattering process is completely influenced by them. Therefore, the background fields and the brane dynamics motivated and stimulated us to compute scattering of a closed string state from a dressed-dynamical Dp-branes. In other words, simultaneous application of the background fields and tangential dynamics prominently enriches the parametric structure of the scattering process and enables us to adjust the strength of the amplitude. Note that at least one of the background fields should be introduced for breaking the Lorentz symmetry inside the brane worldvolume to receive a meaningful tangential dynamics. Besides, comparison of the scatterings from stable and unstable dynamical branes motivated us to introduce a tachyonic background too. In addition, for preserving the conformal symmetry the Kalb-Ramond state has been chosen as incident and scattered states. Furthermore, since this state is massless and has antisymmetric polarization tensor it simplifies the scattering equations. Finally, obtaining effective actions for the dressed-dynamical stable and unstable target branes completes our motivation for choosing the foregoing setup.

This paper is organized as follows. In Sec. 2, we introduce a boundary state which is associated to a non-stationary Dp-brane with background fields. In Sec. 3, we give a brief review of the operator formalism for scattering of closed strings from a Dp-brane. Then, we calculate the scattering amplitude of the Kalb-Ramond state from our Dp-brane. In Sec. 4, we obtain two DBI-like actions via the scattering amplitudes. Section 5 is devoted to the conclusions.

2. Boundary state of a dynamical-dressed Dp-brane

For computing the boundary state corresponding to a rotating-moving Dp-brane with the internal and background fields we begin with the following action for closed string

$$
\begin{aligned}
S = &- \frac{1}{4\pi\alpha'} \int_{\Sigma} d^2\sigma \left(\sqrt{-g} g^{ab} G_{\mu\nu} \partial_a X^\mu \partial_b X^\nu + \epsilon^{ab} B_{\mu\nu} \partial_a X^\mu \partial_b X^\nu \right) \\
&+ \frac{1}{2\pi\alpha'} \int_{\partial\Sigma} d\sigma \left(A_\alpha \partial_\sigma X^\alpha + \omega_{\alpha\beta} J_\tau^{\alpha\beta} + T^2 \left(X^\alpha \right) \right) ,
\end{aligned}
\tag{2.1}
$$

where the set $\{x^\alpha|\alpha = 0, 1, \ldots, p\}$ specifies the directions along the Dp-brane worldvolume. Σ and $\partial\Sigma$ indicate the worldsheet of the closed string and its boundary, respectively. g_{ab} and $G_{\mu\nu}$ are the metrics of the worldsheet and spacetime. We consider flat spacetime with the metric $G_{\mu\nu} = \eta_{\mu\nu} = \mathrm{diag}(-1, 1, \ldots, 1)$. For the $U(1)$ gauge field, which lives in the brane worldvolume, we utilize the gauge $A_\alpha = -\frac{1}{2}F_{\alpha\beta}X^\beta$ with the constant field strength $F_{\alpha\beta}$. We apply a constant Kalb-Ramond field $B_{\mu\nu}$, and the tachyon profile is chosen as $T^2(X) = \frac{1}{2}U_{\alpha\beta}X^\alpha X^\beta$ where the matrix $U_{\alpha\beta}$ is symmetric and constant [34], [35]. The tangential linear motion and rotation of the brane are given by the antisymmetric constant angular velocity $\omega_{\alpha\beta}$, and the angular momentum density is $J_\tau^{\alpha\beta} = X^\alpha\partial_\tau X^\beta - X^\beta\partial_\tau X^\alpha$. Note that because of the internal and background fields the Lorentz symmetry in the worldvolume of the brane has been explicitly lost. Hence, this tangential dynamics is meaningful.

Vanishing the variation of the action defines the equation of motion and the following equations for the boundary state

$$
\begin{aligned}
&\left(\mathcal{K}_{\alpha\beta}\partial_\tau X^\beta + \mathcal{F}_{\alpha\beta}\partial_\sigma X^\beta + B_{\alpha i}\partial_\sigma X^i + U_{\alpha\beta}X^\beta\right)_{\tau=0}|B_x\rangle = 0\,,\\
&\left(X^i - y^i\right)_{\tau=0}|B_x\rangle = 0\,,
\end{aligned}
\tag{2.2}
$$

where $\mathcal{K}_{\alpha\beta} = \eta_{\alpha\beta} + 4\omega_{\alpha\beta}$, and $\mathcal{F}_{\alpha\beta} = B_{\alpha\beta} - F_{\alpha\beta}$ is the total field strength. The set $\{x^i|i = p+1, \ldots, 25\}$ represents the vertical directions to the brane worldvolume, and the parameters $\{y^i|i = p+1, \ldots, 25\}$ exhibit the brane location.

