



Predicting neutrino oscillations with “bi-large” lepton mixing matrices

Peng Chen^{a,*}, Gui-Jun Ding^b, Rahul Srivastava^c, José W.F. Valle^c

^a College of Information Science and Engineering, Ocean University of China, Qingdao 266100, China

^b Interdisciplinary Center for Theoretical Study and Department of Modern Physics, University of Science and Technology of China, Hefei, Anhui 230026, China

^c AHEP Group, Institut de Física Corpuscular – CSIC/Universitat de València, Parc Científic de Paterna, C/Catedrático José Beltrán, 2 E-46980 Paterna (Valencia) - Spain

ARTICLE INFO

Article history:

Received 12 March 2019

Received in revised form 3 April 2019

Accepted 8 April 2019

Available online 11 April 2019

Editor: A. Ringwald

ABSTRACT

We propose two schemes for the lepton mixing matrix $U = U_l^\dagger U_\nu$, where $U = U_l$ refers to the charged sector, and U_ν denotes the neutrino diagonalization matrix. We assume U_ν to be CP conserving and its three angles to be connected with the Cabibbo angle in a simple manner. CP violation arises solely from the U_l , assumed to have the CKM form, $U_l \simeq V_{\text{CKM}}$, suggested by unification. Oscillation parameters depend on a single parameter, leading to narrow ranges for the “solar” and “accelerator” angles θ_{12} and θ_{23} , as well as for the CP phase, predicted as $\delta_{\text{CP}} \sim \pm 1.3\pi$.

© 2019 The Authors. Published by Elsevier B.V. This is an open access article under the CC BY license (<http://creativecommons.org/licenses/by/4.0/>). Funded by SCOAP³.

1. Introduction

After decades of hard work, the origin of flavor mixing and CP violation remains one of the most important challenges in particle physics. Understanding the flavor problem would help us to get a glimpse on physics beyond the standard model. Several approaches have been pursued to find an adequate and predictive description of lepton mixing. Using flavor symmetries from first principles [1] one can obtain “top down” restrictions on neutrino mixing within fundamental theories of neutrino mass [2–6]. Alternatively, one may make educated phenomenological guesses as to what the pattern of lepton mixing should look like. Specially influential were the ideas of mu-tau symmetry and the Tri-Bimaximal (TBM) lepton mixing ansatz proposed by Harrison, Perkins and Scott [7–9]. The latter predicts the three mixing angles as $\sin^2 \theta_{23} = 1/2$, $\sin^2 \theta_{12} = 1/3$, $\sin^2 \theta_{13} = 0$, while the Dirac CP phase vanishes. However, the precise measurements of the reactor angle $\theta_{13} \sim 8.5^\circ$ in Daya Bay [10], RENO [11] and Double Chooz [12] now exclude TBM as a realistic lepton mixing pattern. The discrepancy between experiment and the prediction of TBM led people to pursue new lepton mixing structures. One method is to modify the TBM pattern based on flavor or CP symmetries such as in ref. [13,14].

A more phenomenological approach is to explore new neutrino mixing patterns [15–17]. Recently a “bi-large” mixing scheme has

been proposed in Ref. [18] assuming that $\sin \theta_{13} \simeq \lambda$ where λ is the Cabibbo angle. A generalization of this pattern was proposed in Ref. [19], taking the Cabibbo angle as a universal seed for quark and lepton mixing. Such schemes may emerge from Grand Unified Theories (GUTs) and flavor symmetry [20]. The good features of such bi-large mixing patterns deserve further investigation.

In this paper we will propose two “bi-large” lepton mixing schemes and investigate their phenomenological implications. For definiteness, we assume normal ordered neutrino masses throughout this paper, since inverted ordering is disfavored at more than 3σ [21].¹ As in Ref. [19], we assume that the charged lepton diagonalization matrix is CKM-like, given in terms of the Wolfenstein parameters λ and A , whose values we take from the PDG as $\lambda = 0.22453$ and $A = 0.836$ [23]. In our ansatz the three mixing angles characterizing the neutrino diagonalization matrix are related with λ in a very simple manner. We obtain tight predictions for the physical lepton mixing angles and the CP phase. These are contrasted with current experiments and used to make projections for upcoming long baseline oscillation experiments.

