Research Article

Unquenching the Quark Model in a Nonperturbative Scheme

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In recent years, the discovery in quarkonium spectrum of several states not predicted by the naive quark model has awakened a lot of interest. A possible description of such states requires the enlargement of the quark model by introducing quark-antiquark pair creation or continuum coupling effects. The unquenching of the quark models is a way to take these new components into account. In the spirit of the Cornell Model, this is usually done by coupling perturbatively a quark-antiquark state with definite quantum numbers to the meson-meson channel with the closest threshold. In this work we present a method to coupled quark-antiquark states with meson-meson channels, taking into account effectively the nonperturbative coupling to all quark-antiquark states with the same quantum numbers. The method will be applied to the study of the X(3872) resonance and a comparison with the perturbative calculation will be performed.

1. Introduction

Constituent quark models (CQM) have been extremely successful in describing the properties of hadrons such as the spectrum and the magnetic moments. However, since the earliest days of the hadron spectroscopy, it was realized [1] that such models neglect the contribution of higher Fock components (virtual $q\bar{q}$ pairs, unitary loops, or hadron-hadron channels) predicted by QCD. Unquenching the quark model is a way to incorporate these components, in a similar way as unquenched lattice theories included dynamical quarks instead of static quarks (quenched theories). It is worth noticing, though, that the name "unquenched quark model" refers to different approaches to include these higher Fock components. Thus, Tornqvist and collaborators [2, 3] use a "unitarized" quark model to incorporate the effects of two meson channels to $c\bar{c}$ and $b\bar{b}$ spectrum. Van Beveren and Rupp [4, 5] showed the influence of continuum channels on the properties of hadrons, in a model which describes the meson as a system of one or more closed quark-antiquark states in interaction with several two meson channels. More recently, Bijker and Santopinto [6] developed an unquenched quark model for baryons, in which a constituent quark model is modified to include, as a perturbation, a QCD-inspired quark-antiquark creation mechanism. Under this assumption, the baryon wave function consists of a zeroth-order three-valence quark configuration plus a sum over baryon-meson loops.

The components beyond the naive constituent quark model became more relevant since 2003, when the $X(3872)$ and other XYZ states were discovered. At that time hadron spectroscopy begins to measure hadron states near two particle thresholds and in this situation loop corrections are relevant.

The $X(3872)$ resonance has been studied using different versions of the unquenched quark model in Ref. [7–9]. Eichten *et al.* [7] considered the influence of meson-meson channels using the Cornell coupled channels scheme [10], obtaining a perturbative mass shift of the $c\bar{c}$ configuration below threshold and an additional decay width for configurations above threshold. Using the Resonance Spectrum expansion [11] Coito *et al.* showed that the $X(3872)$ is compatible with a description in terms of a regular 2^3P_1 charmonium state with a renormalized mass, via opened and closed decay channels. Finally, in Ref. [12], it was shown that one can describe the $X(3872)$ as a $\chi_{c1}(2P)$ interpreted as a

$c\bar{c}$ core plus higher Fock components due to perturbative coupling to the meson-meson continuum.

A different point of view from the references mentioned above is presented in Ref. [13]. The authors of Ref. [13] also study the coupling of the $q\bar{q}$ states with meson-meson channels, but in this case the meson-meson interaction which originates by the underlying quark-quark interaction is fully taken into account, in contrast to other studies such as Ref. [14]. The approach of Ref. [13] allows to have bound states not only in the $q\bar{q}$ channels but, under certain circumstances, in the meson-meson channels. They start from a coupled channels approach for the $J^{PC} = 1^{++}$ channel, to study the influence of the $q\bar{q}$ channels in the meson-meson one. Solving the coupling for the $q\bar{q}$ states one arrives to a Schrödinger-type equation in which the meson-meson interaction gets modified by the coupling with the bare $q\bar{q}$ states, generating a resonance which can be identified with the $X(3872)$. Besides this state, one gets an orthogonal one which can be identified with the $X(3940)$. Both are different combinations of $q\bar{q}$ and meson-meson states.