Introducing the solution of the equation of motion $(\partial_\tau^2 - \partial_\sigma^2)X^\mu(\sigma,\tau) = 0$, i.e.,

$$
X^\mu(\sigma,\tau) = x^\mu + 2\alpha' p^\mu\tau + \frac{i}{2}\sqrt{2\alpha'}\sum_{m\neq 0}\frac{1}{m}\left(\alpha_m^\mu e^{-2im(\tau-\sigma)} + \tilde{\alpha}_m^\mu e^{-2im(\tau+\sigma)}\right),
$$

into Eqs. (2.2) yields these equations in terms of the closed string oscillators and zero modes. Solutions of the resulted equations are denoted by the product $|B_x\rangle = |B\rangle^{(0)} \otimes |B\rangle^{(\mathrm{osc})}$ where the partial states have the features [33],

$$
\begin{aligned}
|B\rangle^{(0)} = &\frac{T_p}{2\sqrt{\det(U/2)}}\int_{-\infty}^{\infty}\exp\Bigg[i\alpha'\sum_{\alpha\neq\beta}\left(U^{-1}\mathcal{K} + \mathcal{K}^T U^{-1}\right)_{\alpha\beta}\,p^\alpha p^\beta\\
&+\frac{i}{2}\alpha'\left(U^{-1}\mathcal{K} + \mathcal{K}^T U^{-1}\right)_{\alpha\alpha}\,(p^\alpha)^2\Bigg]\prod_{\alpha=0}^{p}\left(|p^\alpha\rangle dp^\alpha\right)\\
&\times\prod_{i=p+1}^{25}\left[\delta\left(x^i - y^i\right)|p^i = 0\rangle\right],
\end{aligned}
\tag{2.3}
$$

$$
|B\rangle^{(\mathrm{osc})} = \prod_{n=1}^{\infty}[\det M_{(n)}]^{-1}\exp\left[-\sum_{m=1}^{\infty}\left(\frac{1}{m}\alpha_{-m}^\mu S_{(m)\mu\nu}\tilde{\alpha}_{-m}^\nu\right)\right]|0\rangle_\alpha|0\rangle_{\tilde{\alpha}}\,,
\tag{2.4}
$$

where T_p is the tension of the Dp-brane. The momentum and center-of-mass position of the closed string, in the worldvolume subspace, possess the significant relation

$$
p^\alpha = -\frac{1}{2\alpha'}\left(\mathcal{K}^{-1}U\right)^\alpha{}_\beta\, x^\beta.
$$

The matrix $S_{(m)}$ is defined by

$$S_{(m)\mu\nu} = \left((M^{-1}_{(m)} N_{(m)})_{\alpha\beta}, -\delta_{ij}\right) ,$$
$$M_{(m)\alpha\beta} = \mathcal{K}_{\alpha\beta} - \mathcal{F}_{\alpha\beta} + \frac{i}{2m} U_{\alpha\beta} ,$$
$$N_{(m)\alpha\beta} = \mathcal{K}_{\alpha\beta} + \mathcal{F}_{\alpha\beta} - \frac{i}{2m} U_{\alpha\beta} . \tag{2.5}$$

In fact, receiving the solution (2.4) through the coherent state method imposes the following conditions on the input parameters $\{B_{\alpha\beta}, F_{\alpha\beta}, U_{\alpha\beta}, \omega_{\alpha\beta}\}$,

$$\eta\mathcal{F} - \mathcal{F}\eta + 4(\omega\mathcal{F} + \mathcal{F}\omega) = 0,$$
$$\eta U - U\eta + 4(\omega U + U\omega) = 0. \tag{2.6}$$

Since we shall use the covariant formulation the conformal ghosts should be also introduced. Thus, we apply the total boundary state $|B\rangle = |B\rangle^{(0)} \otimes |B\rangle^{(\text{osc})} \otimes |B\rangle^{(\text{gh})}$, where the ghost part is the following state [36],

$$|B\rangle^{(\text{gh})} = \exp\left[\sum_{m=1}^{\infty} (c_{-m}\tilde{b}_{-m} - b_{-m}\tilde{c}_{-m})\right] \frac{c_0 + \tilde{c}_0}{2} |q = 1\rangle|\tilde{q} = 1\rangle . \tag{2.7}$$

This state is independent of the background fields and the brane dynamics.

3. Scattering amplitude

3.1. Scattering of many strings from a Dp-branes

Each brane manifestly couples to all closed string states through its corresponding boundary state. This implies that a D-brane is a source (sink) for emitting (absorbing) any closed string state. The source-sink property of the D-branes gives an essential role to them in the processes of string-brane scatterings.

For acquiring the on-shell scattering amplitude of closed strings from a D-brane we should calculate overlap of an outgoing state and the boundary state, associated with the D-brane, via the closed string propagator and vertex operators. Therefore, the tree-level scattering amplitude of $n + 2$ closed strings from a D-brane is given by [5–9],

$$\mathcal{A} = \langle V|c_{-1}\tilde{c}_{-1} \int d^2z \int d^2z_1 \mathcal{V}_1(z_1, \bar{z}_1) \ldots \int d^2z_n \mathcal{V}_n(z_n, \bar{z}_n)\mathcal{V}'(z, \bar{z})(b_0 + \tilde{b}_0)\mathcal{D}|B\rangle, \tag{3.1}$$

where the integrals run over the upper half of the complex plane, and $\mathcal{D}$ is the closed string propagator. The ghost factor $b_0 + \tilde{b}_0$ has been inserted to remove the factor $(c_0 + \tilde{c}_0)/2$ of the ghost boundary state (2.7). The ghost modes c_{-1} and $\tilde{c}_{-1}$ are also eliminated by the outgoing state and the state (2.7). Hence, we don't need to worry about the ghost sector.