2. Bi-large: pattern I

In this section we propose our first “bi-large” lepton mixing pattern. Within the standard parameterization the three angles of the neutrino diagonalization matrix are assumed to be given as

* Corresponding author.

E-mail addresses: pche@mail.ustc.edu.cn (P. Chen), dinggj@ustc.edu.cn (G.-J. Ding), rahulsri@ific.uv.es (R. Srivastava), valle@ific.uv.es (J.W.F. Valle).

¹ A slightly stronger limit can be obtained from cosmology [22].

$$\sin \theta_{23}^{\nu} = 1 - \lambda, \quad \sin \theta_{12}^{\nu} = 2\lambda, \quad \sin \theta_{13}^{\nu} = \lambda, \quad (1)$$

and the Dirac CP phase is taken as $\delta_{CP}^{\nu} = \pi$.² In this case the neutrino diagonalization matrix can be approximated as

$$U_{\nu} \simeq \begin{pmatrix} 1 - \frac{5\lambda^2}{2} & 2\lambda & -\lambda \\ \lambda - 2\sqrt{2}\lambda^{\frac{3}{2}} & \sqrt{2}\lambda - \frac{\lambda^{\frac{3}{2}}}{2\sqrt{2}} & 1 - \lambda - \frac{\lambda^2}{2} \\ 2\lambda + \sqrt{2}\lambda^{\frac{3}{2}} & -1 + \lambda & \sqrt{2}\lambda - \frac{\lambda^{\frac{3}{2}}}{2\sqrt{2}} \end{pmatrix}. \quad (2)$$

If the charged leptons are taken diagonal then this will imply that the leptonic mixing parameters are the same as eq. (1): $\sin^2 \theta_{23} = \sin^2 \theta_{23}^{\nu} \simeq 0.601$, $\sin^2 \theta_{12} = \sin^2 \theta_{12}^{\nu} \simeq 0.202$ and $\sin^2 \theta_{13} = \sin^2 \theta_{13}^{\nu} \simeq 0.0504$, and lie outside the 3σ experimental range [21]. However, corrections are expected from the charged lepton diagonalization matrix [24]. Following Ref. [19] we assume that the bi-large pattern arises from the simplest SO(10) model where the charged and the down-type quarks have roughly the same mass. Then the lepton diagonalization matrix is naturally of the CKM type [25]

$$U_l = R_{23}(\theta_{23}^{CKM}) \Phi R_{12}(\theta_{12}^{CKM}) \Phi^{\dagger} \simeq \begin{pmatrix} 1 - \frac{\lambda^2}{2} & \lambda e^{-i\phi} & 0 \\ -\lambda e^{i\phi} & 1 - \frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3 e^{i\phi} & -A\lambda^2 & 1 \end{pmatrix}, \quad (3)$$

where $\sin \theta_{23}^{CKM} = A\lambda^2$ and $\sin \theta_{12}^{CKM} = \lambda$, where λ and A are the Wolfenstein parameters, R_{ij} is the i - j real rotation matrix, and $\Phi = \text{diag}(e^{-i\phi/2}, e^{i\phi/2}, 1)$ where ϕ is a free phase. For convenience we set $\phi \in (-\pi, \pi]$ throughout this paper. The elements of the lepton mixing matrix $U = U_l^{\dagger} U_{\nu}$ are all given in terms of just one free parameter ϕ , leading to a very high degree of predictivity.

To leading order the three mixing angles and the Jarlskog invariant J_{CP} are given by

$$\begin{aligned} \sin^2 \theta_{13} &\simeq 4\lambda^2(1 - \lambda) \cos^2 \frac{\phi}{2}, \\ \sin^2 \theta_{12} &\simeq 2\lambda^2(2 - 2\sqrt{2}\lambda \cos \phi + \lambda), \\ \sin^2 \theta_{23} &\simeq (1 - \lambda)^2 - 2\sqrt{2}A\lambda^{\frac{5}{2}} - 2\lambda^3(1 + 2\cos \phi), \\ J_{CP} &\simeq -2\sqrt{2}\lambda^{\frac{5}{2}} \sin \phi. \end{aligned} \quad (4)$$

The fact that the above parameters depend on just one free parameter ϕ , leads to strong correlations.