Some models, as our previous work in Ref. [13], choose a particular $q\bar{q}$ state which gets modified by the interaction with the two meson channels. Usually, the states "closer" to the threshold of the meson-meson channel are chosen, but sometimes it is difficult to decide which ones are relevant. Moreover, in some models, this state is modified perturbatively.

In an attempt to improve these caveats, we have developed a new scheme in which the contributions of the complete tower of radial excitations corresponding to a given J^{PC} value of $q\bar{q}$ states are automatically taken into account. A similar approach was already done in Ref. [14], in which a model that can be solved analytically was used, but with no meson-meson interaction included. The rationale of the method is to leave the $q\bar{q}$ radial wave function as unknown to be determined dynamically. Afterwards, the contribution of each "quenched" state is determined by expanding the obtained wave function in a bare quark-antiquark base. In this way, one incorporates from scratch all possible $q\bar{q}$ states and also allows the deformation of the bare $q\bar{q}$ wave function due to interaction with the meson-meson channels. Although the method is general for baryons and mesons, for the sake of clarity in this paper we will develop it only for mesons.

The paper is organized as follows. In Section 2 we first discuss the coupled channel formalism and the coupling mechanism between the different channels and the basic ingredients of the constituent quark model. Results and comments are presented in Section 3. Finally, we summarize the main achievements of our calculation in Section 4.

2. The Model

2.1. The Constituent Quark Model. The constituent quark model used in this work has been extensively described elsewhere [15, 16] and therefore we will only summarize here its most relevant aspects. The chiral symmetry of the original QCD Lagrangian appears spontaneously broken in nature and, as a consequence, light quarks acquire a dynamical mass.

The simplest Lagrangian invariant under chiral rotations must therefore contain chiral fields and can be expressed as

$$\mathscr{L} = \overline{\psi}\left(i\not{\partial} - M\left(q^2\right)U^{\gamma_5}\right)\psi \qquad (1)$$

where $U^{\gamma_5} = e^{i(\lambda_a/f_\pi)\phi^a\gamma_5}$ is the Goldstone boson fields matrix and $M(q^2)$ is the dynamical (constituent) mass. This Lagrangian has been derived in Ref. [17] as the low-energy limit in the instanton liquid model. In this model the dynamical mass vanishes at large momenta and it is frozen at low momenta, for a value around 300 MeV. Similar results have also been obtained in lattice calculations [18]. To simulate this behavior we parametrize the dynamical mass as $M(q^2) = m_q F(q^2)$, where $m_q \simeq 300\ MeV$, and

$$F\left(q^2\right) = \left[\frac{\Lambda_\chi^2}{\Lambda_\chi^2 + q^2}\right]^{1/2}. \qquad (2)$$

The cut-off Λ_χ fixes the chiral symmetry breaking scale.

The Goldstone boson field matrix U^{γ_5} can be expanded in terms of boson fields,

$$U^{\gamma_5} = 1 + \frac{i}{f_\pi}\gamma^5\lambda^a\pi^a - \frac{1}{2f_\pi^2}\pi^a\pi^a + \cdots \qquad (3)$$

The first term of the expansion generates the constituent quark mass while the second gives rise to a one-boson exchange interaction between quarks. The main contribution of the third term comes from the two-pion exchange which has been simulated by means of a scalar exchange potential.

In the heavy quark sector chiral symmetry is explicitly broken and this type of interaction does not act. However it constrains the model parameters through the light meson phenomenology and provides a natural way to incorporate the pion exchange interaction in the open charm dynamics.

Below the chiral symmetry breaking scale quarks still interact through gluon exchanges described by the Lagrangian

$$\mathscr{L}_{gqq} = i\sqrt{4\pi\alpha_s}\,\overline{\psi}\gamma_\mu G_c^\mu\lambda_c\psi, \qquad (4)$$

where λ_c are the SU(3) color generators and G_c^μ is the gluon field. The other QCD nonperturbative effect corresponds to confinement, which prevents from having colored hadrons. Such a term can be physically interpreted in a picture in which the quark and the antiquark are linked by a one-dimensional color flux-tube.