We can relocate the propagator to the left and consider its effect on the outgoing state, hence, it disappears from the amplitude [4], [5]. Thus, we receive the following convenient amplitude which is ghost-free

$$\mathcal{A} = \frac{\alpha'}{4\pi} \langle V_x| \int d^2z \int d^2z_1 \ldots \int d^2z_n \mathcal{V}_1(z_1, \bar{z}_1) \ldots \mathcal{V}_n(z_n, \bar{z}_n)\mathcal{V}'(z, \bar{z})|B\rangle^{(0)} \otimes |B\rangle^{(\text{osc})} , \tag{3.2}$$

where $|V_x\rangle$ is the matter part of the outgoing state. In this formula the ranges of all integrals are outside of the unit circle, i.e. $|z_l| > 1$ with $z_l \in \{z, z_1, \ldots, z_n\}$.

Note that from the point of view of the low-energy effective action a D-brane is a charged massive object and, therefore, its presence inevitably induces a curvature into the spacetime. However, the string-brane scattering amplitudes which are calculated in the Minkowski spacetime comprise information for the dynamics in an effective curved spacetime, e.g. see [2].

3.2. Elastic scattering of the Kalb-Ramond state

Now we construct the amplitude concerning the elastic scattering of the Kalb-Ramond string state from a single dynamical-dressed Dp-brane. Our calculations will be in the t-channel. Using the characteristic feature of the worldsheet duality it is possible to recast the amplitude in the s-channel too. Note that when the vertex operators approach to each other the t-channel case occurs. The s-channel appears when one of the vertex operators approaches to the boundary of the string worldsheet.

The tree-level amplitude regarding the scattering of the Kalb-Ramond state from our Dp-brane is specified by

$$\mathcal{A}_{\rm KR} = \frac{\alpha'}{4\pi}\langle B_{\rm KR}|V_{\rm KR}^{(0,0)}(\zeta, k)|B\rangle^{(0)} \otimes |B\rangle^{(\rm osc)} \,. \tag{3.3}$$

The vertex operator, associated with this state in the (0,0) picture, has the structure

$$V_{\rm KR}^{(0,0)}(\zeta, k) = \int\limits_{|z|>1} d^2z\, \mathcal{V}_{\rm KR}^{(0,0)}(\zeta, k; z, \bar{z}) \,,$$

$$\mathcal{V}_{\rm KR}^{(0,0)}(\zeta, k; z, \bar{z}) = \frac{\sqrt{2}\kappa}{\pi\alpha'}\zeta_{\mu\nu}\partial X^\mu \bar{\partial} X^\nu e^{ik.X} \,, \tag{3.4}$$

where the polarization tensor $\zeta_{\mu\nu}$ is antisymmetric, and $\kappa = (2\pi)^{7/2}(\alpha')^2 g_s/\sqrt{2}$ is the gravitational constant and g_s is the string coupling. The amplitude (3.3) elaborates the physics of a single Dp-brane which interacts with two closed strings.

By introducing the Kalb-Ramond state into Eq. (3.3) the amplitude, for outgoing and incoming states with the momenta k_1 and k_2 and polarizations $\zeta_{(1)\mu\nu}$ and $\zeta_{(2)\mu\nu}$ respectively, possesses the form

$$\mathcal{A}_{\rm KR} = \frac{\alpha'}{4\pi}\langle \mathbf{1}_x| \left(V_{\rm KR}^{(0,0)}(\zeta_1, k_1)\right)^\dagger V_{\rm KR}^{(0,0)}(\zeta_2, k_2)|B\rangle^{(0)} \otimes |B\rangle^{(\rm osc)} \,, \tag{3.5}$$

where the vacuum $|\mathbf{1}_x\rangle$ is the matter part of the total vacuum $|\mathbf{1}\rangle = |\mathbf{1}_x\rangle \otimes |\mathbf{1}_{\rm gh}\rangle$. Combining Eqs. (3.4) and (3.5) eventuates to the equation

$$\begin{aligned}\mathcal{A}_{\rm KR} = {}& \frac{\kappa^2}{2\pi^3\alpha'}\zeta_{(1)\mu\nu}\zeta_{(2)\mu'\nu'}\int d^2z_1 \int d^2z_2 \\ & \times \langle \mathbf{1}_x|\Big[e^{-ik_1\cdot X_1(z_1,\bar{z}_1)}(\bar{\partial}_1 X_L^\nu(\bar{z}_1))^\dagger(\partial_1 X_R^\mu(z_1))^\dagger\Big] \\ & \times \Big[\partial_2 X_R^{\mu'}(z_2)\bar{\partial}_2 X_L^{\nu'}(\bar{z}_2)e^{ik_2\cdot X_2(z_2,\bar{z}_2)}\Big]|B\rangle^{(0)} \otimes |B\rangle^{(\rm osc)}. \end{aligned} \tag{3.6}$$

This amplitude clearly is due to the two worldsheets that are semi-infinite cylinders.