The predictions for the oscillation parameters are shown in Fig. 1 and Fig. 2. Requiring $\sin^2 \theta_{13}$ to lie inside the allowed 3σ range implies that the value of ϕ/π should be inside the range of $\pm[0.749, 0.779]$. This severely restricts the allowed ranges for the θ_{12} , θ_{23} and δ_{CP} . The resulting ranges for the oscillation parameters become

$$\begin{aligned} 0.0196 &\leq \sin^2 \theta_{13} \leq 0.0241, \\ 0.302 &\leq \sin^2 \theta_{12} \leq 0.309, \\ 0.572 &\leq \sin^2 \theta_{23} \leq 0.574, \\ 1.303 &\leq \delta_{CP}/\pi \leq 1.307, \end{aligned} \quad (5)$$

where we have chosen the positive solution of ϕ in order to be compatible with the present data of δ_{CP} . One sees that, given θ_{13} , we find that the resulting allowed ranges for the other mixing angles and CP violation phase are very narrow. We perform a conventional χ^2 analysis including the information on δ_{CP} . The χ^2

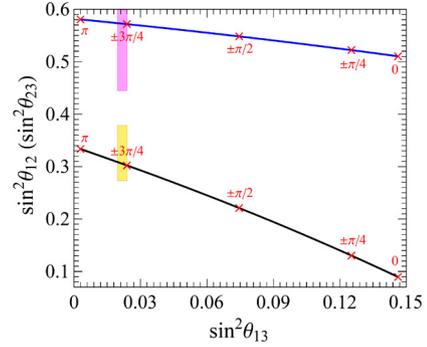


Fig. 1. Predicted correlations involving the three mixing angles in pattern I. “Solar” and “accelerator” angles $\sin^2 \theta_{12}$ (lower, black line) and $\sin^2 \theta_{23}$ (upper, blue line) correlate with the “reactor” mixing parameter $\sin^2 \theta_{13}$. The yellow and the magenta boxes represent the current 3σ ranges of the mixing angles [21]. The cross symbols correspond to $\phi = 0, \pi, \pm\pi/4, \pm\pi/2, \pm 3\pi/4$, respectively.

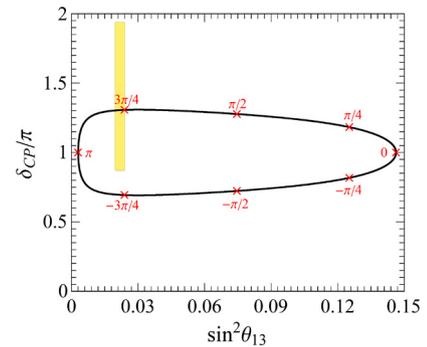


Fig. 2. Predicted correlation between the Dirac phase δ_{CP} and the “reactor” mixing parameter $\sin^2 \theta_{13}$ in pattern I. The yellow box is the current 3σ range from the global fit [21]. The crosses correspond to $\phi = 0, \pi, \pm\pi/4, \pm\pi/2, \pm 3\pi/4$, respectively.

takes the minimum value $\chi_{\min}^2 = 2.366$ when $\phi = 0.766\pi$, leading to the following values for the physical mixing parameters,

$$\begin{aligned} \sin^2 \theta_{23} &= 0.573, & \sin^2 \theta_{13} &= 0.0216, \\ \sin^2 \theta_{12} &= 0.306, & \delta_{CP} &= 1.305\pi, \end{aligned} \quad (6)$$

where $\sin^2 \theta_{13}$, $\sin^2 \theta_{12}$ and δ_{CP} are inside the 1σ range while $\sin^2 \theta_{23}$ is inside the 2σ range of [21], hence fitting very well the experimental results. One sees that the three mixing angles and the Dirac phase are in very good agreement with the current experimental values [21]. It is also remarkable that, starting from a CP conserving U_{ν} in eq. (2), we obtain a CP violating phase that lies very close to the best fit value.