Lattice calculations have shown that, as far as the quarks get separated, virtual quark anti-quark pairs tend to modify the confinement potential, giving rise at some scale to a breakup of the color flux-tube [19]. Coupled channels and screening potential represent similar physics [20]. However, our approach is to couple those channels whose thresholds are near to the $q\bar{q}$ states we are studying in detail; therefore it should not be treated perturbatively, and, furthermore, averaging the effects of the rest of the thresholds through the following screened confinement potential,

$$V_{CON}\left(\vec{r}_{ij}\right) = \left\{-a_c\left(1 - e^{-\mu_c r_{ij}}\right) + \Delta\right\}\left(\vec{\lambda_i^c} \cdot \vec{\lambda_j^c}\right). \qquad (5)$$

Explicit expressions for these interactions and the value of the parameters are given in Ref. [15, 16].

2.2. The Unquenched Meson Spectrum. Although over the time the procedure to incorporate new Fock components to the $q\bar{q}$ wave function has received several names (unitarized quark model [2], resonance spectrum expansion [11], and coupled channel formalism [1]) the basic idea behind the unquenched meson model is to assume that a hadron wave function with fixed J^P quantum numbers combines a zeroth-order configuration plus a sum over the possible higher Fock components due to the creation of $q\bar{q}$ pairs:

$$|\Psi_A\rangle = \mathcal{N}|A\rangle + \sum_{BClj}\int d\vec{K}k^2 dk \left|BClj, \vec{K}, k\right\rangle$$
$$\times \frac{\left\langle BClj, \vec{K}, k\left|T^+\right|A\right\rangle}{E_0 - E_B - E_C} \tag{6}$$

where T^+ stands for the operator which couples the different components, usually the $^3P_0$ quark-antiquark pair creator, and $|A\rangle$ is an eigenstate of the bare Hamiltonian $H_0|A\rangle = E_0|A\rangle$.

In practice what is done is to assume that the first Fock component, namely, the $q\bar{q}$ structure, is renormalized via the influence of nearby meson-meson channels and, thus, coupled channels calculation is performed including the bare $q\bar{q}$ state and the meson-meson channels. Solving the coupling with the meson-meson channels one obtains for the mass shift of the meson $|A\rangle$

$$\Sigma(E_A) = \sum_{BClj}\int dk^2 dk \frac{\left|\left\langle BClj, \vec{K}, k\left|T^+\right|A\right\rangle\right|^2}{E_A - E_{BC}} \tag{7}$$

where E_{BC} is the kinetic energy of the meson-meson pair.

As stated in the introduction, this method has two important shortcomings. First of all, it focuses the study on the modification of the $q\bar{q}$ channel, avoiding the study of the meson-meson channel where interesting structures may appear. Second, it is chosen from the beginning one $q\bar{q}$ channel to be modified, neglecting those $q\bar{q}$ channels which may be generated in the interaction with the meson-meson channel.

For these reasons, we have developed a new scheme in which the contributions of all states are initially taken into account, being the dynamics the responsible of selecting the contribution of each bound state.

The Hamiltonian we consider

$$H = H_0 + V \tag{8}$$

is the sum of an "unperturbed" part H_0 and a second part V which couples a $q\bar{q}$ system to a continuum made of meson-meson states.

Instead of expanding the wave function of the $q\bar{q}$ system in eigenstates of the H_0 Hamiltonian and then solving the coupled channels equation with the meson-meson channels, we use a general wave function for the $q\bar{q}$ system to solve the coupled channels problem and then develop the solution of the $q\bar{q}$ system in the base of the bare $q\bar{q}$ states. In this way, the dynamics of the system is the element which determines the contribution of each bare state to the eigenstate, obtained from the two-body Schrödinger equation which includes the effect of the dynamics of the nearby meson-meson channels.