From now on, for simplification, we consider a perpendicular incident and reflection of the incoming and outgoing states. The equation of motion, in terms of the complex coordinates of the worldsheet, is $\partial_z\partial_{\bar{z}}X^\mu(z,\bar{z})=0$. Using the closed string solution of it, i.e. $X^\mu(z,\bar{z})=X^\mu_{\rm R}(z)+X^\mu_{\rm L}(\bar{z})$, with

$$X^\mu_{\rm R}(z)=\frac{1}{2}x^\mu-\frac{i}{2}\alpha' k^\mu\ln(z)+i\sqrt{\frac{\alpha'}{2}}\sum_{m\neq 0}\frac{\alpha^\mu_m}{mz^m}\ ,$$
$$X^\mu_{\rm L}(\bar{z})=\frac{1}{2}x^\mu-\frac{i}{2}\alpha' k^\mu\ln(\bar{z})+i\sqrt{\frac{\alpha'}{2}}\sum_{m\neq 0}\frac{\tilde{\alpha}^\mu_m}{m\bar{z}^m}\ ,$$

and defining the operators

$$\eta_r=-\sqrt{\frac{\alpha'}{2}}\sum_{m\neq 0}^{\infty}\frac{k_r}{m}\cdot\frac{\alpha_{-m}}{\bar{z}_r^m}\ ,$$
$$\tilde{\eta}_r=-\sqrt{\frac{\alpha'}{2}}\sum_{m\neq 0}^{\infty}\frac{k_r}{m}\cdot\frac{\tilde{\alpha}_{-m}}{z_r^m}\ , \tag{3.7}$$

with $r=1,2$, Eq. (3.6) finds the form

$$\begin{aligned}\mathcal{A}_{\rm KR}=\frac{\alpha'\kappa^2}{8\pi^3}\,&\zeta_{(1)\mu\nu}\zeta_{(2)\mu'\nu'}\int d^2z_1\int d^2z_2\langle\mathbf{1}_x|\Bigg\{e^{-ik_1\cdot x_1}e^{\eta_1+\tilde{\eta}_1}\\
&\times\Bigg[-\sqrt{\frac{\alpha'}{2}}\sum_{m=1}^{\infty}\left(\frac{k_1^\nu\alpha^\mu_m}{z_1\bar{z}_1^{-m+1}}+\frac{k_1^\nu\alpha^\mu_{-m}}{z_1\bar{z}_1^{m+1}}+\frac{k_1^\mu\tilde{\alpha}^\nu_m}{z_1^{-m+1}\bar{z}_1}+\frac{k_1^\mu\tilde{\alpha}^\nu_{-m}}{z_1^{m+1}\bar{z}_1}\right)\\
&-\sum_{m_1=1}^{\infty}\sum_{m_2=1}^{\infty}\left(\frac{\tilde{\alpha}^\nu_{m_1}\alpha^\mu_{m_2}}{z_1^{-m_1+1}\bar{z}_1^{-m_2+1}}+\frac{\tilde{\alpha}^\nu_{-m_1}\alpha^\mu_{-m_2}}{z_1^{m_1+1}\bar{z}_1^{m_2+1}}+\frac{\tilde{\alpha}^\nu_{m_1}\alpha^\mu_{-m_2}}{z_1^{-m_1+1}\bar{z}_1^{m_2+1}}+\frac{\tilde{\alpha}^\nu_{-m_1}\alpha^\mu_{m_2}}{z_1^{m_1+1}\bar{z}_1^{-m_2+1}}\right)\Bigg]\\
&\times\Bigg[-\sqrt{\frac{\alpha'}{2}}\sum_{m=1}^{\infty}\left(\frac{k_2^{\nu'}\alpha^{\mu'}_{-m}}{z_2^{-m+1}\bar{z}_2}+\frac{k_2^{\nu'}\alpha^{\mu'}_m}{z_2^{m+1}\bar{z}_2}+\frac{k_2^{\mu'}\tilde{\alpha}^{\nu'}_{-m}}{z_2\bar{z}_2^{-m+1}}+\frac{k_2^{\mu'}\tilde{\alpha}^{\nu'}_m}{z_2\bar{z}_2^{m+1}}\right)\\
&-\sum_{m_1=1}^{\infty}\sum_{m_2=1}^{\infty}\left(\frac{\alpha^{\mu'}_{-m_1}\tilde{\alpha}^{\nu'}_{-m_2}}{z_2^{-m_1+1}\bar{z}_2^{-m_2+1}}+\frac{\alpha^{\mu'}_{m_1}\tilde{\alpha}^{\nu'}_{m_2}}{z_2^{m_1+1}\bar{z}_2^{m_2+1}}+\frac{\alpha^{\mu'}_{m_1}\tilde{\alpha}^{\nu'}_{-m_2}}{z_2^{m_1+1}\bar{z}_2^{-m_2+1}}+\frac{\alpha^{\mu'}_{-m_1}\tilde{\alpha}^{\nu'}_{m_2}}{z_2^{-m_1+1}\bar{z}_2^{m_2+1}}\right)\Bigg]\\
&\times e^{ik_2\cdot x_2}e^{\eta_2^\dagger+\tilde{\eta}_2^\dagger}\Bigg\}|B\rangle^{(0)}\otimes|B\rangle^{(\rm osc)}\ .\end{aligned} \tag{3.8}$$