3. Bi-large: pattern II

We now turn to our second example. Again we take the Dirac CP phase as $\delta_{CP}^{\nu} = \pi$ but now assume the neutrino mixing angles in the standard parameterization to be given by

$$\sin \theta_{13}^{\nu} = 1\lambda, \quad \sin \theta_{12}^{\nu} = 2\lambda, \quad \sin \theta_{23}^{\nu} = 3\lambda. \quad (7)$$

To order λ^2 the neutrino mixing matrix of such “1-2-3” bi-large mixing pattern is written as

$$U_{\nu} \simeq \begin{pmatrix} 1 - \frac{5}{2}\lambda^2 & 2\lambda & -\lambda \\ -2\lambda + 3\lambda^2 & 1 - \frac{13}{2}\lambda^2 & 3\lambda \\ \lambda + 6\lambda^2 & -3\lambda + 2\lambda^2 & 1 - 5\lambda^2 \end{pmatrix}. \quad (8)$$

As theoretical motivation this time we consider the framework of SU(5) Grand Unified models. In the simplest SU(5) GUTs the lepton

² The Majorana phases are taken to be zero, hence our ansatz is just for oscillation physics, with hardly any predictivity for neutrinoless double beta decay experiments.

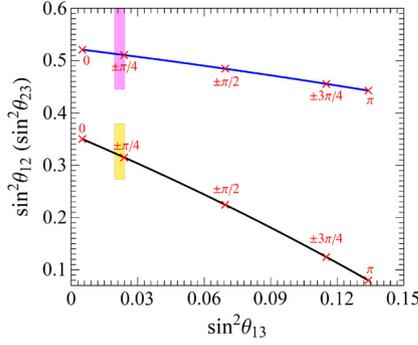


Fig. 3. Predicted correlations involving the three mixing angles in pattern II. “Solar” and “accelerator” angles $\sin^2 \theta_{12}$ (lower, black line) and $\sin^2 \theta_{23}$ (upper, blue line) correlate with the “reactor” mixing parameter $\sin^2 \theta_{13}$. The yellow and the magenta boxes represent the current 3σ ranges of the mixing angles [21]. The cross symbols correspond to $\phi = 0, \pi, \pm\pi/4, \pm\pi/2, \pm3\pi/4$, respectively.

and down quark mass matrices obey the relation $M_e \sim M_d^T$. As in the previous section, this suggests us to adopt a CKM-type lepton diagonalization matrix

$$U_l = \Phi^\dagger R_{12}^T(\theta_{12}^{CKM}) \Phi R_{23}^T(\theta_{23}^{CKM}) \simeq \begin{pmatrix} 1 - \frac{\lambda^2}{2} & -\lambda e^{i\phi} & A\lambda^3 e^{i\phi} \\ \lambda e^{-i\phi} & 1 - \frac{\lambda^2}{2} & -A\lambda^2 \\ 0 & A\lambda^2 & 1 \end{pmatrix}, \quad (9)$$

with $\phi \in (-\pi, \pi]$. Then to leading order, the mixing angles and J_{CP} obtained from the lepton mixing matrix $U = U_l^\dagger U_\nu$ are given by

$$\begin{aligned} \sin^2 \theta_{13} &\simeq \lambda^2 - 6\lambda^3 \cos \phi, \\ \sin^2 \theta_{12} &\simeq \lambda^2 (5 + 4 \cos \phi), \\ \sin^2 \theta_{23} &\simeq 9\lambda^2 + 6\lambda^3 (A + \cos \phi), \\ J_{CP} &\simeq -3\lambda^3 \sin \phi. \end{aligned} \quad (10)$$

As before, requiring $\sin^2 \theta_{13}$ to lie in the allowed 3σ range severely restricts the consistency ranges for the other oscillation parameters θ_{12} , θ_{23} and δ_{CP} , as follows

$$\begin{aligned} 0.0196 &\leq \sin^2 \theta_{13} \leq 0.0241, \\ 0.315 &\leq \sin^2 \theta_{12} \leq 0.323, \\ 0.511 &\leq \sin^2 \theta_{23} \leq 0.513, \\ 1.266 &\leq \delta_{CP}/\pi \leq 1.274. \end{aligned} \quad (11)$$

Taking into account the current experimental data, the χ^2 takes the minimum value $\chi_{\min}^2 = 3.162$ when $\phi = 0.233\pi$, and the mixing parameters are

$$\begin{aligned} \sin^2 \theta_{23} &= 0.512, & \sin^2 \theta_{13} &= 0.0216, \\ \sin^2 \theta_{12} &= 0.319, & \delta_{CP} &= 1.270\pi. \end{aligned} \quad (12)$$

One sees that $\sin^2 \theta_{13}$, $\sin^2 \theta_{12}$ and δ_{CP} are inside the 1σ range, while $\sin^2 \theta_{23}$ is inside the 2σ range given by current global oscillation fits. The results are displayed in Figs. 3 and 4. As before, one sees that the predictions fit very well with the observed oscillation parameter values.