The meson wave functions to be used all along this work will be expressed using the Gaussian Expansion Method [21] (GEM), expanding the radial wave function in terms of basis functions

$$\left|\Phi_\alpha(r)\right\rangle = \sum_{n=1}^{n_{max}} c_n^\alpha \left|\phi_{nl}^G(r)\right\rangle \tag{9}$$

where α refers to the channel quantum numbers and $|\phi_{nl}^G(r)\rangle$ are Gaussian trial functions with ranges in geometrical progression. This choice is useful for optimizing the ranges with a small number of free parameters [21]. In addition, the density of the distribution of the Gaussian ranges in geometrical progression at small ranges is suitable for making the wave function correlate with short range potentials.

To introduce higher Fock components in the $q\bar{q}$ wave function we assume that the hadronic state is

$$|\Psi\rangle = |\Phi_\alpha\rangle + \sum_\beta \chi_\beta(P)\left|\phi_A\phi_B\beta\right\rangle \tag{10}$$

where $|\Phi_\alpha\rangle$ is the $q\bar{q}$ wave function, ϕ_M are $q\bar{q}$ eigenstates describing the A and B mesons, $|\phi_A\phi_B\beta\rangle$ is the two-meson state with β quantum numbers coupled to total J^{PC} quantum numbers, and $\chi_\beta(P)$ is the relative wave function between the two mesons in the molecule. The meson-meson interaction will be derived from the qq interaction using the Resonating Group Method (RGM) [22].

We must notice that, although the Gaussian Expansion Method of the $q\bar{q}$ wave functions can be also used for the mesons constituting the molecular states, we will assume that the wave functions of these mesons are simple solutions of the Schrödinger two-body equation.

In order to couple both sectors, we use the QCD-inspired $^3P_0$ model [23], which gives a clear picture of the physical mechanism of the coupling. In this model, a quark pair is created from the vacuum with the vacuum quantum numbers. After the pair creation, a recombination of the quark-antiquark of the initial meson with the $^3P_0$ pair follows to give the two final mesons. The $^3P_0$ quark-antiquark pair-creation operator can be written as [24]

$$\mathcal{T} = -3\sqrt{2}\gamma'\sum_\mu \int d^3p\, d^3p'\, \delta^{(3)}\left(p + p'\right)$$
$$\times \left[\mathcal{Y}_1\left(\frac{p - p'}{2}\right)b_\mu^\dagger(p)\, d_\nu^\dagger\left(p'\right)\right]^{C=1, I=0, S=1, J=0} \tag{11}$$

where μ ($\nu = \bar{\mu}$) are the quark (antiquark) quantum numbers and $\gamma' = 2^{5/2}\pi^{1/2}\gamma$ with $\gamma = g/2m$ is a dimensionless constant which gives the strength of the $q\bar{q}$ pair creation from the vacuum.

Finally, in the context of the $^3P_0$ model, the transition operator $h_{\beta\alpha}(P)$ can be defined from the $\mathcal{T}$ operator as

$$\langle \phi_{M_1} \phi_{M_2} \beta | \mathcal{T} | \Phi_\alpha \rangle = P h_{\beta\alpha}(P) \delta^{(3)}\left(\vec{P}_{\text{cm}}\right) \quad (12)$$

where P is the relative momentum of the two-meson state.

Now, we use Eq. (9) to decompose $h_{\beta\alpha}(P)$ as

$$h_{\beta\alpha}(P) = \sum_{n=1}^{n_{max}} c_n^\alpha h_{\beta\alpha}^n(P) \quad (13)$$

Then, the coefficients c_n^α of the $q\bar{q}$ meson wave function and the eigenenergy E are determined from the coupled-channel equations

$$\sum_{\alpha,n}\left[\mathcal{H}_{n'n}^{\alpha'\alpha} - \mathcal{G}_{n'n}^{0\alpha'\alpha}(E)\right] c_n^\alpha = \sum_n E N_{n'n}^{\alpha'}c_n^{\alpha'}$$

$$\sum_\beta \int H_{\beta'\beta}\left(P',P\right)\chi_\beta(P)P^2 dP + \sum_\alpha h_{\beta'\alpha}\left(P'\right) \quad (14)$$