According to Ref. [16] (Appendix 7.A) and applying the oscillating part of the boundary state, we receive the identity

$$\langle\mathbf{1}_x|e^{\eta_1+\tilde{\eta}_1}e^{\eta_2^\dagger+\tilde{\eta}_2^\dagger}|B'\rangle^{(\rm osc)}=\exp\Big[\langle\mathbf{1}_x|\eta_1\tilde{\eta}_1\eta_2^\dagger\tilde{\eta}_2^\dagger|B'\rangle^{(\rm osc)}\Big], \tag{3.9}$$

where the state $|B'\rangle^{(\rm osc)}$ is similar to Eq. (2.4) without the overall infinite product. The exponent part of the right-hand side is simplified as

$$\begin{aligned}&\langle\mathbf{1}_x|\eta_1\tilde{\eta}_1\eta_2^\dagger\tilde{\eta}_2^\dagger\exp\left(-\sum_{n=1}^{\infty}\frac{1}{n}\alpha^\mu_{-n}S_{(n)\mu\nu}\tilde{\alpha}^\nu_{-n}\right)|\mathbf{1}_x\rangle\\
&=-\frac{\alpha'^2}{4}k_{1\mu}k_{1\nu}k_{2\mu'}k_{2\nu'}\sum_{n=1}^{\infty}\left[\frac{1}{n^2}\left(S^{\mu\nu}_{(n)}S^{\mu'\nu'}_{(n)}+S^{\mu\nu'}_{(n)}S^{\mu'\nu}_{(n)}\right)\left(\frac{z_1\bar{z}_1}{z_2\bar{z}_2}\right)^n\right].\end{aligned} \tag{3.10}$$

Thus, the quantity in Eq. (3.9) takes the value

$$\exp\left[\langle \mathbf{1}_x|\eta_1\tilde{\eta}_1\eta_2^\dagger\tilde{\eta}_2^\dagger|B'\rangle^{(\mathrm{osc})}\right]=\exp\left[-\frac{\alpha'^2}{4}\sum_{n=1}^{\infty}\frac{\lambda_{(n)}}{n^2}\left(\frac{z_1\bar{z}_1}{z_2\bar{z}_2}\right)^n\right], \tag{3.11}$$

where

$$\lambda_{(n)}=k_{1\mu}k_{1\nu}k_{2\mu'}k_{2\nu'}\left(S_{(n)}^{\mu\nu}S_{(n)}^{\mu'\nu'}+S_{(n)}^{\mu\nu'}S_{(n)}^{\mu'\nu}\right). \tag{3.12}$$

In fact, the scattering of strings from the branes drastically provides reliable keys for extracting some essential quantities such as the sizes of the branes. Hence, the exponential factor of Eq. (3.11), which is a portion of the scattering amplitude, defines the characteristic length of the system as the order $\sqrt{\alpha'}$. This length effectively indicates the thickness of the target D-brane.

Eq. (3.11) implies that the scattering amplitude exponentially depends on the factors $\{\alpha'^2\lambda_{(n)}/n^2\,|n\in\mathbb{Z}^+\}$. We shall compute the amplitude approximately, i.e. we consider the limit $\alpha'\to 0$ such that for all mode numbers of string the inequality $\alpha'^2\lambda_{(n)}/n^2<<1$ to be valid.

By applying a convenient choice for the positions of the vertex operators as $z_1=iy$ and $z_2=i$, e.g. see Ref. [37] and [38], we can use the well-known integral representation of the Euler beta-function

$$\int_0^1 t^{a-1}(1-t)^{b-1}dt=\frac{\Gamma(a)\Gamma(b)}{\Gamma(a+b)}. \tag{3.13}$$

This enables us to simplify the integration over the variable y.

We analyze the amplitude for two prominent cases: the unstable target D-brane and the stable one. These special cases are due to the presence and absence of the background tachyon field, respectively. Note that the tachyon condensation phenomenon drastically imposes a collapse to the brane.