4. long baseline oscillations

The lepton mixing matrix in both cases discussed above only depends on one free parameter ϕ . As we saw, the one-parameter nature of both *ansatze* leads to tight correlations amongst the oscillation parameters and predict very narrow ranges for the “solar”

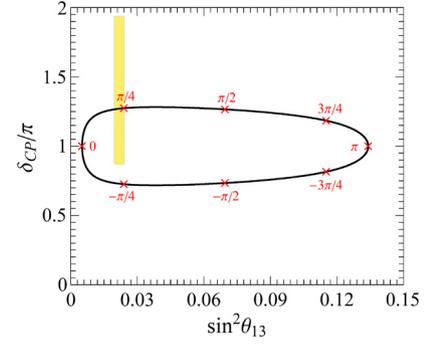


Fig. 4. Predicted correlation between the Dirac phase δ_{CP} and the “reactor” mixing parameter $\sin^2 \theta_{13}$ in pattern II. The yellow box is the current 3σ range from the global fit [21]. The crosses correspond to $\phi = 0, \pi, \pm\pi/4, \pm\pi/2, \pm3\pi/4$, respectively.

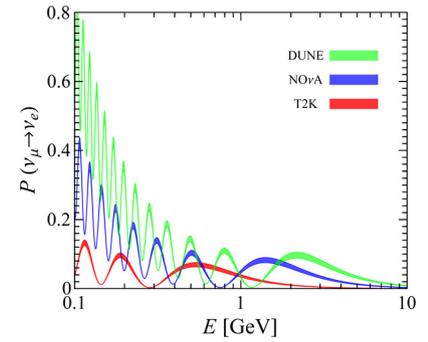


Fig. 5. The $\nu_\mu \rightarrow \nu_e$ transition probability versus energy for pattern I for the T2K, NOvA and DUNE experiments. The mixing angles and Dirac CP phase are taken within the currently allowed 3σ range.

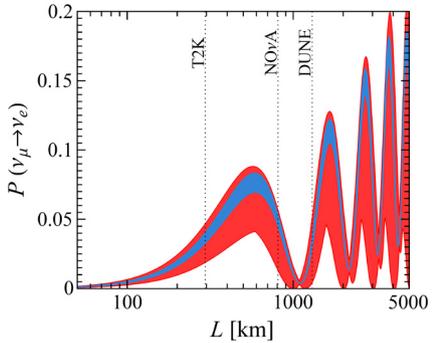


Fig. 6. $\nu_\mu \rightarrow \nu_e$ transition probability versus distance taking the mixing angles and Dirac CP phase within the currently allowed 3σ range. The broad band (red) refer to a generic scenario, whereas the thin band (blue) corresponds to the bi-large pattern I prediction.

and “accelerator” angles θ_{12} and θ_{23} . This translates into phenomenological implications for the expected neutrino and anti-neutrino appearance probabilities in neutrino oscillation experiments [26,27]. To illustrate the implications of our mixing patterns for future long baseline oscillation experiments we present the resulting oscillation probabilities in Figs. 5 and 6.

One sees that indeed the expected oscillation probabilities are tightly restricted, indicating that our bi-large mixing patterns should be testable at the upcoming long baseline oscillation experiments. In particular, the CP asymmetry, displayed in Fig. 7, is very tightly predicted as compared to the generic three-neutrino oscillation scheme. This is seen by comparing the thin band (blue) with the broad band (red) in the figure.

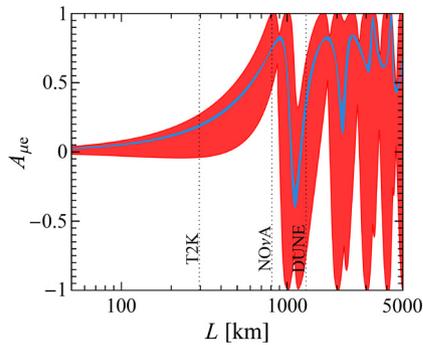


Fig. 7. The CP asymmetry $A_{\mu e} = \frac{P(\nu_{\mu} \rightarrow \nu_e) - P(\bar{\nu}_{\mu} \rightarrow \bar{\nu}_e)}{P(\nu_{\mu} \rightarrow \nu_e) + P(\bar{\nu}_{\mu} \rightarrow \bar{\nu}_e)}$ versus distance taking the mixing angles and Dirac CP phase within the currently allowed 3σ range. The broad band refer to a generic scenario, whereas the thin band refers to the bi-large pattern I prediction.