$$= E\chi_{\beta'}\left(P'\right)$$

where $N_{n'n}^{\alpha'}$ is the normalization matrix of the GEM trial Gaussian functions, $\mathcal{H}_{n'n}^{\alpha'\alpha}$ is the interquark Hamiltonian that defines the bare $q\bar{q}$ meson spectrum, and $H_{\beta'\beta}$ is the RGM Hamiltonian for the two meson states, obtained from the $q\bar{q}$ interaction. In the previous equations we have defined the perturbative mass shift $\mathcal{G}_{n'n}^{0\alpha'\alpha}$ as

$$\mathcal{G}_{n'n}^{0\alpha'\alpha}(E) = -\delta^{\alpha'\alpha}\delta_{n'n}\sum_\beta \int h_{\alpha'\beta}^{n'}(P)\chi_\beta(P)P^2 dP, \quad (15)$$

which codifies the coupling with molecular states. Apparently, the perturbative mass shift does not mix different $q\bar{q}$ channels, so the only operator which mixes them is the potential $V_{n'n}^{\alpha'\alpha}$ in $\mathcal{H}_{n'n}^{\alpha'\alpha}$. However, as will be detailed below, the molecular wave function is dependent on different meson channels.

The previous coupled channel equations can be solved in a more elegant way through the T matrix, solution of the Lippmann-Schwinger equation,

$$T^{\beta'\beta}\left(P',P;E\right) = V^{\beta'\beta}\left(P',P;E\right)$$

$$+ \sum_{\beta''} \int V^{\beta'\beta''}\left(P',P'';E\right) \quad (16)$$

$$\cdot \frac{1}{E - E_{\beta''}\left(P''\right)} T^{\beta''\beta}\left(P'',P;E\right)P''^2 dP''$$

where $V^{\beta'\beta}(P',P)$ is the RGM potential.

Using the T matrix, we end up with a Schrödinger-like equation for the c_n^α coefficients,

$$\sum_{\alpha,n}\left[\mathcal{H}_{n'n}^{\alpha'\alpha} - \mathcal{G}_{n'n}^{\alpha'\alpha}(E)\right] c_n^\alpha = \sum_n E N_{n'n}^{\alpha'}c_n^{\alpha'}. \quad (17)$$

In the previous equation we identify $\mathcal{G}_{n'n}^{\alpha'\alpha}$ as the energy-dependent complete mass-shift matrix

$$\mathcal{G}_{n'n}^{\alpha'\alpha}(E) = \sum_\beta \int dq\, q^2 \frac{\overline{h}_{\alpha'\beta}^{n'}(q,E)h_{\beta\alpha}^n(q)}{q^2/2\mu - E - i0^+}, \quad (18)$$

where $\overline{h}^{\alpha'\beta}$ is the $^3P_0$ potential dressed by the RGM meson-meson interaction,

$$\overline{h}^{\alpha\beta'}(p,E) = h_{\alpha\beta'}(p)$$

$$- \sum_\beta \int d^3q \frac{T^{\beta'\beta}(p,q,E)h_{\alpha\beta}(q)}{q^2/2\mu - E - i0^+}, \quad (19)$$

which can be decomposed in the GEM basis as $h^{\alpha\beta}$ in Eq. (13).

In order to find molecular states above and below thresholds in the same formalism we have to analytically continue all the potentials for complex momenta. Therefore, resonances are solutions of Eq. (17), with the pole position solved by the Broyden method [25].

The molecular wave function is related with the c_n^α coefficients of the meson $q\bar{q}$ state,

$$\chi_\beta(p) = -\sum_{\alpha,n} \frac{\overline{h}_{\alpha\beta}^n(p,E)c_n^\alpha(E)}{p^2/2\mu - E - i0^+} \quad (20)$$

with the normalization given by

$$1 = \sum_{\alpha,\alpha',n,n'} c_{n'}^{\alpha'} N_{n'n}^{\alpha'\alpha}c_n^\alpha + \langle \chi_\beta | \chi_\beta \rangle \quad (21)$$

The partial decay widths can be defined through the complete S-matrix of the mix channel, as detailed in [26].