3.2.1. Presence of the tachyonic background field

In the presence of the tachyon field the amplitude (3.8) can be approximately written as

$$\mathcal{A}_{\mathrm{KR}}\approx\mathcal{A}_{(0)\mathrm{KR}}+\mathcal{A}_{(1)\mathrm{KR}}\;, \tag{3.14}$$

where $\mathcal{A}_{(0)\mathrm{KR}}$ is the zero slope limit (i.e. $\alpha'\to 0$) part,

$$\begin{aligned}\mathcal{A}_{(0)\mathrm{KR}}=&\frac{\alpha'\kappa^2T_pV_{p+1}\prod_{n=1}^{\infty}[\det(\mathcal{K}-\mathcal{F}+\frac{i}{2n}U)]^{-1}}{2(2\pi)^{28-p}\sqrt{\det(U/2)\det\mathcal{W}}}(\gamma_{\mathrm{EM}}-1)\\&\times\Bigg\{-\zeta_{(1)ij}\zeta_{(2)ij}+\sum_{n=1}^{\infty}\Big[\zeta_{(1)i\alpha}\zeta_{(2)i\beta}\left(S_{(n)}^{\alpha\beta}+S_{(n)}^{\beta\alpha}\right)\\&+\zeta_{(1)\alpha\beta}\zeta_{(2)\alpha'\beta'}\left(S_{(n)}^{\alpha\beta'}S_{(n)}^{\alpha'\beta}+S_{(n)}^{\alpha\beta}S_{(n)}^{\alpha'\beta'}\right)\Big]\Bigg\},\end{aligned} \tag{3.15}$$

where $\gamma_{\mathrm{EM}}=0.577\cdots$ is the Euler-Mascheroni number which was entered via a regularization scheme, and the matrix $\mathcal{W}_{\alpha\beta}$ has the definition

$$\mathcal{W}_{\alpha\beta}=\begin{cases}-i\alpha'(U^{-1}\mathcal{K}+\mathcal{K}^TU^{-1})_{\alpha\beta}, & \text{if } \alpha=\beta,\\ -2i\alpha'(U^{-1}\mathcal{K}+\mathcal{K}^TU^{-1})_{\alpha\beta}, & \text{if } \alpha\neq\beta,\end{cases}$$

$$= i\alpha' \left[-2\left(U^{-1}\mathcal{K} + \mathcal{K}^T U^{-1}\right)_{\alpha\beta} + \left(U^{-1}\mathcal{K} + \mathcal{K}^T U^{-1}\right)_{\alpha\alpha} \delta_{\alpha\beta} \right]. \tag{3.16}$$

As we see in this approximation the amplitude completely is independent of the energies and momenta of the incoming and outgoing strings.

The next α'-correction of the amplitude has the feature

$$\mathcal{A}_{(1)\mathrm{KR}} = \frac{2\kappa^2 T_p V_{p+1} \prod_{n=1}^{\infty}[\det(\mathcal{K} - \mathcal{F} + \frac{i}{2n}U)]^{-1}}{(2\pi)^{28-p}\sqrt{\det(U/2)\det\mathcal{W}}} \left(\frac{\alpha'}{2}\right)^{3/2} \gamma_{\mathrm{EM}}$$
$$\times \left(k_2^i - k_1^i\right) \Big\{ -\zeta_{(1)ij}\zeta_{(2)jj'}k_2^{j'} - \zeta_{(1)ij}\zeta_{(2)j0}k_2^0$$
$$+ \left[\zeta_{(1)\alpha\beta}\zeta_{(2)i0}k_2^0 + \zeta_{(1)i\beta}\zeta_{(2)\alpha j}k_2^j + \zeta_{(1)i\beta}\zeta_{(2)\alpha 0}k_2^0\right] \left(\sum_{n=1}^{\infty} S_{(n)}^{\alpha\beta}\right) \Big\}. \tag{3.17}$$

In fact, the string scattering gives rise a recoil to the brane, which can be partly seen by the difference of the momenta in the amplitude, i.e. $k_2^i - k_1^i$. Eqs. (3.15) and (3.17) elucidate that the tachyon field induces infinite partial amplitudes. This is an effect of the scattering from an unstable brane.

We should note that derivations of Eqs. (3.15) and (3.17) generally are on the basis of the quantum mechanical techniques, specially we used the commutation relations $[x^\mu, p^\nu] = i\eta^{\mu\nu}$ and $[\alpha_m^\mu, \alpha_n^\nu] = [\tilde{\alpha}_m^\mu, \tilde{\alpha}_n^\nu] = m\eta^{\mu\nu}\delta_{m+n,0}$. Besides, some computational techniques of the Appendix 7.A of Ref. [16] have been applied. Since the scattering amplitude (3.14) was approximately computed the explicit forms of the Γ-functions disappeared.