5. Conclusion

In this letter we have proposed two bi-large-type lepton mixing schemes. They make definite assumptions on the two factors that comprise the lepton mixing matrix $U = U_l^\dagger U_\nu$, where $U = U_l$ comes from the charged sector while U_ν describes the neutrino diagonalization matrix. We assume U_ν to be CP conserving and its three angles to be related with the Cabibbo angle in a simple way, given as $\sin\theta_{13}^\nu = \lambda$, $\sin\theta_{12}^\nu = 2\lambda$ and $\sin\theta_{23}^\nu = 1 - \lambda$ (pattern I) or 3λ (pattern II), with the Dirac CP phase taken at the CP conserving value $\delta_{CP}^\nu = \pi$. CP violation arises only from the U_l factor, assumed to have the CKM form, $U_l \simeq V_{CKM}$, as expected in the simplest Grand Unified models. The Dirac CP phase is predicted as $\delta_{CP} \sim \pm 1.3\pi$, the positive value being very close to its current best fit value. The mixing angles also depend on a single parameter ϕ . The good measurement of the “reactor” angle leads to tight correlations that predict narrow ranges for the “solar” and “atmospheric” angles θ_{12} and θ_{23} in good agreement with current oscillation data. The predictions should be testable at the upcoming long baseline oscillation experiments. Moreover, the structure of the two patterns is very simple, consistent with unification scenarios, and suggestive of novel model building approaches involving Abelian family symmetries [20].

Acknowledgements

PC is supported by National Natural Science Foundation of China under Grant No 11847240 and China Postdoctoral Science Foundation Grant No 2018M642700. GJD acknowledges the support of the National Natural Science Foundation of China under Grant No 11835013. RS and JV are supported by the Spanish grants SEV-2014-0398 and FPA2017-85216-P (AEI/FEDER, UE), PROMETEO/2018/165 (Generalitat Valenciana) and the Spanish Red Consolider MultiDark FPA2017-90566-REDC.