3. Calculations, Results, and Discussions

In order to compare the results of the proposed scheme with the perturbative one we perform a similar calculation as in Ref. [13], namely, a coupled channel calculation including the 1^{++} $q\bar{q}$ sector and the $D^0\overline{D}^{0*}$ and $D^\pm D^{*\mp}$ channels. We, first of all, perform a calculation with the same parameters as [13] in the charge basis

$$\left|D^\pm D^{*\mp}\right\rangle = \frac{1}{\sqrt{2}}\left(\left|D\overline{D}^* I = 0\right\rangle - \left|D\overline{D}^* I = 1\right\rangle\right) \quad (22)$$

$$\left|D^0\overline{D}^{*0}\right\rangle = \frac{1}{\sqrt{2}}\left(\left|D\overline{D}^* I = 0\right\rangle + \left|D\overline{D}^* I = 1\right\rangle\right) \quad (23)$$

Isospin symmetry is explicitly broken taking the experimental threshold difference into account in our equations and solving for the charged and the neutral components.

One can see in Table 2 that when we use the value of $\gamma = 0.26$ we get three states with energies close to the $X(3940)$, the $X(3872)$, and the $c\bar{c}$ $M = 3510$ MeV/c^2. The first two states are mixtures of $c\bar{c}$ and $D\overline{D}^*$ components, being the $X(3940)$ predominantly $c\bar{c}$ and the $X(3872)$ mostly $D\overline{D}^*$. The third state is clearly a $q\bar{q}$ state. If we project the $q\bar{q}$ component in the base of bare $c\bar{c}$ states (Table 3) we realize that this third state is an almost pure 1^3P_1 $c\bar{c}$ state whereas the other two are predominantly 2^3P_1. In order to compare with the bare H_0 spectrum we show in Table 1 the charmonium states in the vicinity of the $X(3872)$, only up to the first radial excitation.

TABLE 1: H_0 spectrum for the $J^{PC} = 1^{++}$ sector in the vicinity of the $X(3872)$ resonance. A more complete description of the bare charmonium spectrum can be found in Ref. [27].

$c\bar{c}$ State	Mass [MeV]
1^3P_1	3503.9
2^3P_1	3947.4

One can analyze the isospin content of the $D\bar{D}^*$ component of the different states. The only one which shows a sizable $I = 1$ component is the one we have identified with the $X(3872)$. The other two are basically $I = 0$ states. Notice that all the interactions are isospin conserving interactions and the reason to have a non-zero $I = 1$ component for the $X(3872)$ is that the binding energy of the $D\bar{D}^*$ component for this state is smaller than the isospin breaking of the $D^{(*)0}$ and $D^{(*)\pm}$ masses which enhances the $D^0\bar{D}^{*0}$ component at large distances.

As in Ref. [13], we have fine-tuned the $^3P_0$ parameter to get the right binding energy of the $X(3872)$. The results are shown in the second part of Table 2. We get again three states. The first one, with a mass of $M = 3943.9$ MeV/c^2, can be identified with the $X(3940)$ [28] and is a mixture of 57% of $q\bar{q}$ and 43% of $D\bar{D}^*$ molecule with isospin $I = 0$. The $q\bar{q}$ component is basically a 2^3P_1 state. The second state has now the right energy of the $X(3872)$ resonance. The $D^0\bar{D}^{*0}$ clearly dominates its structure with a 94% probability, giving a 55% probability for the isospin 0 component and 39% for isospin 1, which is enhanced in this case because the binding energy is much smaller compared with mass difference of the neutral and charged $D\bar{D}^*$ components. As the isospin breaking is a threshold effect, it grows as we get closer to the $X(3872)$ physical mass. The $q\bar{q}$ component is, as in the previous state, a 2^3P_1 bare state. Finally we get an almost pure $I = 0$ 1^3P_1 state with a mass of $M = 3480.75$ MeV/c^2.

The scenario drawn by these results consists of two bare states (a molecule and a 2^3P_1 $q\bar{q}$ state), which mix together to give the two physical states, and a third state which participates slightly into the game, although, as we will see, its contribution is important. The result is similar to those of Ref. [13], within the uncertainties of the model, but now it appears dynamically without the need to make educated guesses about the bare states involved in the calculation. In this way we are confident that all the physics is included into the model.