3.2.2. *Absence of the tachyonic background field*

For a stable brane, i.e. in the absence of the tachyon field, the matrix $S_{(n)}^{\alpha\beta}$ and the infinite product in Eqs. (3.15) and (3.17) will be free of the string mode numbers. Therefore, the scattering amplitude possesses the form

$$\mathcal{A}'_{\mathrm{KR}} \approx \mathcal{A}'_{(0)\mathrm{KR}} + \mathcal{A}'_{(1)\mathrm{KR}}, \tag{3.18}$$

whit the ingredients

$$\mathcal{A}'_{(0)\mathrm{KR}} = \frac{\alpha'\kappa^2 T_p V_{p+1}\sqrt{\det(\mathcal{K} - \mathcal{F})}}{2(2\pi)^{28-p}}(\gamma_{\mathrm{EM}} - 1)$$
$$\times \Big\{ -\zeta_{(1)ij}\zeta_{(2)ij} + \zeta_{(1)i\alpha}\zeta_{(2)i\beta}\left(S^{\alpha\beta} + S^{\beta\alpha}\right)$$
$$+ \zeta_{(1)\alpha\beta}\zeta_{(2)\alpha'\beta'}\left(S^{\alpha\beta'}S^{\alpha'\beta} + S^{\alpha\beta}S^{\alpha'\beta'}\right) \Big\}, \tag{3.19}$$

$$\mathcal{A}'_{(1)\mathrm{KR}} = \frac{2\kappa^2 T_p V_{p+1}\sqrt{\det(\mathcal{K} - \mathcal{F})}}{(2\pi)^{28-p}} \left(\frac{\alpha'}{2}\right)^{3/2} \gamma_{\mathrm{EM}}$$
$$\times \left(k_2^i - k_1^i\right) \Big\{ -\zeta_{(1)ij}\zeta_{(2)jj'}k_2^{j'} - \zeta_{(1)ij}\zeta_{(2)j0}k_2^0$$
$$+ \left[\zeta_{(1)\alpha\beta}\zeta_{(2)i0}k_2^0 + \zeta_{(1)i\beta}\zeta_{(2)\alpha j}k_2^j + \zeta_{(1)i\beta}\zeta_{(2)\alpha 0}k_2^0\right] S^{\alpha\beta} \Big\}. \tag{3.20}$$

The matrix $S^{\alpha\beta}$ has the definition

$$S^{\alpha\beta} = \left((\mathcal{K} - \mathcal{F})^{-1}(\mathcal{K} + \mathcal{F})\right)^{\alpha\beta}. \tag{3.21}$$

As a special case let quench the magnetic part of the total field strength, i.e. $\mathcal{F}_{\bar{\alpha}\bar{\beta}} = 0$ with $\bar{\alpha}, \bar{\beta} \in \{1, 2, \cdots, p\}$. Besides, stop the spatial rotation of the brane, i.e. $\omega_{\bar{\alpha}\bar{\beta}} = 0$. Thus, the square root factor of the scattering amplitude reduces to $\sqrt{1 - V^2 - E^2 + 2\vec{E} \cdot \vec{V}}$, where the components of the linear velocity of the brane and the total electric field are given by $V_{\bar{\alpha}} = 4\omega_{0\bar{\alpha}}$ and $E_{\bar{\alpha}} = \mathcal{F}_{0\bar{\alpha}}$. We observe that for our setup the extra term $2\vec{E} \cdot \vec{V}$ is nonzero, while it is absent for the conventional transverse dynamics of the branes. However, by adjusting the electric field and brane velocity such that $|\vec{E} - \vec{V}| \to 1$ the scattering amplitude obviously goes to zero. Similarly, for the case $\vec{E} - \vec{V} \to \vec{0}$ the prefactor of the amplitude tends to its maximum value.

Scatterings from both unstable and stable branes manifestly demonstrate that the polarization elements $\zeta_{(1)ij}$ and $\zeta_{(2)ij}$ do not mix with the matrices $S^{\alpha\beta}_{(n)}$ and $S^{\alpha\beta}$. This implies that if the incident state has a polarization with only $\zeta_{(1)ij} \neq 0$ then the matrices $S^{\alpha\beta}_{(n)}$ and $S^{\alpha\beta}$ do not appear in the scattering amplitudes. Physically this means that an incoming state with this special polarization cannot completely explore the structure of the target brane.

4. The DBI-like actions

The Dirac-Born-Infeld (DBI) action and its extended versions are the low energy effective actions of the tachyon and massless fields. In other words, these actions elaborate the interactions between the foregoing fields and the corresponding D-brane. We observe that the square root factor in Eqs. (3.19) and (3.20) clearly indicates a generalized DBI Lagrangian, associated with the stable D-brane. The corresponding DBI-like action, by including the dilaton field, possesses the feature

$$S^{(\omega)}_{\text{DBI}} = -T_p \int d^{p+1}\xi\, e^{-\phi} \sqrt{-\det\left[\tilde{G}_{\alpha\beta} - \tilde{B}_{\alpha\beta} + F_{\alpha\beta} + 4\omega_{\alpha\beta}\right]}\,, \tag{4.1}$$

where $\tilde{G}_{\alpha\beta}$ and $\tilde{B}_{\alpha\beta}$ are pullbacks of $G_{\mu\nu}$ and $B_{\mu\nu}$ on the brane worldvolume, respectively. This action explicitly comprises the effect of the brane dynamics. In the static gauge $\{\xi^\alpha = x^\alpha | \alpha = 0, 1, \cdots, p\}$ we obtain $\tilde{G}_{\alpha\beta} = \eta_{\alpha\beta}$ and $\tilde{B}_{\alpha\beta} = B_{\alpha\beta}$. Hence, in this gauge for $\phi = 0$ and constant $B_{\alpha\beta}$ and $F_{\alpha\beta}$ the Lagrangian in Eq. (4.1), up to a constant factor, reduces to the square root factor of Eqs. (3.19) and (3.20).