References

- [1] H. Ishimori, T. Kobayashi, H. Ohki, Y. Shimizu, H. Okada, M. Tanimoto, Non-Abelian discrete symmetries in particle physics, *Prog. Theor. Phys. Suppl.* 183 (2010) 1–163, arXiv:1003.3552 [hep-th].
- [2] K.S. Babu, E. Ma, J.W.F. Valle, Underlying $A(4)$ symmetry for the neutrino mass matrix and the quark mixing matrix, *Phys. Lett. B* 552 (2003) 207–213, arXiv:hep-ph/0206292 [hep-ph].
- [3] G. Altarelli, F. Feruglio, Discrete flavor symmetries and models of neutrino mixing, *Rev. Mod. Phys.* 82 (2010) 2701–2729, arXiv:1002.0211 [hep-ph].
- [4] P. Chen, et al., Warped flavor symmetry predictions for neutrino physics, *J. High Energy Phys.* 01 (2016) 007, arXiv:1509.06683 [hep-ph].
- [5] S. Morisi, J.W.F. Valle, Neutrino masses and mixing: a flavour symmetry roadmap, *J. Fortsch. Phys.* 61 (2012) 466–492, arXiv:1206.6678 [hep-ph].
- [6] S.F. King, C. Luhn, Neutrino mass and mixing with discrete symmetry, *Rep. Prog. Phys.* 76 (2013) 056201, arXiv:1301.1340 [hep-ph].
- [7] P. Harrison, W. Scott, Mu - tau reflection symmetry in lepton mixing and neutrino oscillations, *Phys. Lett. B* 547 (2002) 219–228.
- [8] P.F. Harrison, D.H. Perkins, W.G. Scott, Tri-bimaximal mixing and the neutrino oscillation data, *Phys. Lett. B* 530 (2002) 167, arXiv:hep-ph/0202074 [hep-ph].
- [9] P.F. Harrison, W.G. Scott, Symmetries and generalizations of tri - bimaximal neutrino mixing, *Phys. Lett. B* 535 (2002) 163–169, arXiv:hep-ph/0203209 [hep-ph].
- [10] Daya Bay Collaboration, F.P. An, et al., Measurement of electron antineutrino oscillation based on 1230 days of operation of the Daya Bay experiment, arXiv:1610.04802 [hep-ex].
- [11] RENO Collaboration, M.Y. Pac, Recent results from RENO, arXiv:hep-ex/1647948, 2018.
- [12] Double Chooz Collaboration, Y. Abe, et al., Improved measurements of the neutrino mixing angle θ_{13} with the double Chooz detector, *J. High Energy Phys.* 10 (2014) 086, arXiv:1406.7763 [hep-ex], Erratum: *J. High Energy Phys.* 02 (2015) 074.
- [13] P. Chen, et al., Realistic tri-bimaximal neutrino mixing, *Phys. Rev. D* 98 (2018) 055019, arXiv:1806.03367 [hep-ph].
- [14] P. Chen, et al., CP symmetries as guiding posts: revamping tri-bi-maximal mixing-I, arXiv:1812.04663 [hep-ph].
- [15] H. Minakata, A.Y. Smirnov, Neutrino mixing and quark-lepton complementarity, *Phys. Rev. D* 70 (2004) 073009.
- [16] C.H. Albright, A. Dueck, W. Rodejohann, Possible alternatives to tri-bimaximal mixing, *Eur. Phys. J. C* 70 (2010) 1099–1110, arXiv:1004.2798 [hep-ph].
- [17] F. Plentinger, W. Rodejohann, Deviations from tribimaximal neutrino mixing, *Phys. Lett. B* 625 (2005) 264–276.
- [18] S. Boucenna, et al., Bi-large neutrino mixing and the Cabibbo angle, *Phys. Rev. D* 86 (2012) 051301, arXiv:1206.2555 [hep-ph].
- [19] S. Roy, et al., The Cabibbo angle as a universal seed for quark and lepton mixings, *Phys. Lett. B* 748 (2015) 1–4, arXiv:1410.3658 [hep-ph].
- [20] G.-J. Ding, S. Morisi, J.W.F. Valle, Bi-large neutrino mixing and Abelian flavor symmetry, *Phys. Rev. D* 87 (2013) 053013, arXiv:1211.6506 [hep-ph].
- [21] P.F. de Salas, et al., Status of neutrino oscillations 2018: 3σ hint for normal mass ordering and improved CP sensitivity, *Phys. Lett. B* 782 (2018) 633–640, arXiv:1708.01186 [hep-ph], <http://globalfit.astroparticles.es/>.
- [22] P.F. De Salas, S. Gariazzo, O. Mena, C.A. Ternes, M. Tórtola, Neutrino mass ordering from oscillations and beyond: 2018 status and future prospects, *Front. Astron. Space Sci.* 5 (2018) 36, arXiv:1806.11051 [hep-ph].
- [23] Particle Data Group Collaboration, M. Tanabashi, et al., Review of particle physics, *Phys. Rev. D* 98 (2018) 030001.
- [24] J.A. Acosta, A. Aranda, J. Virrueta, CP violating phase from charged-lepton mixing, *J. High Energy Phys.* 04 (2014) 134, arXiv:1402.0754 [hep-ph].
- [25] A.Y. Smirnov, X.-J. Xu, Neutrino mixing in $SO(10)$ GUTs with a non-Abelian flavor symmetry in the hidden sector, *Phys. Rev. D* 97 (9) (2018) 095030, arXiv:1803.07933 [hep-ph].
- [26] P. Pasquini, S.C. Chulíá, J.W.F. Valle, Neutrino oscillations from warped flavor symmetry: predictions for long baseline experiments T2K, NOvA and DUNE, *Phys. Rev. D* 95 (9) (2017) 095030, arXiv:1610.05962 [hep-ph].
- [27] S.S. Chatterjee, P. Pasquini, J.W.F. Valle, Probing atmospheric mixing and leptonic CP violation in current and future long baseline oscillation experiments, *Phys. Lett. B* 771 (2017) 524–531, arXiv:1702.03160 [hep-ph].