Although not addressed in Ref. [13], we have calculated with our wave functions the two decay rates that represent a challenge in the description of the $X(3782)$ structure, namely,

$$R_I = \frac{\mathcal{B}\left(X(3872) \longrightarrow \omega J/\psi\right)}{\mathcal{B}\left(X(3872) \longrightarrow \pi^-\pi^+ J/\psi\right)} \tag{24}$$

$$R_\gamma = \frac{\mathcal{B}\left(X(3872) \longrightarrow \psi(2S)\gamma\right)}{\mathcal{B}\left(X(3872) \longrightarrow J/\psi\gamma\right)} \tag{25}$$

R_I has been measured by BaBar Collaboration [29] obtaining $R_I = 0.8 \pm 0.3$. A value of $R_\gamma = 2.46 \pm 0.64 \pm 0.29$ has been recently reported by LHCb Collaboration [30].

Whereas R_I, which showed a large isospin symmetry breaking, has been used to justify that the $X(3872)$ should be hadronic molecule, the high value of R_γ was interpreted as a strong evidence that the $X(3872)$ cannot be a pure hadronic molecule, based on the claim that the contributions of the molecular component to this ratio should be small [31].

Our $X(3872)$ wave function allows for a simultaneous description of both ratios. To calculate R_γ we have assumed that the decay proceeds through the $q\bar{q}$ component, obtaining the value $R_\gamma = 0.39$ which is clearly far from the experimental value. A similar result is obtained in [32], where it is shown that if the 1^3P_1 states are neglected the ratio approaches to the experimental value. In our approach the 1^3P_1 appears automatically. Then, it seems that a more careful calculation of this decay in the line of Ref. [33, 34] should be done.

The obtained value for the first ratio is $R_I = 1.5$ that is close to the experimental result, similar to the values obtained in Ref. [35–37].

4. Summary

In this work we present a method to coupled quark-antiquark states with meson-meson channels, taking into account effectively the nonperturbative coupling to all quark-antiquark states with the same quantum numbers. Instead of expanding the wave function of the $q\bar{q}$ system in eigenstates of the H_0 Hamiltonian and then solving the coupled channels equation with the meson-meson channels, we use a general wave function for the $q\bar{q}$ system to solve the coupled channels problem and then develop the solution of the $q\bar{q}$ system in the base of the bare $q\bar{q}$ states. The method is applied to the coupling of the $X(3872)$ resonance to the 1^{++} $q\bar{q}$ states and the results were compared with those of the perturbative calculation of Ref. [13].

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Acknowledgments

TABLE 2: Mass and channel probabilities for the three states in the present approach using the two values of the $^3P_0$ strength parameter as explained in the text.

γ^3P_0	M (MeV)	$c\bar{c}$	$D^0\overline{D}^{*0}$	$D^{\pm}D^{*\pm}$	I=0	I=1
	3948.89	56.71%	22.47%	20.82%	43.10%	0.25%
0.260	3867.36	30.22%	51.37%	18.40%	64.72%	5.06%
	3468.29	95.70%	2.18%	2.12%	4.30%	0.0%
	3944.58	57.03%	22.07%	20.89%	42.72%	0.43%
0.218	3871.76	3.62%	93.99%	2.39%	54.98%	39.34%
	3478.55	96.84%	1.60%	1.56%	3.16%	0.0%

TABLE 3: Decomposition of the $c\bar{c}$ component of the states in Table 2 in the bare $c\bar{c}$ basis.

γ^3P_0	M (MeV)	$c\bar{c}$	1^3P_1	2^3P_1	3^3P_1	4^3P_1
	3948.89	56.71%	1.61%	96.33%	1.28%	0.78%
0.260	3867.36	30.22%	1.80%	98.14%	0.06%	0.0%
	3468.29	95.70%	99.99%	0.01%	0.0%	0.0%
	3944.58	57.03%	0.23%	99.49%	0.28%	0.0%
0.218	3871.76	3.62%	2.11%	97.75%	0.14%	0.0%
	3478.55	96.84%	100.0%	0.0%	0.0%	0.0%

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