In the same way, the prefactor of Eqs. (3.15) and (3.17) indicates the following effective action

$$S^{(\omega,T)}_{\text{DBI}} = -T_p \int d^{p+1}\xi\, e^{-\phi} V(T) \sqrt{-\det\left[\tilde{G}_{\alpha\beta} - \tilde{B}_{\alpha\beta} + F_{\alpha\beta} + 4\omega_{\alpha\beta}\right]}\, \mathcal{G}(\bar{U})\,, \tag{4.2}$$

where our tachyon profile implies $\bar{U}_{\alpha\beta} = 2\left(\partial_\alpha T \partial_\beta T + T \partial_\alpha \partial_\beta T\right)$. The functional $\mathcal{G}(\bar{U})$ is given by

$$\mathcal{G}(\bar{U}) = \frac{i(-2i\alpha')^{(p+1)/2}}{\sqrt{\det(\bar{U}\bar{\mathcal{W}})}} \det\Gamma\left[\mathbf{1} + \frac{i}{2}\left(\tilde{G} - \tilde{B} + F + 4\omega\right)^{-1} \bar{U}\right]. \tag{4.3}$$

The symbol "Γ" represents the gamma-function, and the matrix $\bar{\mathcal{W}}$ possesses the form (3.16) with $\bar{U}$ instead of U. The variable $V(T)$ is the tachyon potential, and the literature has proposed several forms for it. For a non-BPS brane it is well-known that the tachyon potential is an even functional of the tachyon field T, and as $T \to \pm\infty$ it vanishes. Note that for acquiring this action we applied the following regularization schemes

$$\prod_{n=1}^{\infty} a \to \frac{1}{\sqrt{a}},$$
$$\prod_{n=1}^{\infty} \left(1+\frac{a}{n}\right) \to \frac{1}{\Gamma(a+1)}. \tag{4.4}$$

We observe that the kinetic term of the tachyon field is not under the square root. There are various effective actions in which the kinetic term of the tachyon is out of the square root, e.g. see [39–42]. In addition to these features, there are some other extended forms for the tachyonic part of the actions. For example, some of the generalized actions can be found in the Refs. [39–47]. However, for a stationary brane and in the absence of the massless fields the action (4.2) reduces to an effective action for the tachyon field. Similarly, in the absence of the tachyon field and for the tachyon potential with $V(0) = 1$ the action (4.2) accurately reduces to the action (4.1), as expected.

As we know the physical tension of a D-brane is proportional to the inverse of the closed string coupling g_s. This fact implies that the D-branes are essential part of the non-perturbative string theory. Therefore, the brane effective actions (4.1) and (4.2), which originated from the string-brane scattering, clarify that such scatterings prepare a remarkable intuition on the non-perturbative string theory.

Note that the unstable branes under the tachyon condensation phenomenon collapse into the closed string vacuum or drastically decay to the stable configurations with lower dimensions [34,47–49].

5. Conclusions

For calculating the scattering amplitude we applied the string operator formalism. In this reliable method an amplitude is computed by evaluating the correlation function of the vertex operators which are corresponding to the string states, presented in the scattering process.

We acquired the scattering amplitude of the Kalb-Ramond state from a Dp-brane with the following background fields: a $U(1)$ gauge potential with constant field strength, a constant antisymmetric field and a quadratic tachyon field. The brane has a rotation and a linear motion. The amplitude extremely depends on the background fields and the brane dynamics. The variety of the input parameters $\{B_{\alpha\beta}, F_{\alpha\beta}, U_{\alpha\beta}; \omega_{\alpha\beta}; p\}$ gave a general feature to the amplitude. The strength of the scattering can be accurately adjusted by these variables. For example, for a stable target brane with an internal electric field and a linear motion this strength was adjusted to the zero value and to the maximum value.

The scattering from an unstable brane was represented by infinite partial amplitudes. Scatterings from both kind of the stable and unstable branes elucidate that for a particular polarization of the incoming state the scattering amplitudes are independent of the worldvolume matrices $S_{(n)}^{\alpha\beta}$ and $S^{\alpha\beta}$, which include information about the target branes. In other words, structure of a target D-brane can be partially investigated by such incident states.

We observed that at the zero slope limit the scattering amplitude is independent of the energies and momenta of the incoming and outgoing string states. By adding the α'-corrections dependence on these quantities is restored. Besides, we received the characteristic length of the system as the order $\sqrt{\alpha'}$, which can be interpreted as the effective thickness of the target brane.

As the final result, the scattering amplitudes enabled us to receive effective actions which are corresponding to the stable and unstable dynamical target Dp-branes. For the unstable brane, due to the initial tachyon profile, the kinetic term of the tachyon field was recast in a complicated functional. These actions, because of the essential role of the D-branes, may shed light on the non-perturbative string theory, and may possibly lead to a deeper understanding of the D-branes substantial properties.

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